Fractions and Decimals

Online Interview
Classroom Activities

Department of Education
Victoria

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1. Cuisenaire and Pattern Blocks Comparisons

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions
Students find equivalent representations of common fractions

Mathematical focus:

- Fractions as part-whole
- Moving from the part to the whole and the whole to the part and from the part to the part.

Related interview tasks:

- 1: Pie Task
- 2: Pattern Blocks
- 7: Draw me a Whole.

1.1 Introductory Activity: Cuisenaire

Materials:

- One set of Cuisenaire rods per group
- Paper for recording results.

Instructions:

Firstly, provide an opportunity for free exploration while the students are getting used to the materials, i.e. their size and colour. If the students are unfamiliar with Cuisenaire you may need to establish the colour names for each rod, depending on the type of set (and fading!), the colours can be different from one to another.

Ask the students the following questions one at a time and share their reasoning.

- What fraction of the brown rod is the red rod? (ANS: 1/4)
- If the crimson is 2/3, what is the whole? (ANS: dark green)
- If the brown rod is 4/3, what rod is one? (ANS: dark green)
- If the dark green is 1/2, what is 3/4? (ANS: blue)
- If the blue rod is 1 1/2, what is 2/3? (ANS: pink)

Individually, or in groups, the students can then come up with their own 'brilliant question' to pose to the rest of the class. Problem posing is an important component of problem solving.

This second set of questions may represent a development in complexity for flexibly moving from the whole to the part, the part to the whole, and the part to the part.

- How many reds placed in a row would be the same length as the brown?

(ANS: 4)

- What fraction of the orange rod is the green?

(ANS: 1/2)

- If light green is 1/3, what is the whole?

(ANS: blue)

- If dark green is 3/4, what is the whole?

(ANS: brown)

- If dark green is one what fraction is pink?

(ANS: 2/3)

- If blue is 3/2, what rod is one?

(ANS: dark green)

- If orange is 5/4, what value is the brown rod?

(ANS: one)

- If pink is 1/3, which rod is 5/6?

(ANS: orange)

- If an orange and a red is 3/4, what is the value of the red rod?

(ANS: 1/8)

- If is two oranges are 5/3, then which rod is 1/2?

(ANS: dark green)

1.2 Follow-up Activity: Pattern Blocks

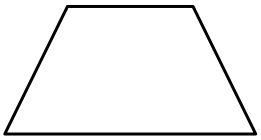
Materials:

- An assortment of pattern blocks per student or group.

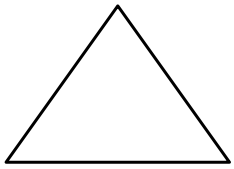
Instructions:

Assume a value for a particular block then pose questions about other blocks as suggested below.

The rhombus has a value of one
Name the values of the hexagon, triangle and trapezium.
Which block is equal to 1 1/2 ?



The trapezium has a value of1/2
What now is the value of a triangle?
Make a model of 7/6.
What is the value of 2 hexagons?



A triangle has the value of 1/3.
Using the least blocks possible, show 9 x 1/3. Explain.
Make a model of 5 1/3; you cannot use a trapezium or a triangle.
Individually or in groups the students can come up with their own question to pose to the class.

Source:

Swan, P., & White, G. (2006). *Developing mathematics with pattern blocks.* Western Australia: RIC Publications (p. 31).

2. Tangram Parts

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions
Students find equivalent representations of common fractions

Mathematical focus:

- Naming and writing fractions from a part-whole representation
- Moving from the whole to the part and emphasising a situation where not all partitions are of the same size.

Related interview tasks:

- 1: Fraction Pie
- 2: Pattern Blocks.

Materials:

- Tangram task sheet
- Tangram pieces (optional).

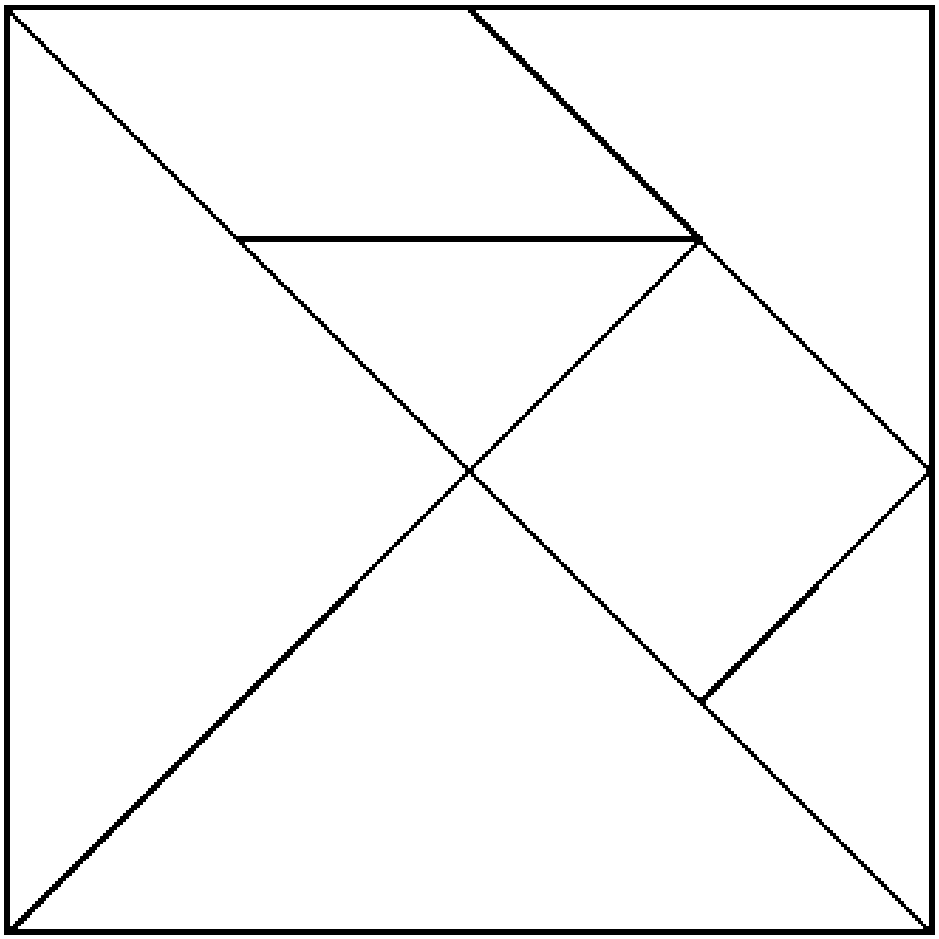
Instructions:

For each Tangram piece name the fraction it represents of the whole set. If Tangram pieces are available students might use these to assist in finding a solution.

2.1 Follow-up Activity: Tangram Parts

- Assume the large triangle has a value of one. What now is the value of the whole set and of each individual piece?
- Assume the smallest triangle has a value of 1/3. What now is the value of the whole set and of each individual piece?

2.2 Activity Sheet: Tangram Parts



What fraction of the large square does each Tangram piece represent?

3. Partitioning Geoboards

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions

Mathematical focus:

- Fractions as part-whole (moving from the whole to the part)
- Naming and writing a variety of fractions.

Related interview task:

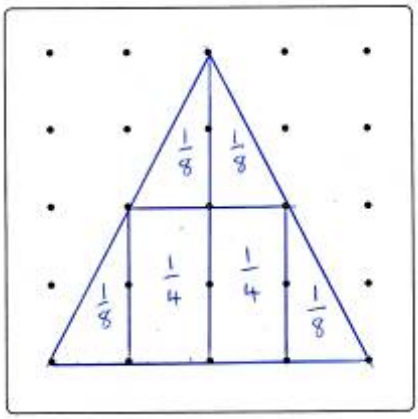
- 1: Fraction Pie
- 2: Pattern Blocks
- 7: Draw me a Whole.

Materials:

- Geoboard
- Rubber bands
- Pencil
- Ruler
- One page of dot paper per student.

Instructions:

- Students are asked to make a shape on the geoboard with a single rubber band and give this shape the value of one.
- They then copy this shape onto the dot paper.
- By joining some of the dots within the larger shape, they make smaller shapes and give these a value with respect to one.
- Encourage students to make each whole progressively more complex.



Source:

Pengelly, H. (1992). *Making sense of fractions* Gosford: Ashton Scholastic (p. 50)

4. Colour in Fractions

Links to Victorian Essential Learning Standards:

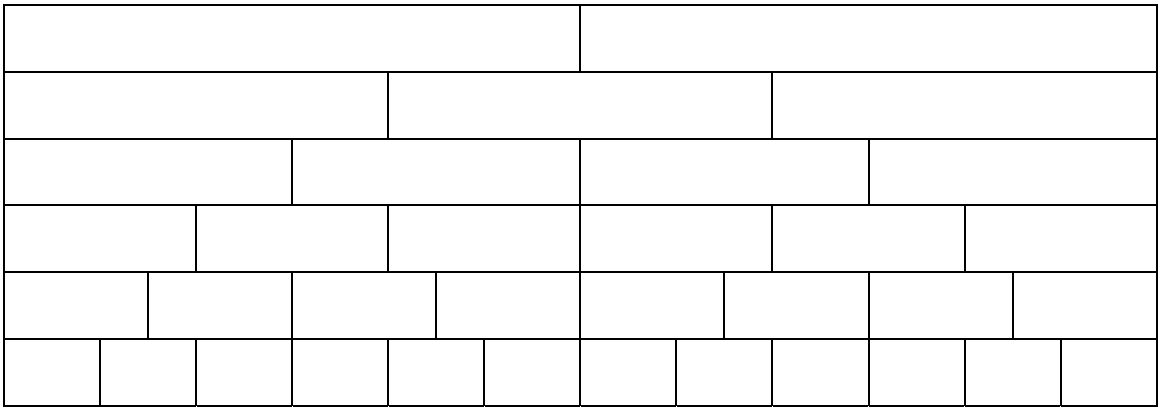
Level 3 Number
Students develop fraction notation and compare simple common fractions such as 3/4 > 2/3 using physical models. Students add and subtract simple common fractions with the assistance of physical models.

Mathematical focus:

- In this game students will be confronted with equivalent fractions and improper fractions.
- As a consequence of creating the fractions with the two dice it is hoped the students will be more aware of the purpose of the numerator and denominator such that: In the fraction *a*/*b*, *b* is the name or size of the part (e.g. fifths have this name because five equal parts can fill a whole) and *a* is the number of parts of that name or size.
- If we have 4/3, the three tells the name or size of the parts (thirds) and the 4 tells us that we have four of those thirds (or 1 1/3). The teacher might notice that as students are playing this game some may be incorrectly reading fractions such as 3/4 as 'three-fours' or 'four-threes' – an indication that their understanding of the numerator and denominator is not yet in place.
- The game also lends itself to a gentle introduction to addition of fractions, as students come to see (particularly as the game gets tight) that they may have rolled 3/6, but this enables them to shade in, for example 1/12, 1/6 and 1/4, which of course is equivalent to 3/6 or 1/2.

Related interview tasks:

- 1: Fraction Pie
- 2: Pattern Blocks
- 3: Dots Array.



Instructions:

- Students have dice that create fractions up to twelfths and a fraction wall. They colour in sections of the wall that correspond to the fractions that they roll with the dice.
- Players in turn throw both dice.
- They make a fraction with the first die being the numerator.
- Each horizontal strip on the game board is one whole. So this game board is equivalent to six wholes or six.
- Students then colour the equivalent of the fraction shown. For example, if they throw 2 and */4, then they can colour in:

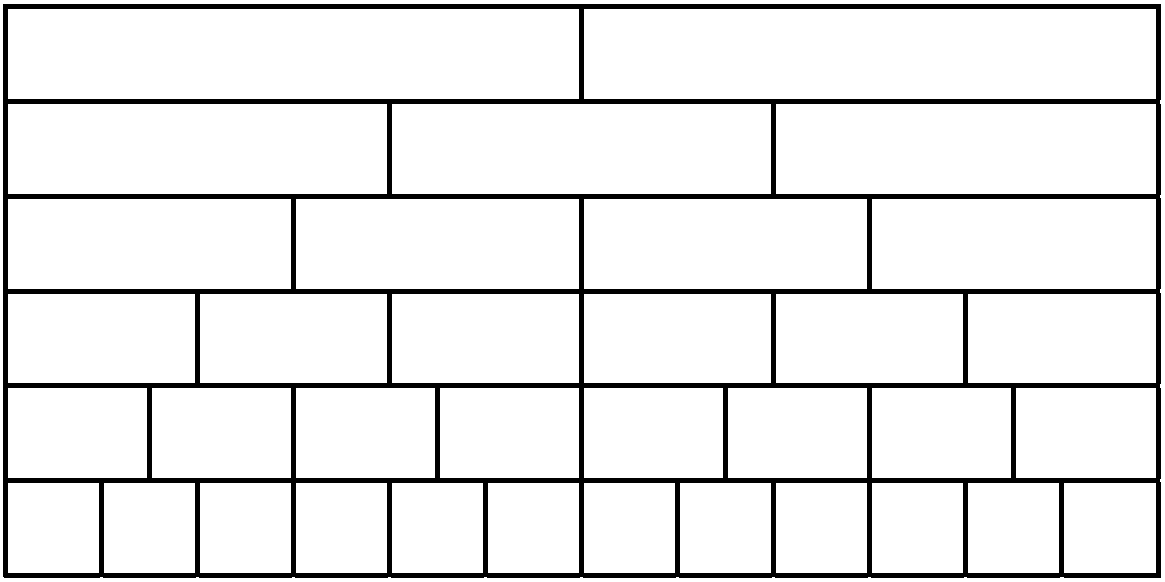
$\frac{2}{4}$ of one line or $\frac{4}{8}$ of one line or $\frac{1}{4}$ of one line and $\frac{2}{8}$ of another

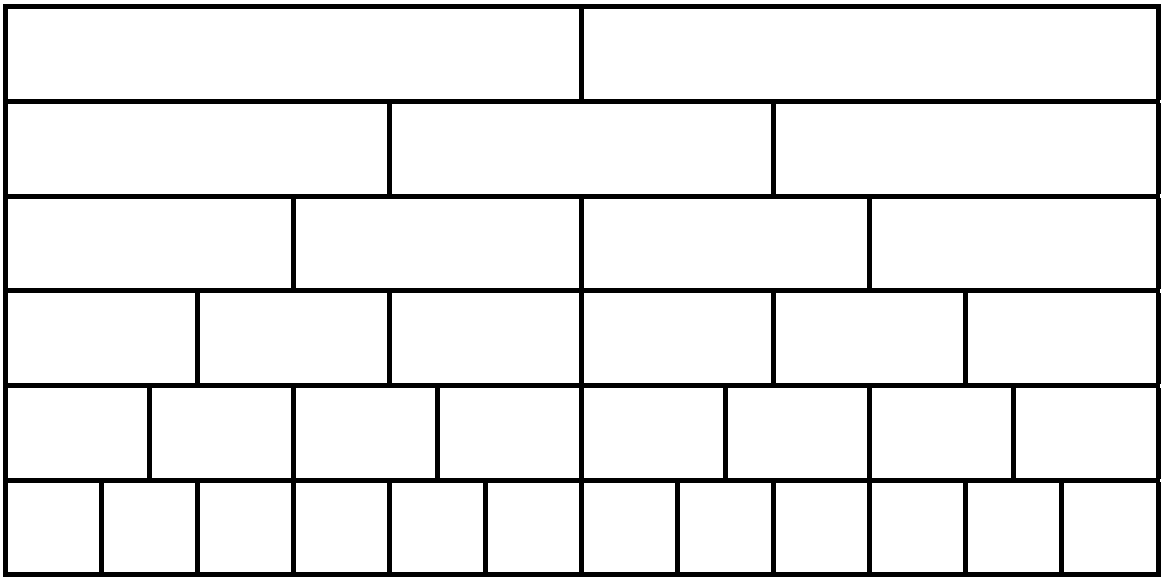
or any other combination that is the same as $\frac{2}{4}$.

- A 'brick' within the wall cannot be cut to make the required fraction.
- If a player is unable to use their turn, they 'pass'.
- The first player who colours in their whole wall is the winner.

There can be problems posed based on the game and class discussion of strategies. Many teachers have found it important to ask students to write down a table of what they rolled in one column and what they shaded in, in another. This provides some accountability and also enables them to observe patterns.

4.1 Activity Sheet: Colour In





5. Find me a Partner

Links to Victorian Essential Learning Standards:

Level 4 Number.
Students comprehend the size and order of small numbers.
Students model common fractions and decimals.
Students place integers, decimals and common fractions on a number line.

Mathematical focus:

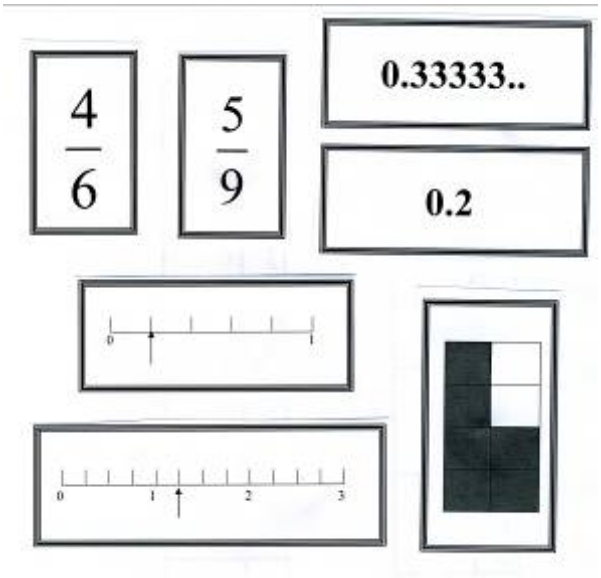
- Connecting concepts with symbols and representations
- Equivalency.

Related interview task:

- 1: Fraction Pie
- 3: Dots Array
- 5: Fractions on a Number Line
- 10: Decimals on a Number Line
- 14: Connecting Fractions, Decimals and Percentages.

Materials:

A set of cards with a variety of fractions, decimals, part-whole representations (continuous and discrete) and number lines (see activity sheet for a larger set).



Instructions:

Give each student a card from the set and present the following instructions:

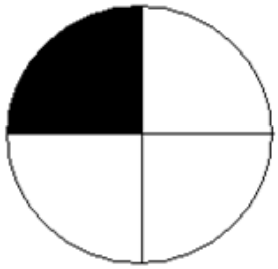
- Tell the person next to you everything you know about your number
- Find one or more students whose cards, when added to yours, equal some whole number
- Find a partner that has an equivalent number
- Finally the whole class is asked to put themselves in order from smallest to largest.

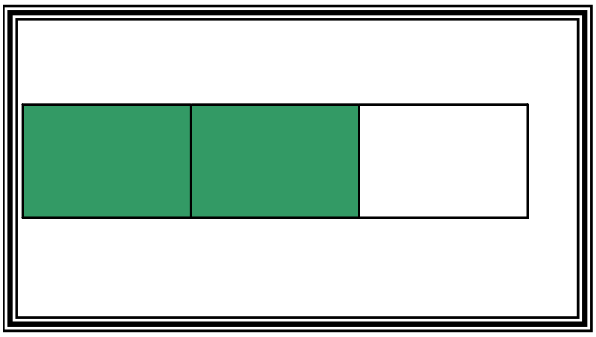
5.1 Activity Sheet: Find me a Partner

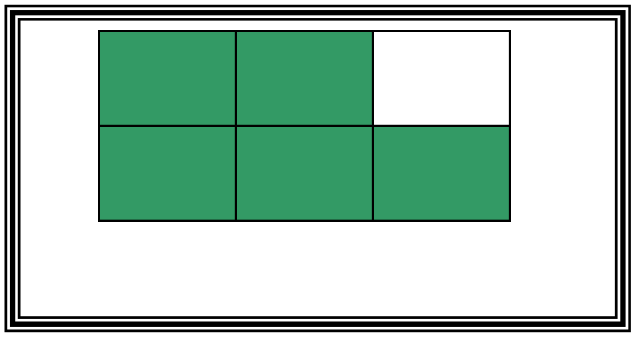
0.33333..	**0.2**
1.75	**0.25**
$\underline{0.1}$	**0.5**

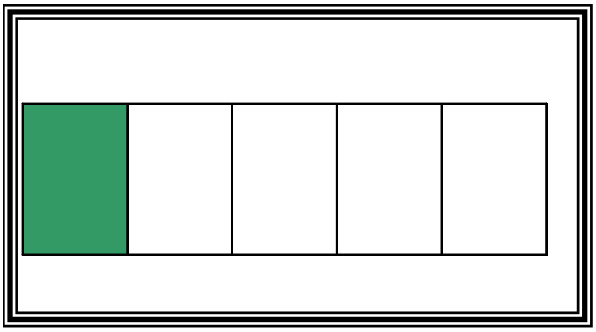
$\frac{1}{3}$	$\frac{3}{4}$	$\frac{1}{6}$	$\frac{2}{5}$
$\frac{4}{5}$	$\frac{4}{6}$	$\frac{2}{8}$	$\frac{3}{8}$

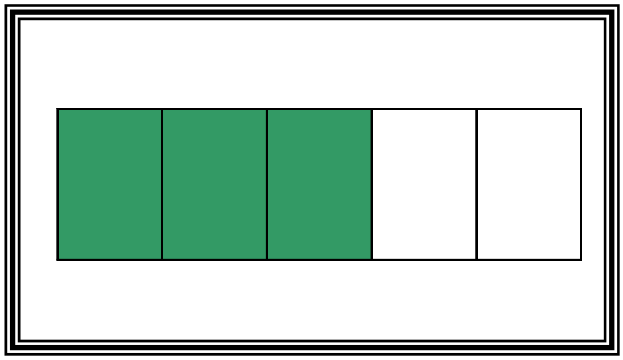
$\frac{5}{9}$	$\frac{3}{10}$	$\frac{6}{10}$

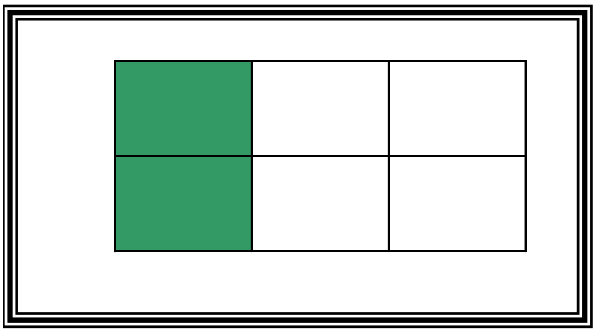


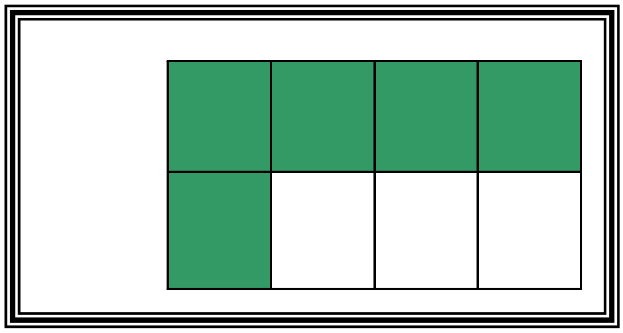


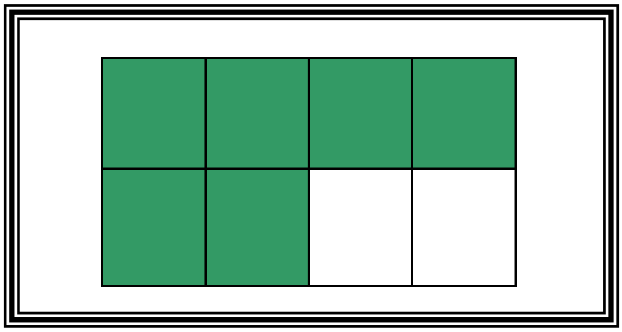


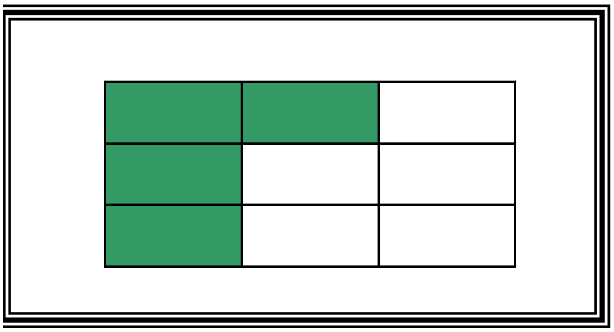


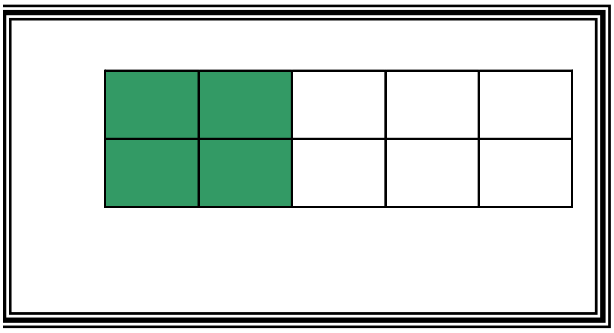


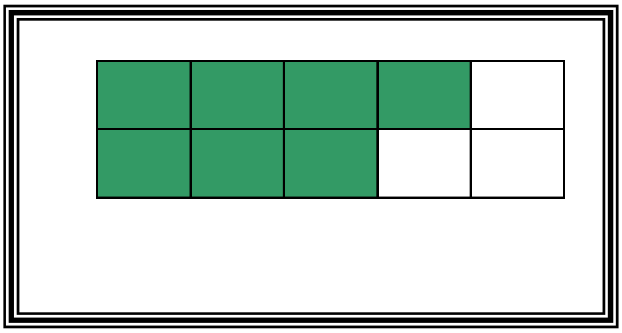


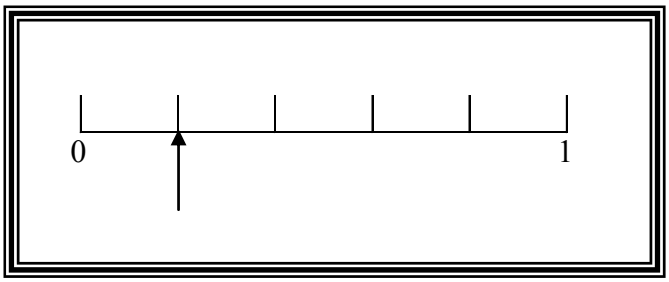








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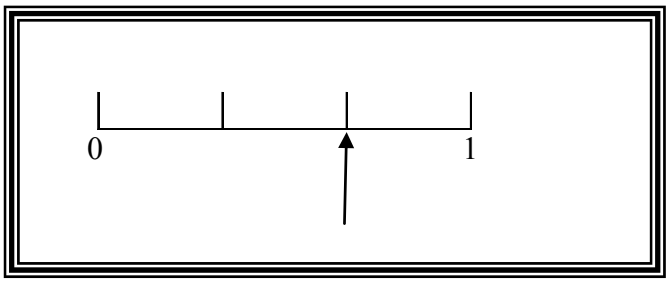
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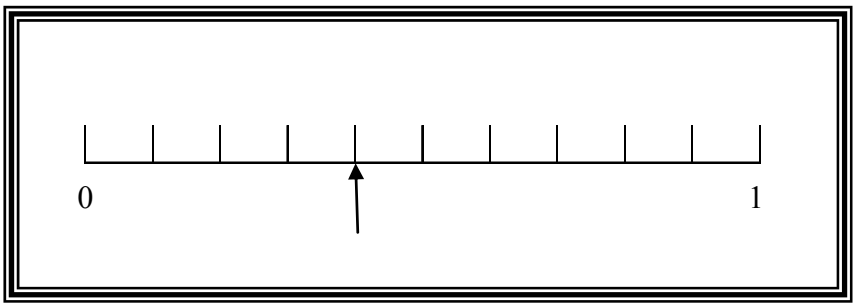
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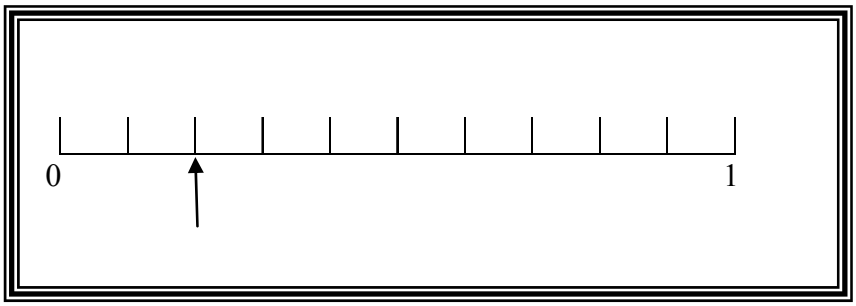
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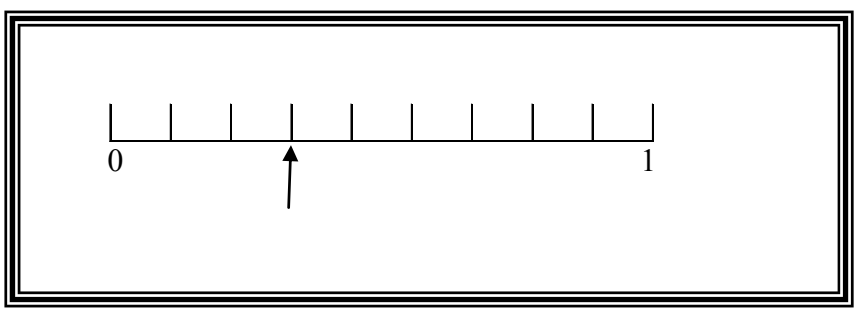
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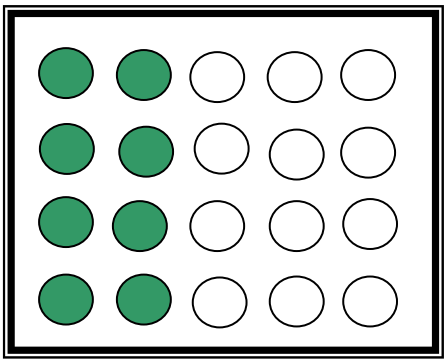
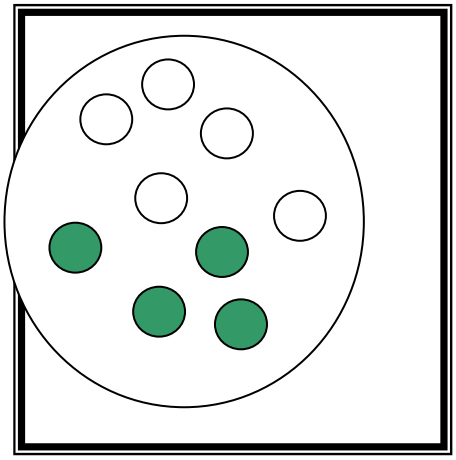
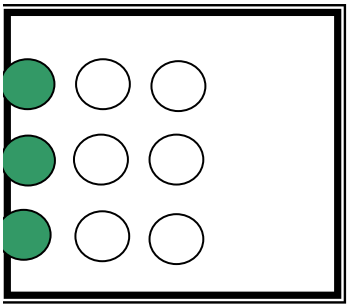
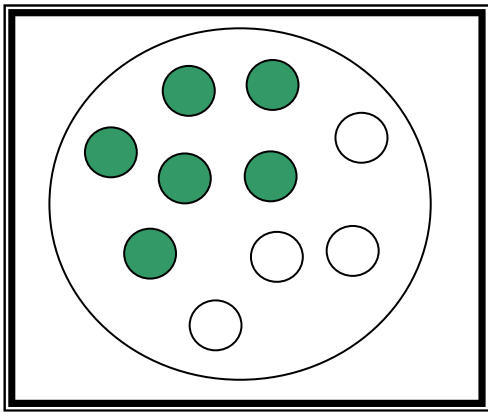
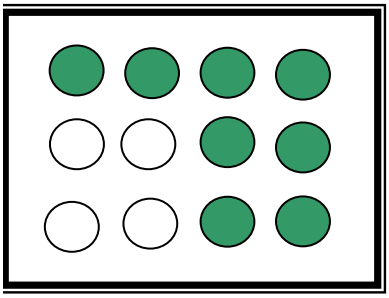
$\frac{5}{2}$	$\frac{3}{2}$	$\frac{4}{3}$
$\frac{5}{3}$	$\frac{5}{4}$	$\frac{7}{4}$
$\frac{10}{6}$	$\frac{6}{4}$	$\frac{10}{8}$











6. Draw me Three-quarters

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions
Students place common fractions on a number line.
Students find equivalent representations of common fractions.

Mathematical focus:

- Connecting concepts with symbols and different representations.

Related interview tasks:

- 1: Fraction Pie
- 3: Dots Array
- 5: Fractions on a Number Line
- 14: Connecting Fractions, Decimals and Percentages.

Materials:

- Large sheets of blank white paper (for group work) or A4 paper for pairs or individual work
- Pencils or textas.

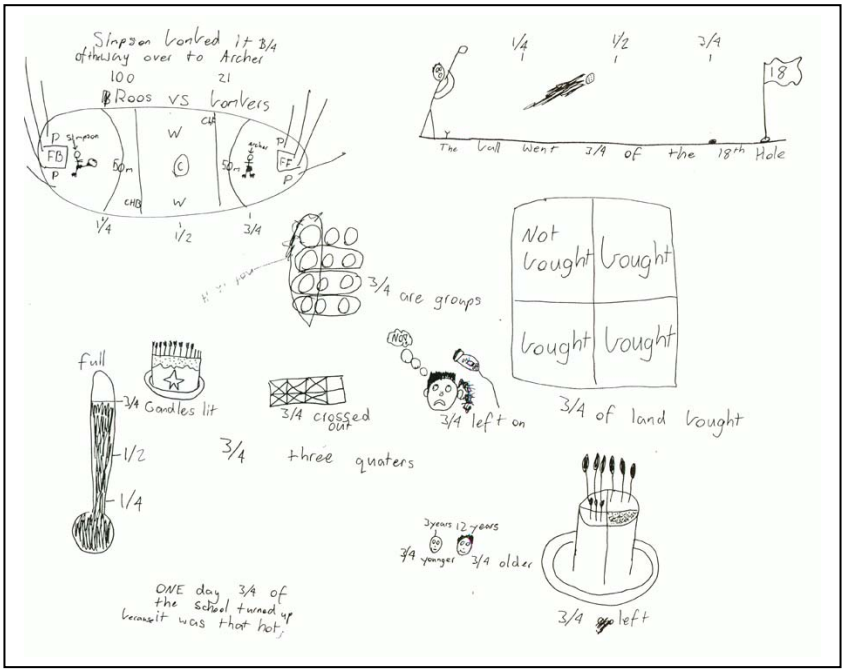
Instructions:

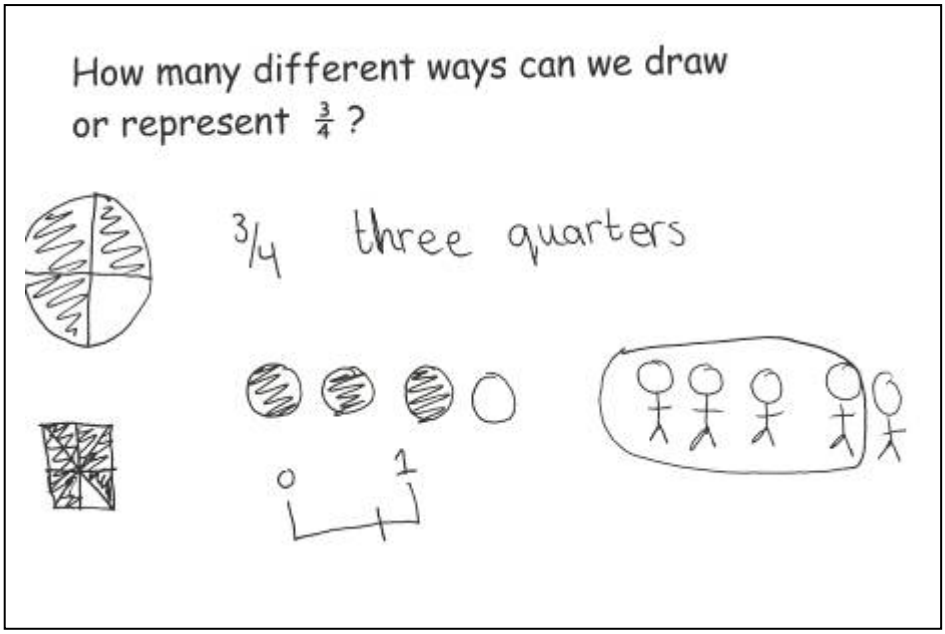
- Ask the students to write or draw what comes to mind when they think of three-quarters
- Invite them to draw as many different ways to represent three quarters as they can
- Have the students explain their drawings.

Below is an example of one student's breadth of understanding when asked to draw what comes to mind when he thinks of 3/4.

The example below is more typical of student work samples that have been collected.

The 'pie' representation is common; however the number line is rarely used. This work sample also demonstrates that the student possibly has the misunderstanding about fractions we call 'Gap Thinking'. Note that the student has drawn four out of five people and shaded seven out of eight parts of the square. They possibly believe that 3/4, 4/5 and 7/8 are the same number as they each are 'one piece less than a whole'.





6.1 Follow up Activity: Everything About my Fraction

Source:

Downton, A., Knight, R., Clarke, D., & Lewis, G. (2006*). Mathematics assessment for learning: Rich tasks & work samples* page 77. Melbourne, Victoria: Maths Teaching and Learning Centre, Australian Catholic University, & Catholic Education Office.

7. Sticky Numbers

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions
Students comprehend the size and order of small numbers.
Students model integers (positive and negative whole numbers and zero), common fractions and decimals.
Students use decimals, ratios and percentages to find equivalent representations of common fractions.

Mathematical focus:

- Connecting concepts with symbols, including improper fractions
- Benchmarking
- Understanding the meaning attached to the numerator and denominator
  For example: in the fraction *a*/*b*, *b* is the name or size of the part (e.g. fifths have this name because five equal parts can fill a whole) and *a* is the number of parts of that name or size. So if we have 7/3, the three tells the name or size of the parts (thirds) and the seven tells us that we have seven-thirds (or 2 1/3).

Related interview task:

- 3: Dots Array
- 8: Construct a Sum
- 9: Fraction Pairs
- 14: Connecting Fractions, Decimals and Percentages.

Materials:

- Sticky labels with a fraction on each (enough for one per student).
  Examples of fractions could be: 1/3, 3/4, 5/4, 2/3, 5/6, 1/7, 9/10, 4/3, 3/2, 1/2, 3/5, 2/8, 4/6, 8/10, 6/3, etc. It is important to include improper fractions in the set and an assortment of denominators.

Instructions:

- Place a number on the back of each student
- Each student must determine the number on their back by asking another student a question about their number
- The question must be answered with either 'yes' or 'no' and only one question can be asked at a time
- The student then moves to a different student to ask a new question
- When a student has determined their number they then place the sticky number on their chest or arm and then continue to answer questions for other students.

NOTE: The teacher can answer questions and assist students to make sense of what they know.
A good question for the teacher to pose after the activity has progressed for a few minutes is "What do you already know about your number?" Sometimes students have been given incorrect information about their number or have misunderstood information.

When all students have removed their number from their back discuss the following questions:

What questions were helpful for determining your number?

Some good questions might be:

- Is my number more/less than a half?
- Is my number more/less than one?
- Is my number a unit/non-unit fraction?
- Is my numerator/denominator even/odd?
- Is my denominator bigger/smaller than fifths?

What questions were unhelpful for determining your number?

Some unhelpful questions might be:

- Is my number even?
- Is my number 3/4?
- Is there a two in my number?

7.1 Follow up Activity: Sticky Numbers

Include mixed numbers, decimals and percentages in the set.

8. Fraction 'Multo'

Links to Victorian Essential Learning Standards:

Level 4 Number
Students multiply fractions.

Mathematical focus:

- Multiplying fractions with whole numbers
- Determining equivalent improper fractions and mixed numbers
- Also students may determine the likelihood of rolling certain numbers and will form opinions about the 'best' game board.

Related interview task:

- 4: Simple Operators.

Materials:

- An activity sheet for each student
- Two dice (one standard six-sided die and one with fractions 1/2,1/3, 1/4, 1/5, 1/6, 1/8)
- Transparent counters.

Instructions:

- The students play this game in pairs or in small groups.
- Each student has his or her own game board (see the activity sheet for Fraction 'Multo').
- The aim of the game is to be the first to cover four numbers in a row (horizontally, vertically or diagonally).
- To begin, each student must prepare their own game board by recording one number in each box on their grid. Each number is taken from the list of numbers provided in the table at the top of the activity sheet. No number should be repeated. This means seven numbers from the list will be left off each grid.
- Each student takes it in turns to throw the pair of dice.
- The whole number and fraction are then multiplied and the result or an equivalent number is covered with a transparent counter on the board of the student who threw the dice. For example if 1/5 and 3 are thrown the student will cover 3/5 on their own game board. If they were to throw 1/6 and 4 then 2/3 would be covered.
- If the number which is called does not exist on their board (and neither does its equivalent), or the number has already been covered, then that student misses a turn.
- The play then continues with the next person.

1/8	1/6	1/5	1/4	1/3	1/2	2/5	2/3	3/4	3/8	3/5	4/5
5/8	5/6	1	2	3	1 1/3	1 1/2	1 1/4	1 2/3	2 1/2	1 1/5	

8.1 Follow-up Activities: Fraction 'Multo'

- Change the dice to ten-sided dice (e.g., 0 — 9 and ignore the zero; or 1 — 10) and get the students to generate the list of possible results and play the game again. Possibly with a larger grid.
- Consider the probability of each result occurring. Also, pose the question: "Having done this how might you change your board to improve your chance of winning?"

8.2 Follow-up Activity: Fraction 'Multo'

$\frac{1}{8}$	$\frac{1}{6}$	$\frac{1}{5}$	$\frac{1}{4}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{4}{5}$
$\frac{5}{8}$	$\frac{5}{6}$	1	2	3	$1\frac{1}{3}$	$1\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{2}{3}$	$2\frac{1}{2}$	$1\frac{1}{5}$	

9. Draw the Spinner

Links to Victorian Essential Learning Standards:

Level 4 Working Mathematically
Students develop and test conjectures.
Students understand that a few successful examples are not sufficient proof and recognise that a single counter-example is sufficient to invalidate a conjecture.

Level 4 Measurement, Chance & Data
Students understand that experimental estimates of probabilities converge to the theoretical probability in the long run.

Level 5 Number
Students write equivalent fractions for a fraction given in simplest form.
Students find equivalent representations of fractions as decimals, ratios and percentages.

Mathematical focus:

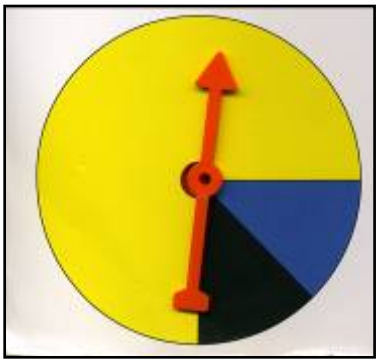
- To see the connections between fractions, decimals, percentages and chance in a game situation.

Related interview task:

- 1: Fraction Pie.

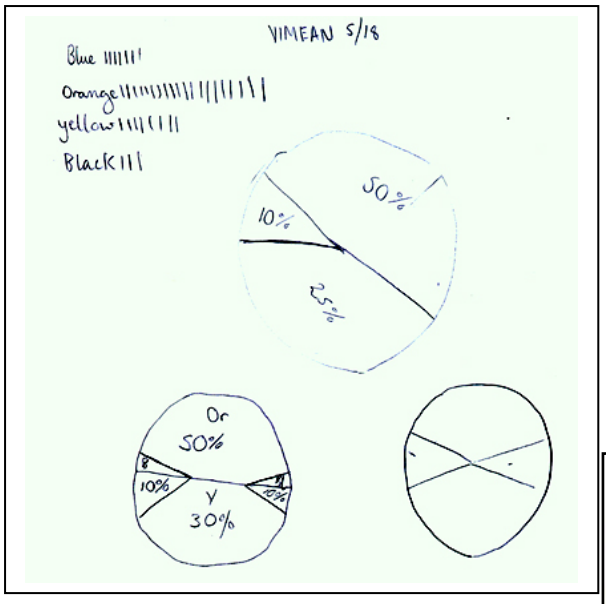
Materials:

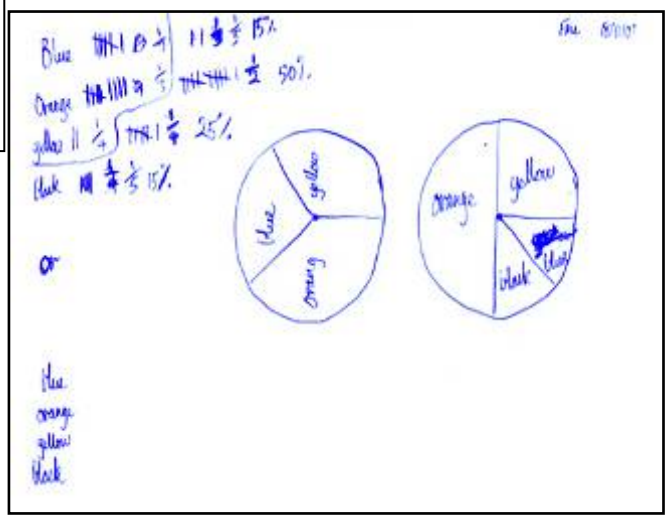
- Coloured spinners, broken up into a variety of colours (see Ron Smith's book for samples)
- Calculators (optional).



Instructions:

- Show the class a sample spinner, pointing out a circular spinner broken up into four, five or six sectors of different colours. Explain that the one you are showing them is a sample, and is *not* the one which will be used initially in the activity.
- Explain that you are going to turn the spinner 10 times (without showing it to them) and call out each result.
- The students will record each result in a way that enables them to tally the 10 rolls.
- Students are then asked to draw a spinner which might have led to these results.
- Now indicate that you are going to turn the same spinner another 10 times, giving a total of 20 results.
- Again, ask the students to draw a predicted spinner beside the other one.
- For students in Years 6-8, ask them to attempt to label the parts as fractions and percentages.
- Discuss with students how they made the connections between fractions and percentages.





Some student work samples

Source:

Smith, R. (2003). *Certain number: Teaching Number through chance and data activities.* Melbourne: Perfect Number Consultancy.

10. Benchmarking Fractions

Links to Victorian Essential Learning Standards:

Level 4 Number
Students model common fractions.
Students comprehend the size and order of small numbers.

Mathematical focus:

- Fractions as a number.
- Students are required to order or compare the relative size of fractions using number sense strategies in preference to algorithms such as converting to common denominators. It is also hoped the students will become more aware of the purpose of the numerator and denominator such that: In the fraction *a*/*b*, *b* is the name or size of the part (e.g. fifths have this name because 5 equal parts can fill a whole) and *a* is the number of parts of that name or size. So if we have 7/4, the four tells the name or size of the parts (fourths or quarters), and the seven tells us that we have seven quarters (or 1 3/4).
- The teacher might notice that some students may be incorrectly reading fractions such as 3/4 as 'three-fours' or 'four-threes' – an indication that their understanding of the numerator and denominator is not yet in place.

Related interview task:

- 8: Construct a Sum
- 9: Fraction Pairs.

Materials:

- A set of fraction cards (either generated by the teacher or the student).

Instructions:

- Generate some fractions cards such as:

1/3	**12/15**	**2/5**
9/8	**8/5**	**39/40**
4/7	**4/3**	**2/19**
9/9	**12/21**	**1/10**

*(remember to include improper fractions to the set)

Instructions cont…

- Sort the cards into three groups ‘near 0’, ‘near 1/2’, ‘near 1’
- Ask students to explain their thinking
- Order the set from smallest to largest
- Choose a fraction card and make a new fraction card that when added to the fraction you chose, makes one (or some whole number).

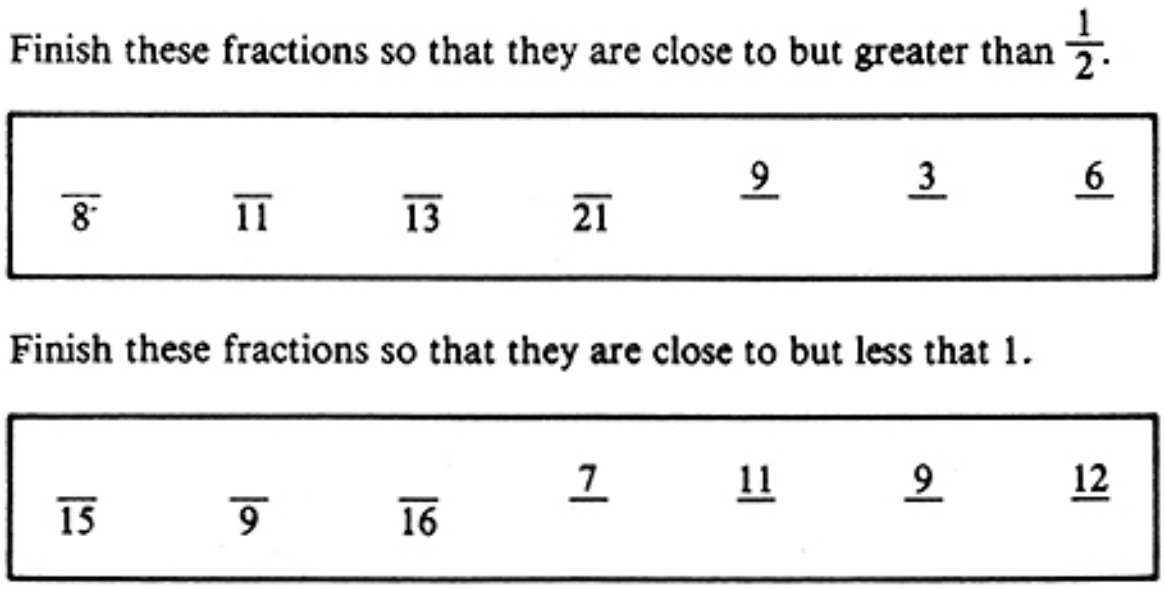
Source:

McIntoch, A., Reys, B., Reys, R., & Hope, J. (1997). *Number sense: Simple effective number sense experiences: Grades 4-6.* USA: Dale Seymour Publications (Pearson Publications) (p. 97-101).

10.1 Follow up activity: Finish these fractions

Instructions:

Finish these fractions so that they are close to one half or one:



Source:

Reys, B. J. (1986). Teaching computational estimation: Concepts and strategies. In H. Schoen & M. Zweng (Eds.), *Estimation and mental computation: 1986 NCTM Yearbook* (p.33). Reston, VA. Natonal Council of Teachers of Mathematics.

10.2 Activity Worksheet: Benchmarking Fractions

Finish these fractions so that they are close to but greater than $\frac{1}{2}$.

$\frac{\ }{8}$	$\frac{\ }{11}$	$\frac{\ }{13}$	$\frac{\ }{21}$	$\frac{9}{\ }$	$\frac{3}{\ }$	$\frac{6}{\ }$

Finish these fractions so that they are close to but less that 1.

$\frac{\ }{15}$	$\frac{\ }{9}$	$\frac{\ }{16}$	$\frac{7}{\ }$	$\frac{11}{\ }$	$\frac{9}{\ }$	$\frac{12}{\ }$

11. Convert and Compare: Fractions to Decimals

Links to Victorian Essential Learning Standards:

Level 4 Number
Students comprehend the size and order of small numbers (to thousandths.
Students use decimals, ratios and percentages to find equivalent representations of common fractions.

Level 5 Number
Students write equivalent fractions for a fraction given in simplest form.
Students know the decimal equivalents for the unit fractions and find equivalent representations of fractions as decimals, ratios and percentages.

Mathematical focus:

- Converting fractions to decimals
- Comparing the relative size of fractions and decimals.

Related interview tasks:

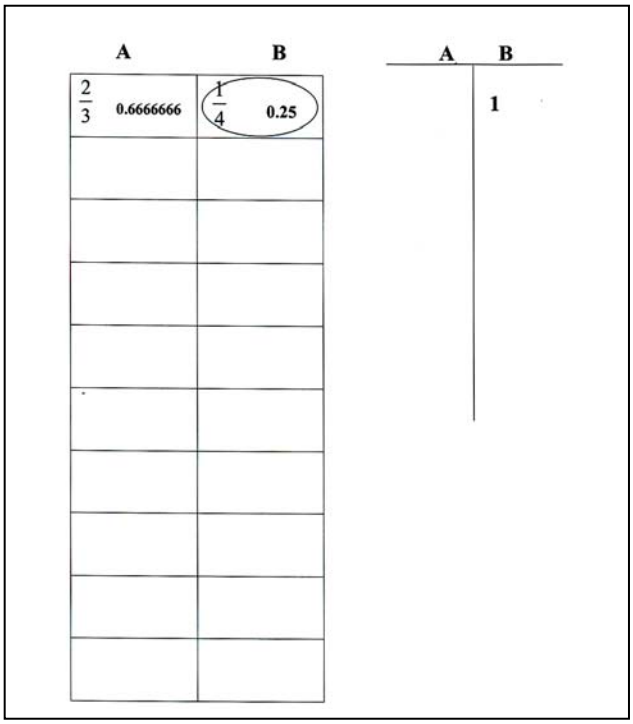
- 8: Construct a Sum
- 9: Fraction Pairs
- 13: Ordering Decimals
- 15: Decimal Comparison Test.

Materials:

- Blank paper or activity sheet
- Two standard six-sided dice
- Calculator
- Pen.

Instructions:

- Playing in pairs, the first player rolls the two dice and produces a fraction using the two numbers thrown.
- As the aim is to produce the smallest number it is assumed the players would opt for a proper fraction, but not always!
- The fraction is then recorded on the grid under Player A. This fraction is then converted to a decimal (using a calculator to divide the numerator by the denominator is permissible) and the decimal is recorded beside the fraction.
- Player B repeats the process and records their fraction and decimal on the grid under Player B.
- The players then agree which number is the smallest and the resultant winner scores one point. If the two numbers are the same then both players score a point.
- Play continues until the grid is full or the time allocated has run out.



After this game the class could have a discussion around some of the following questions:

- When is a fraction greater than one/less than one?
- Which fractions/decimals are easy/hard to compare?
- How do you read 6/2?
- Did you find any different fractions that were equal to the same decimal? (Make a list of these)
- Did you notice anything interesting about the decimals?

This game might provide an opportunity to discuss recurring decimals and how to read and interpret decimals beyond two decimal places.

11.1 Follow-up Activity

This game can easily be adapted to include improper fractions and a larger set of fractions for older students:

- Repeat the game, only this time the largest number wins
- Use two ten-sided dice (0 — 9, ignoring the 0; or 1 — 10).

What you roll is a random event so winning the game is random too, but you can adapt the game to give the students some opportunities to improve their chances of winning. For example:

- Roll three dice and the student has a choice about the numbers they choose
- Roll three dice and the opposing player removes one dice before the fraction is made.

11.2 Activity Sheet: Convert and Compare Fractions to Decimals

A	B
$\frac{2}{3}$ 0.6666666	$\frac{1}{4}$ 0.25

A	B
	1

The smallest number wins!!
The first game has been played for you.

12. Sharing Chocolate

Links to Victorian Essential Learning Standards:

Level 3 Working Mathematically
Students recognise the mathematical structure of problems and use appropriate strategies to find solutions.

Level 5 Working Mathematically
Students formulate conjectures and follow simple mathematical deductions.

Mathematical focus:

- To exemplify the concept of *fraction as division* (or quotient) in a sharing situation.

Related interview task:

- 6: Pizza.

Materials:

- Six blocks of chocolate, preferably non-partitioned chocolate.

Instructions:

- Place three chairs (well separated) out the front of the classroom, with one, two and three blocks of chocolate respectively on them.
- Ask ten students to leave the room. Before they do, explain that when they come back in one at a time, they will be asked to stand behind one chair, knowing that when everyone is in the room, they will be able to share the chocolate with others standing at their chosen chair. The object is to get as much chocolate as possible.
- The students then file in, one at a time, and make their decision about where to stand.
- **Optional:** After eight students are in the room, ask the rest of the class to offer advice as to where person nine, and then later, person 10 should go.
- Once everyone is in the room, ask the group at each chair to explain how much chocolate they should each receive. Most will determine this by dividing each piece into the number of people at the chair. For example, for two blocks and three people, most students will divide each block into three and say 'each person gets one third of each block, so two thirds each'.
- It is possible that some students will immediately see the pattern that x blocks shared between y people means x/y each, but many will not.
- Record the findings on the board (e.g. 2 blocks, 3 people, 2/3 each; 3 blocks, 5 people, 3/5 each, etc.) and encourage students to comment on any patterns they detect.

To ensure that the mathematical intent is clear, pose some similar problems to students to solve, for example: 'If you had 2 pies shared evenly between five people, what fraction of a pie would each get?'

Source:

Clarke, D. M. (2006). Fractions as division: The forgotten notion? *Australian Primary Mathematics Classroom, 11*(3), 4-10.

13. Boys, Girls and Pizza

Links to Victorian Essential Learning Standards:

Level 4 Number
Students comprehend the size and order of small numbers.
Students use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Mathematical focus:

- Fractions as part-whole
- In this task the students are required to form their own partitions and name the fractional parts. They must also keep track of 'the whole' or 'unit'. With sufficient experience at partitioning, some students may connect the portioning with division and then with their fractional answer e.g. three pizzas shared among seven girls means 3 ÷ 7 = $\frac{3}{7}$ and may lead to the generalization that a ÷ b = $\frac{a}{b}$.
- This is the important notion of fractions as division.

Related interview tasks:

- 1: Fraction Pie
- 6: Pizza.

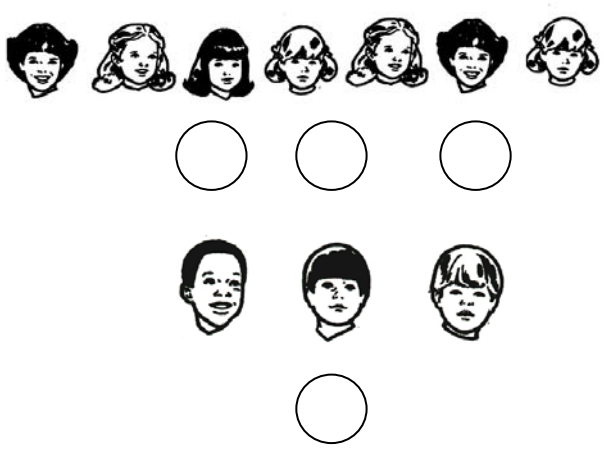
Materials:

- Activity sheet and pencils.

Instructions:

The students could complete this individually or in pairs. Ask them: 'Who gets more pizza, a girl or a boy?' If:

- Three pizzas were shared between seven girls
- One pizza was shared between three boys.



The students record their thinking then share these with the class.

For students who solve this quickly ask the following questions:

- How much pizza does each person get?
- How much more pizza does each girl get? (This is quite a complex question)

13.1 Follow-up Activity: Boys, Girls and Pizza

Adapt the task by changing the numbers so that students have experience with:

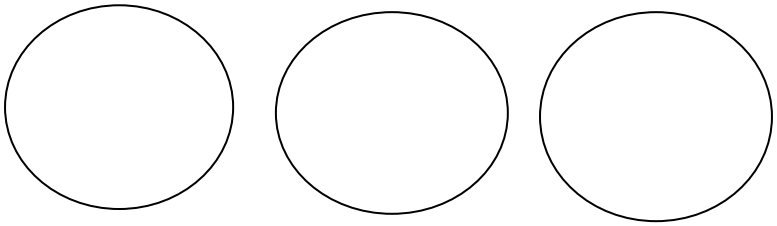
- Unit fractions: three people share 1 pizza - 1/3 each
- Non unit fractions: four people share three sausages - 3/4 each
- Improper fractions: five people share six bags of lollies - 6/5 each
- Mixed numbers: six people share 13 litres of water - 2 1/6 each.

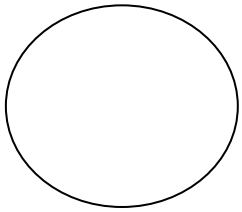
Source:

Lamon, S. J. (2006). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers (2nd Ed.).* Mahwah, NJ: Lawrence Erlbaum.

Lamon, S. J. (2006). *More: In-depth discussion of the reasoning activities in "Teaching fractions and ratios for understanding"* (2nd Ed.). Mahwah, Nj: Lawrence Erlbaum.

13.2 Activity Sheet: Boys, Girls and Pizza





14. Estimating Fractions

Links to Victorian Essential Learning Standards:

Level 4 Number
Students place common fractions on a number line.
Students find equivalent representations of common fractions.

Mathematical focus:

- Fractions as a measure
- The students are required to mentally partition a model of a number line (the rope) into fifths and determine a method for exactly locating a fraction (2/5) on this number line.

Related interview task:

- 5: Fractions on a Number Line.

Materials:

- Large rope (4-5 metres long)
- Three clothes pegs
- Large tape measure (optional).

Instructions:

- Select three volunteers to place a peg two-fifths of the way along the rope from the left-hand side (by estimating).
- Have the class vote on which student's peg they believe is the closest to the correct location.
- Ask the class to propose how they would calculate the exact location of two-fifths and share these solutions.
- For each strategy which is suggested, link it to fraction understanding. For example, a student might take one of the estimates, assume it is 2/5, than fold it over once (effectively making 4/5) and then half a fold (making 5/5 now), and see how close it is to the end of the rope. The mathematical expression for what the student has done is thus 2/5 + 2/5 + 1/5 = 1.

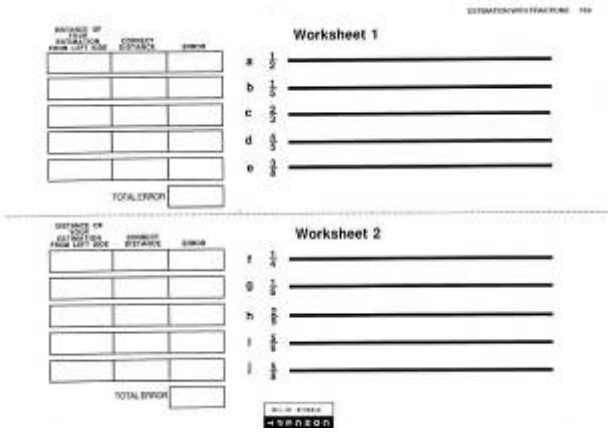
14.1 Follow-up Activity: Estimating Fractions

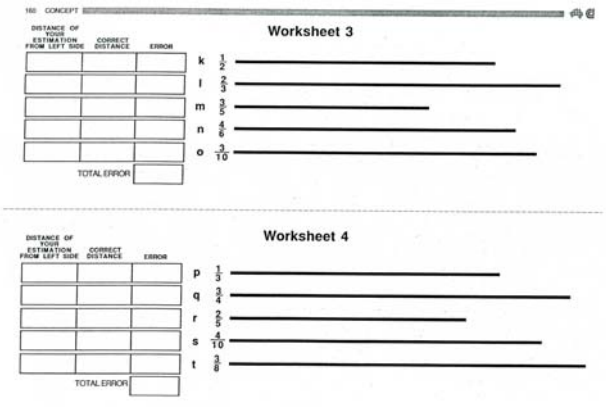
Materials:

- Worksheets 1-4
- Rulers
- Calculators.

Instructions:

- Ask students to estimate how far from the left end of the line would the fraction occur and mark this place with a vertical line (e.g. for the first one, the student marks an estimate of 1/2 way along the line; for the second, 1/5 across from the left).
- Continue for all lines (Worksheet 1 only).
- They then use a ruler to measure the exact distance to the nearest millimetre of each mark and record this in the first column.
- The teacher then provides the exact measure to the closest mm of the exact location of each fraction and provides this information to the students to record in the second column.
- The students then calculate the absolute error from their estimate to the correct solution.
- All errors are added and the total is recorded. The students might then compare their total errors.
- Students who recorded the least amount of error are encouraged to share their strategies. The students now complete Worksheet 2 and compare total errors and improvement.





Source:

Lovitt, C., & Clarke, D. M. (1988). *Mathematics Curriculum and Teaching Program: Activity bank, Vol.1.* Carlton, Victoria: Curriculum Corporation.

14.2 Activity Sheets: Estimating Fractions

Worksheet 1

DISTANCE OF YOUR ESTIMATION FROM LEFT SIDE	CORRECT DISTANCE	ERROR
TOTAL ERROR		

a $\frac{1}{2}$ ——————————

b $\frac{1}{5}$ ——————————

c $\frac{2}{3}$ ——————————

d $\frac{4}{5}$ ——————————

e $\frac{3}{8}$ ——————————

Worksheet 2

DISTANCE OF YOUR ESTIMATION FROM LEFT SIDE	CORRECT DISTANCE	ERROR
TOTAL ERROR		

f $\frac{1}{4}$ ——————————

g $\frac{1}{6}$ ——————————

h $\frac{2}{5}$ ——————————

i $\frac{5}{6}$ ——————————

j $\frac{5}{8}$ ——————————

COZCOWAH

Worksheet 3

DISTANCE OF YOUR ESTIMATION FROM LEFT SIDE	CORRECT DISTANCE	ERROR
TOTAL ERROR		

k $\frac{1}{2}$

l $\frac{2}{3}$

m $\frac{3}{5}$

n $\frac{4}{6}$

o $\frac{3}{10}$

Worksheet 4

DISTANCE OF YOUR ESTIMATION FROM LEFT SIDE	CORRECT DISTANCE	ERROR
TOTAL ERROR		

p $\frac{1}{3}$

q $\frac{3}{4}$

r $\frac{2}{5}$

s $\frac{4}{10}$

t $\frac{3}{8}$

15. Introducing Decimats

Links to Victorian Essential Learning Standards:

Level 4 Number
Students comprehend the size and order of small numbers (to thousandths).

Mathematical focus:

- Decimat as a model for representing decimals
- This activity provides opportunities for the symbols for fractions and decimals to be connected and highlights the place value of decimals to thousandths. This model has the advantage of showing the proportional relationship between the respective place values (e.g. ten thousandths will make one hundredth, and ten hundredths will make one tenth).

Related interview tasks:

- 10: Decimals on a Number Line
- 11: Decimal Density
- 12: Make me a Decimal
- 13: Ordering Decimals
- 14: Connecting Fractions, Decimals and Percentages
- 15: Decimal Comparison Test.

Materials:

- Large decimat per student
- Two dice (one standard six-sided die and one containing these fractions, 1/10, 1/100, 1/100, 1/1000, 1/1000, 1/1000)
- Pencils and paper for recording.

Instructions:

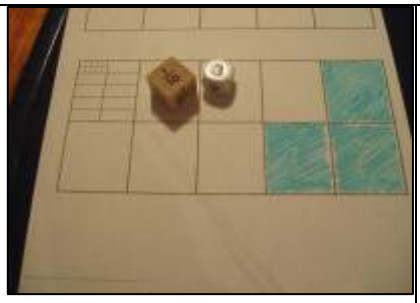
- Player A rolls the two dice and then shades the product of the two numbers displayed.
- Player A then records 'how much of the decimat is shaded'.
- Player B rolls the two dice and continues the game, with each player recording 'how much of the decimat is shaded' after their turn.
- It is permitted for tenths/hundredths to be further divided into hundredths/thousandths if more hundredths/thousandths are required. For example if a player has shaded all of their hundredths and they then roll four hundredths then the player can 'cut' a tenth into 10 equal pieces (therefore creating hundredths) and shade four of these.
- If the product of the roll is greater than the remaining unshaded part of the decimat then the player misses a turn.

The first player to reach one (by shading their complete decimat) or the player closest to one (by shading more of their decimat) after a certain amount of time has elapsed, is the winner.

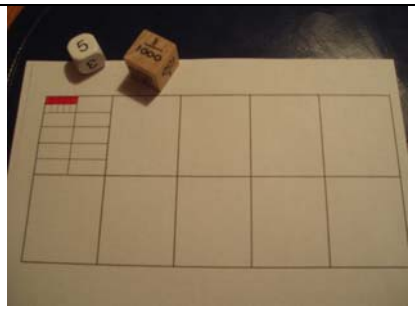
An example game has been shown:

- **Player A has rolled 3 x 1/10, shades 3 tenths and recc**
- **Player B has rolled 5 x 1/1000, shades 5 thousandths records 0.005**
- **Player A now rolls 6 x 1/100, shades 6 hundredths an**

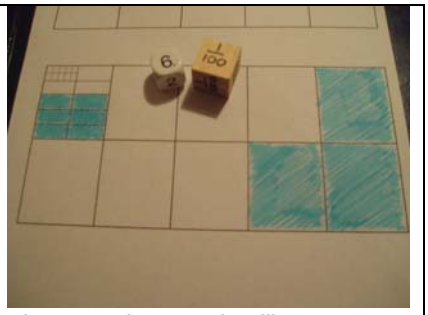
Player A	Player B
0.3	0.06
0.36	



Player A (first roll)



Player B (first roll)



Player A (second roll)

16. Library Books and Where do they Go?

Links to Victorian Essential Learning Standards:

Level 4 Number
Students comprehend the size and order of small numbers (to thousandths).

Mathematical focus:

- Ordering decimals in a real life context.

Related interview tasks:

- 13: Ordering Decimals
- 15: Decimal Comparison Test.

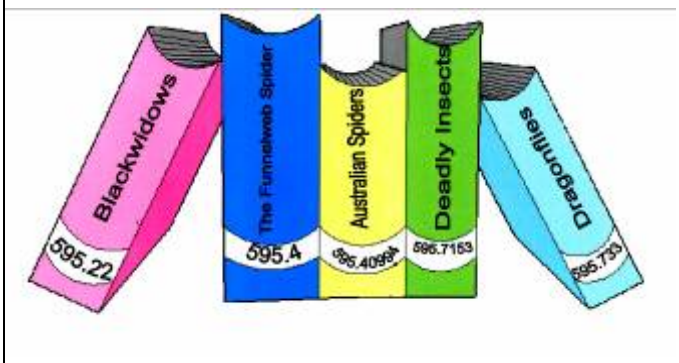
Materials:

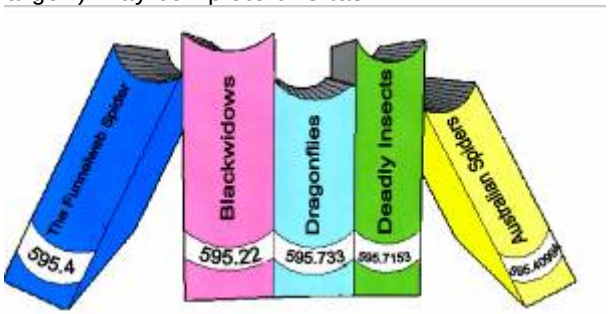
- A set of library books that begin with the same whole number and that have ragged decimals (see example below).

Instructions:

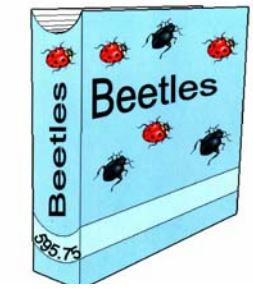
In groups or pairs ask the students to order the books as they should occur on the library shelf.

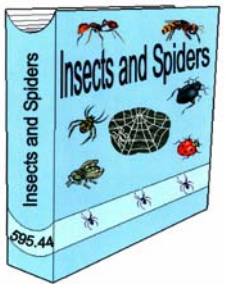
Notice that the first set of books has been ordered incorrectly to demonstrate how a student who believes decimals can be ordered like whole numbers ('longer is larger') may complete this task.





Once the books have been ordered correctly (see below) ask the students where they would place these extra books (Insects and Spiders and Beetles)





17. Sorting Decimals

Links to Victorian Essential Learning Standards:

Level 4 Number
Students comprehend the size and order of small numbers (to thousandths)
Students place integers, decimals and common fractions on a number line.

Mathematical focus:

- Ordering decimals
- Students are required to recognise that a decimal is near 0.5 or near 1. This skill is important for being able to order decimals and make estimations when doing decimal calculations.

Related interview tasks:

- 13: Ordering Decimals
- 15: Decimal Comparison Test.

Materials:

- A set of decimal cards either generated by the teacher or the student.

Instructions:

Generate some fractions cards such as:

0.46	**0.4444**	**0.7**
0.54	**0.8**	**0.32**
0.5234	**0.48**	**0.61**
0.08	**0.007**	**0.501**

- Sort these cards into two groups – decimals less than 0.5 and decimals greater than 0.5.
- Sort these cards into two groups – decimals near zero and decimals near one.
- Find a decimal that proves that proves this statement: 'The relative size of a decimal is not determined by the number of digits it has'.
- Take two decimals from the pile of decimals 'near one' and explain what you know about their sum.
- Find three decimal between 1/2 and 6/10.
- Choose two decimal cards whose sum is more than 0.5 but less than one.

Source:

McIntoch, A., Reys, B., Reys, R., & Hope, J. (1997). *Number sense: Simple effective number sense experiences: Grades 4-6.* USA: Dale Seymour Publications (Pearson Publications) (p. 102-106.)

18. Decimal Maze

Links to Victorian Essential Learning Standards:

Level 4 Number
Students multiply fractions and decimals (to two decimal places) and apply these operations in practical contexts. Students use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Level 5 Working Mathematically
Students formulate conjectures and follow simple mathematical deductions.

Mathematical focus:

- Operating on decimals
- To dispel the misconception that multiplication always increases the value of numbers and division always makes them smaller.

Related interview tasks:

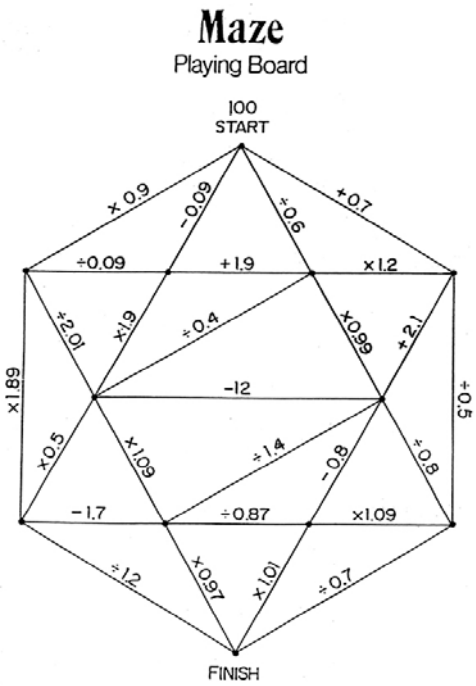
- 16: Decimal Operations.

Materials:

- Maze board
- Two calculators
- One playing piece (counter) per pair of students.

Instructions:

- The aim of the game is to have the smallest number on your calculator when the game ends.
- Both players enter 100 on their calculator and the counter is placed on the START.



- Player A chooses to move the counter along a single line segment and performs the operation on his or her calculator.
- Player B moves the counter from its new position along a single line segment (but not back along the line segment their opponent has just used) and performs this operation on their calculator.
- The game continues with each player in turn moving the counter and performing the operation that is on the line segment they have just passed.
- The calculator is not cleared between turns.
- The game ends when one player reaches the FINISH and winner is the player with the smallest number on their calculator. Alternatively you could choose to end the game after a certain time has elapsed and the player with the smaller number at that point is the winner.

18.1 Follow-up Activities: Decimal Maze

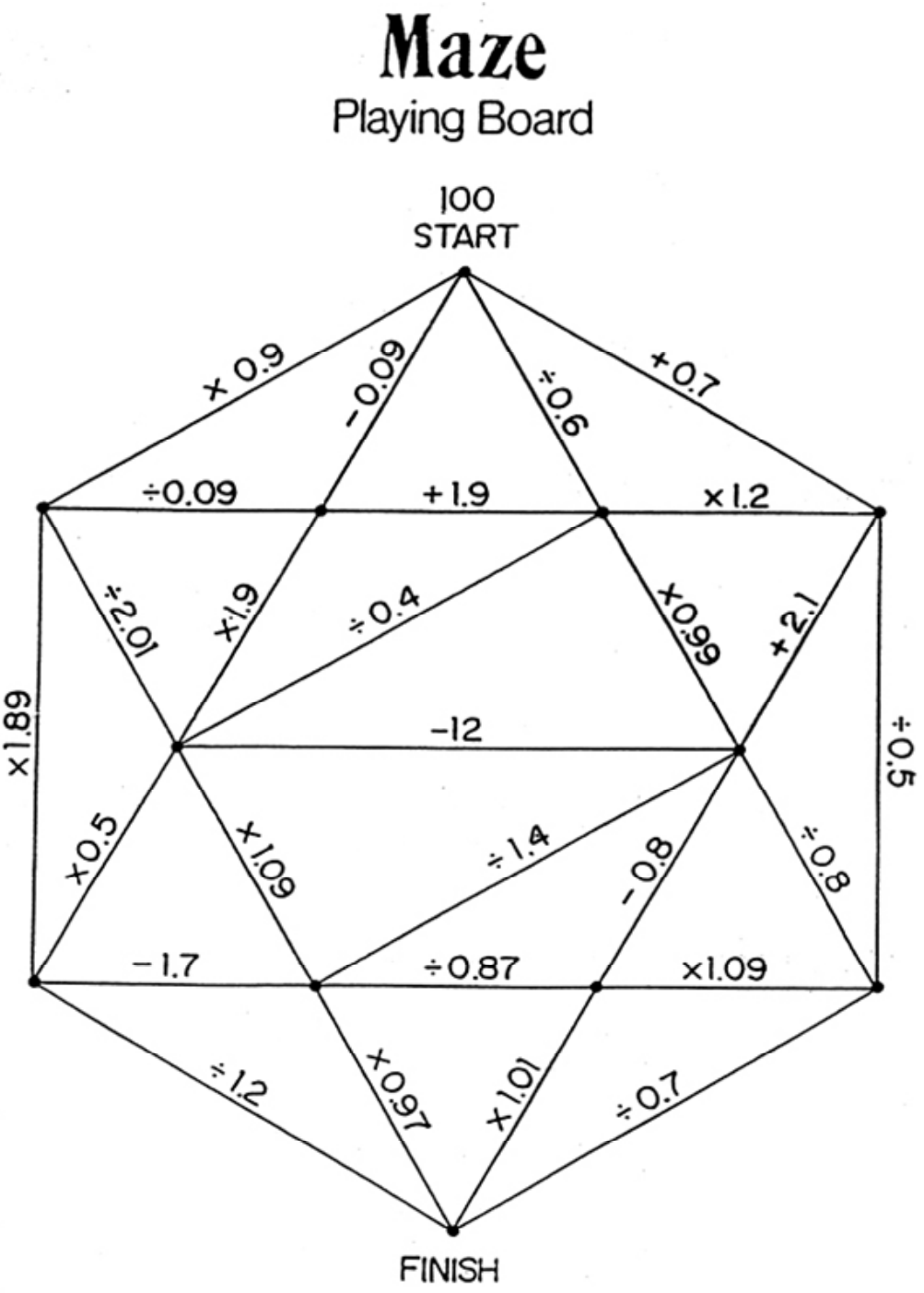
Adapt the game by:

- Changing the aim to the largest number wins, or nearest to a particular number e.g. 127.6
- Playing with three players
- Letting each player have their own counter
- Not allowing any segments to be retraced. The game ends when the finish is reached or there are no further moves possible
- Removing the word 'finish' and asking students to keep moving around the game board until the time is up.

Source:

Morris, J. (1981). *How to develop problem solving using a calculator* Reston, Va: National Council of Teachers of Mathematics. (p.27).

18.2 Activity Sheets: Maze



19. On Target

Links to Victorian Essential Learning Standards:

Level 5 Number
Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding.
Students use efficient mental and/or written methods for arithmetic computation involving rational numbers.
Students use technology for arithmetic computations involving several operations on rational numbers of any size.

Mathematical focus:

- Operating with decimals
- Estimating the product when multiplying by decimals larger (and smaller) than one.

Related interview task:

- 20: Cheese Please.

Materials:

- One calculator and a task sheet per player.

Instructions:

- The aim of the game is to reach the target number in the least amount of turns.
- A target number and a starting number are set prior to starting the game.
- In order to reach the target number the student must multiply the number on display by some other number.
- This process is repeated until the target number is reached (digits after the decimal point do not count).

Here is a sample game:

Guess	Display	Target	Thoughts
	13	**152**	
X 11	143		Too small
X 12	156		Too big, the number must be 11 point something
X 11.2	145.6		Still too small
X 11.9	154.7		Too big
X 11.7	152.1		Close enough!

19.1 Follow-up Activity: On Target

Adapt the game by:

- Changing the target number and/or the starting number
- Making a starting number that is a decimal less than one
- Comparing the final display numbers and determine who achieved the closest result.

Source:

Swan, P. (2007). *Calculators in classrooms: Using them sensibly.* Perth, Western Australia: Author. (p. 49).

20. What's my Number?

Links to Victorian Essential Learning Standards:

Level 5 Number
Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding.
Students use efficient mental and/or written methods for arithmetic computation involving rational numbers.
Students use technology for arithmetic computations involving several operations on rational numbers of any size.

Mathematical focus:

- Operating with decimals
- Estimating the quotient when dividing whole numbers.

Related interview task:

- 20: Cheese Please.

Materials:

- One calculator between two players
- Blank paper (or task sheet) and pen to record results.

Instructions:

- One player chooses a mystery number between one and 100 (begin with two-digit whole numbers only) and enters it into the calculator using the following keystroke sequence:

Mystery Number	÷	Mystery Number	=

- The display should now show 1.
- This has been done while the second player has their back turned or eyes closed.
- The calculator is then passed to the second player whose aim is to determine the 'mystery number'.
- The second player guesses what the mystery number might be and enters this guess into the calculator (remember the calculator is still displaying one, and nothing needs to be cleared), and then presses equals.
- If the display shows 1 then the player has guessed correctly.
- If the display shows some other number then the second player needs to guess again to get closer to one.
- NOTE: Do not clear the display between turns.
- The number of guesses required can be recorded on the task sheet.
- Players swap roles and the first player must attempt to guess the second player's mystery number.

Enter	Display	Thinking
50	1.1627906	Too big. Try a smaller number
32	0.744186	Too small. Try about half way between.
41	0.9534883	Still a bit small
42	0.9767441	Nearly there
43	1	Bingo! The mystery number is 43

20.1 Follow-up Activity

Adapt the game by:

- Choosing a mystery number between one and one hundred that has two decimal places
- Changing the aim so that the mystery number must be obtained in less than seven guesses.

Source:

Swan, P. (2007). Calculators in classrooms: Using them sensibly. Perth, Western Australia: Author (p. 55).

21. Guestimate

Links to Victorian Essential Learning Standards:

Level 4 Number
Students use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Level 5 Number
Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding.

Mathematical focus:

- Operating with decimals
- Estimating the product when multiplying by decimals larger (and smaller) than one.

Related interview task:

- 20: Cheese Please.

Materials:

- One calculator between two players
- Blank paper (or task sheet) and pen to record results.

Instructions:

- The aim of this game is to be the player to reach the target of 100 or as close as possible to it.
- The first player enters a number on the calculator (some 'ugly' number less than 100 is usual but any number could be displayed).
- The second player must multiply this number by another number so that the result will be as close as possible to 100.
- The first player then multiplies this new number, trying to get closer still to 100.
- The players take it in turns until one player hits the target by getting 100 on the calculator (digits after the decimal point do not count, so 100.xxx is fine).
- The game can be recorded on the task sheet or blank paper.

Here is a sample game:

Player	Keys pressed	Display shows	Thoughts
1	37	37	
2	X 2.5	92.5	A bit small
1	X 1.15	106.375	A bit big
2	X 0.9	95.7375	Too low
1	X 1.05	100.52437	I win!

22. Estimating Multiplication of Decimals

Links to Victorian Essential Learning Standards:

Level 4 Structure
Students establish equivalence relationships between mathematical expressions using properties such as the distributive property for multiplication over addition.

Level 5 Number
Students use a range of strategies for approximating the results of computations, such as front-end estimation and rounding.

Mathematical focus:

- Operations with decimals.

Related interview tasks:

- 16: Decimal Operations
- 20: Cheese Please.

Materials:

- Either an over-head projector with the problems on transparency (to be revealed one at a time), or a PowerPoint presentation of the different problems.

Instructions:

- Explain to students that today's activity is about estimating the result when two decimals are multiplied together. Highlight that estimation is a very important skill—approximately 60 per cent of all calculations which adults do in Australia only require an estimate, not an exact answer.
- Students will be given a short amount of time to write down their estimate to a series of calculations. Explain that they won't have time to work out the answers exactly, and that in fact you don't want them to do so. They will be given products such as 4.9 x 3.2, and asked to write down in less than 15 seconds their estimate of the product to one decimal place.
- The teacher reads aloud the five calculations in the set (the problems should also be displayed in some way as well e.g. overhead transparency or PowerPoint).
- The students record their estimates for each.
- Then each result is awarded some points according to the point system below.
- Students will then share their strategies, practise on two more examples, and then complete the second set of calculations.
- Students then score their results from the second set and check for improvement between the two scores.
- Emphasise that quite a few students may score no points the first time, but by sharing approaches and practising, it is likely that almost all will improve.

The points system is as follows:

Estimate:

- Out by 0 or 0.1 (5 points)
- Out by 0.2 or 0.3 (4 points)
- Out by 0.4 or 0.5 (3 points)
- Out by 0.6 or 0.7 (2 points)
- Out by 0.8 or 0.9 (1 point)
- Out by 1.0 or more (0 points).

For example, if a student estimates the product of 2.8 x 3.1 is 9.0 – the error is the difference between 9.0 and 8.7, which is 0.3. Therefore the student scores 4 points for this calculation.

During the discussion between sets, emphasise two things:

a) The common and helpful strategy of rounding up and rounding down before estimating (e.g. 4.9 x 3.2 can be approximated as a bit more than 5 x 3, because the 4.9 is closer to five than the 3.2 is to three).

b) The distributive property of multiplication. In this case for example, 4.9 x 3.2 means 'distributing both the 4 and the 0.9 over the 3 and the 0.2,' so the actual answer is made up of 4 x 3 and 4 x 0.2 and 0.9 x 3 and 0.9 x 0.2. This explains why, for 4.2 x 4.2, the answer, to one decimal place isn't what many students think (16.4 — incorrectly multiplying 4 by 4 and 0.2 by 0.2), but is actually 17.6.

PRACTICE SET:	4.2 x 3.9	(Answer: 16.4)
	3.6 x 4.9	(Answer:17.6)
SET 1	2.8 x 3.1	(Answer: 8.7)
	3.3 x 1.9	(Answer: 6.3)
	4.2 x 2.7	(Answer: 11.3)
	3.5 x 3.8	(Answer: 13.3)
	4.2 x 4.2	(Answer: 17.6)
SET 2:	4.1 x 2.7	(Answer: 11.1)
	3.3 x 1.9	(Answer: 6.3)
	2.9 x 2.9	(Answer: 8.4)
	4.5 x 2.5	(Answer: 11.3)
	2.1 x 3.7	(Answer: 7.8)

Source:

Lovitt, C., & Clarke, D. M. (1989). Mathematics Curriculum and Teaching Program: Activity bank, Vol. 2. Carlton, Victoria: Curriculum Corporation.

23. Money Prize

Links to Victorian Essential Learning Standards:

Level 5 Measurement, Chance & Data
Students measure length, perimeter, area, surface area, mass, volume, and capacity, using suitable units for these measurements in context. Students estimate the accuracy of measurements and give suitable lower and upper bounds for measurement values.

Mathematical focus:

- Apart from length, area, mass, perimeter and capacity this activity requires some ability with the four operations with whole numbers and decimals within the context of money.

Related interview task:

- 16: Decimal Operations.

Materials:

- Empty one litre milk containers
- A balance with 5g and 10g mass pieces
- Metre rulers
- Plenty of coins
- Small rulers.

Instructions:

Problem:

Congratulations! You have won a money prize.

Your prize can be:

- a one litre milk carton filled with 20cent coins
- a square metre of five-cent pieces, with the square metre filled in with one kilogram of $1 coins
- a line of $2 coins one metre long (lying flat and touching).

Using the equipment available investigate and record your procedures for determining the amount of money in each prize.

Source:

Downton, A., Knight, R., Clarke, D., & Lewis, G. (2006*). Mathematics assessment for learning: Rich tasks & work samples*. Melbourne, Victoria: Maths Teaching and Learning Centre, Australian Catholic University, & Catholic Education Office. (p. 138).

24. Ask Marilyn: Shopping and Cereal

Links to Victorian Essential Learning Standards:

Level 4 Number
Students use decimals, ratios and percentages to find equivalent representations of common fractions.
Students use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Mathematical focus:

- Percentages in context.

Related interview task:

- 18: Reserve Bank and Chocolate Milk.

Materials:

- Calculators
- Pen and paper.

Instructions:

Marilyn Vos Savant is claimed to have the highest measured IQ in the world. She is sent complex questions by her readers. Two of the questions she has responded to in the past are included below. Ask students to work together to write a response to the reader.

Ask Marilyn®

BY MARILYN VOS SAVANT

A wholesaler sells a dress for $20. The store marks it up to $40—a markup of 100%. But the dress doesn't sell by the end of the holiday season, and the store discounts it to 50% off. The price is now back to $20.

How can a 100% markup and a 50% reduction result in the same figure?

—Judy and Greg Winski, Lakeland, Fla.

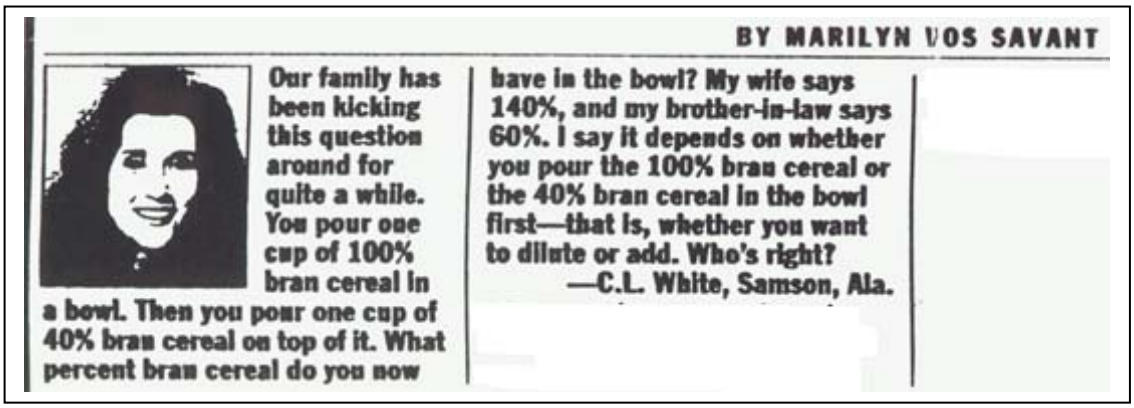
Shopping Problem

This example focuses on a very important mathematical idea (and hence a very important confusion or misconception).

When we think about percentage increase or decrease, we must always focus on the increase or decrease as a percentage of the value we are changing. That is, if we are looking at a 30% increase on $27, we find 30% of $27 and add it on (or just find 130% of $27, giving $35.10.) If we then want to find a 25% reduction on that price, we need to 'act on' the new price, so we would take off 25% off $35.10 (or simply find 75% of $35.10 which would be $26.33 approximately).

This explains why the price in the Marilyn scenario changes, as we are operating on two different bases.

It will be good to provide lots of scenarios of this kind to help with the emphasis on base value.



BY MARILYN VOS SAVANT

Our family has been kicking this question around for quite a while. You pour one cup of 100% bran cereal in a bowl. Then you pour one cup of 40% bran cereal on top of it. What percent bran cereal do you now have in the bowl? My wife says 140%, and my brother-in-law says 60%. I say it depends on whether you pour the 100% bran cereal or the 40% bran cereal in the bowl first—that is, whether you want to dilute or add. Who's right?

—C.L. White, Samson, Ala.

Mixing Bran

This is an example of a problem which could be modelled quite nicely with materials of some kind (e.g. two coloured unifix could model the bran [one colour] in the cereal [another colour]. For example, you could have one mixture that is all brown (100 brown cubes or 100% bran) and another which is 40% bran (40 brown, 60 yellow). When they are combined, you get 140 brown and 60 yellow. This means that there are 140 out of 200 or 70 for every 100 or 70% mixture.

Marilyn's response was: "It's no wonder you guys haven't been able to agree. All three of you are wrong. Regardless of which cereal you pour first, you'll end up with 140 parts bran out of 200 parts cereal. And that translates to 70%."

Students could now pose problems themselves of this kind.

25. Petrol Prices: The Best Discount

Links to Victorian Essential Learning Standards:

Level 5 Working Mathematically
Students formulate conjectures and follow simple mathematical deductions.
Students use variables in general mathematical statements. They substitute numbers for variables (for example, in equations, inequalities, identities and formulas).

Mathematical focus:

- Percentage of a number
- Money
- Graphing linear equations
- Break-even points.

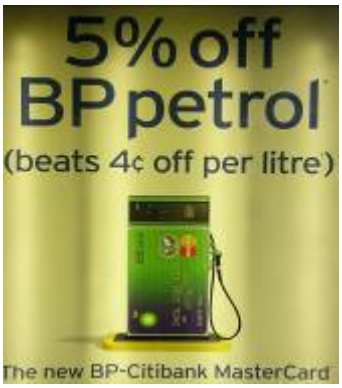
Related interview task:

- 18: Reserve Bank and Chocolate Milk Drink.

Materials:

- Calculators
- Graph paper.

Instructions:



- Discuss with students what they know about petrol discounts and how their families make decisions about where to buy petrol.
- Show students the poster here which claims that 5% off beats 4c off a litre. Explain that the purpose of the lesson is to investigate this claim, and determine for what prices of petrol it is true, and for what prices it is not true.
- Encourage students to create a table to consider the final price for a range of petrol prices, given the two possible discount processes.
- Depending upon the year level, you may wish to encourage students to graph the resulting data. One graph would be of the form $y = 0.95x$, with the other $y = x - 4$.
- Students should eventually find that for any original petrol price of 80 cents a litre or more, the 5% is a better overall deal. Depending upon the year level, this can also be solved by considering the situation where the two are the same, i.e. when $0.95x$ (the case of 5% discount) = $x - 4$ (the case of 4 cents off), where x is the original price in cents.

NOTE: Students find this a very difficult concept. It may be necessary as a 'gentle' lead in to first of all suggest a particular price and talk about what 4 cents a litre off that price would be (e.g. $1.23 per litre), and choose another price (e.g. $1.20 per litre) and discuss what 5% a litre off that price would be.

A related task

It is often helpful for students to provide another example where the same mathematical idea is operating.

Offer the scenario of a federal government considering two possible wage rises: a rise of 2% for everyone or a rise of $20 per week for everyone.

Discuss the ways in which these two possible models would yield different benefits to people with different levels of income.

The $20 per week would provide proportionally a much greater boost to those on low incomes, while a rise in 2% would greatly benefit those on higher incomes.

26. Solving The Case of the Mystery Bones

Links to Victorian Essential Learning Standards:

Level 5 Working Mathematically
Students formulate conjectures and follow simple mathematical deductions.
Students use variables in general mathematical statements. They substitute numbers for variables.

Mathematical focus:

- Proportional reasoning.

Related interview task:

Materials:

- Flexible cm tape measures (one per group of four)
- Long tape secured against wall with masking tape to measure height in cm
- Calculators.

Instructions:

Pose the following scenario:

Police detectives have discovered a human radius bone buried in the ground. (You may need to use the internet or an anatomy book to be sure which bone is the radius bone.) The bone is 25 cm long.

By taking height and radius measurements of five people in your class, see what patterns you notice, and make a prediction of the mystery person's height, giving careful justification for your estimate.

Follow up: Share the table below with your students which shows how to predict height given a particular bone, when you know the gender of the person.

Forensic Table for Height Prediction	
Male Height (cm)	**Female Height (cm)**
3.2 x (humerus) + 67	**3.3 x (humerus) + 60**
3.6 x (radius) + 81	**4.2 x (radius) + 62**
2.3 x (femur) + 64	**2.4 x (femur) + 64**
2.4 x (tibia) + 83	**2.7 x (tibia) + 67**

Pose the following questions, for the students to investigate using the table:

a) Calculate the height estimate for a female with a radius bone of 25 cm

b) Calculate the height estimate for a female with a tibia bone of 44 cm

c) Could the two bones in parts a) and b) belong to the same person? Explain your reasoning

d) The two tallest identical twins in the world are 195 cm tall. Use the table to predict the length of their humerus bones.

Source:

Clarke, D. M. (1996). *The case of the mystery bone: A unit of work on measurement for grades 5 to 8.* North Ryde: The Mathematical Association of New South Wales.

27. Maximum Load in Lifts

Links to Victorian Essential Learning Standards:

Level 5 Working Mathematically
Students develop simple mathematical models for real situations. They develop generalisations by abstracting the features from situations and expressing these in words and symbols. They predict using interpolation (working with what is already known) and extrapolation (working beyond what is already known).

Mathematical focus:

- Data analysis
- The idea of variables
- Proportional reasoning
- Measurement of length, area and volume
- Graphing data.

Related interview task:

- 17: Pod Tunes or New Tunes?

Materials:

- Notebooks
- Calculators
- Measuring tapes (at least three metres long).

Instructions:

There is considerable variation in the listed capacity of lifts in public buildings, in terms of the maximum weight which can be carried and the maximum number of people allowed in the lift at any one time. It is not obvious how these limits are determined. In particular, which variables are involved in determining the maximum limits?

- Ask students to visit a number of public multi-storey buildings with lifts. Discuss in advance which data may be relevant, and prepare a list for data collection.
- These data may include: the maximum weight and persons allowed; height, length, and width of the lift (related to both space and the amount of air available); and the number of floors in the lift (possibly related to the 'strength' of the lift needed.
- Once all the data are collected, encourage students to explore the patterns evident in the data.
- Encourage students to contact lift companies and find out to what extent the variables which they identified are actually relevant in the actual decisions about maximum limits.

28. Size of the Land

Links to Victorian Essential Learning Standards:

Level 5 Measurement, Chance & Data
Students measure length, perimeter, area, surface area, mass, volume, and capacity, using suitable units for these measurements in context. Students estimate the accuracy of measurements and give suitable lower and upper bounds for measurement values.

Mathematical focus:

- Proportional reasoning.

Related interview task:

Materials:

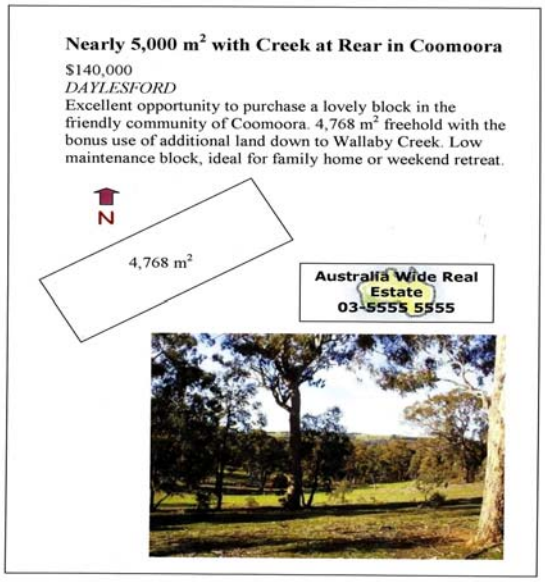
- Calculators
- Grid paper.

Instructions:

Hi Doug,

Can I call on your maths expertise? If, on paper, a block of land is 2 cm × 5.8 cm, and the overall dimensions are 4,768 m^2, how do I work out the actual length and width of the block? My Year 9 maths doesn't allow me to work this out! Hope you can help.

Thanks,
Peter Whitten
Editor, Rally Sport Magazine



As the letter indicates, a person has the actual area of a block of land, and the length and width of the same block when measured on a small plan (we assume the land is a rectangle or close to it). They want to know the actual length and width of the land in metres.

Encourage students to investigate what the dimensions could possibly be.

It is likely that students will use trial and error, possibly trying to find two numbers which multiply together to give 4768, without necessarily taking account of the proportions implied by the 2 cm by 5.8 cm (i.e. one dimension is 2.9 times the other).

28.1 Follow-up Activity: Size of Land

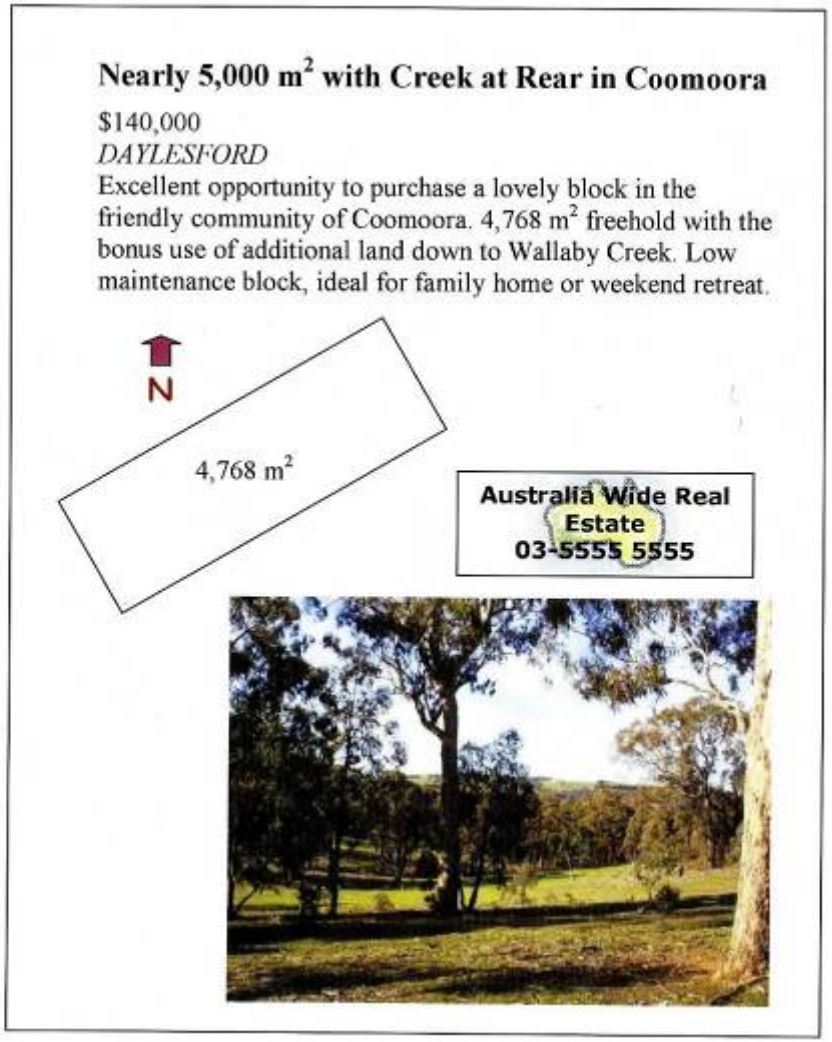
The ratio of the length of a certain rectangle to its width is 4 to 3. Its area is 300 cm^2. What are its length and width?

28.2 Activity Sheet: Size of the Land

Hi Doug,

Can I call on your maths expertise? If, on paper, a block of land is 2 cm × 5.8 cm, and the overall dimensions are 4,768 m^2, how do I work out the actual length and width of the block? My Year 9 maths doesn't allow me to work this out! Hope you can help.

Thanks,
Peter Whitten
Editor, Rally Sport Magazine



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Thanks to David Ebert from Malvern Central School for this contribution.

29. Knitting and Cooking

Links to Victorian Essential Learning Standards:

Level 4 Number
Students apply operations in practical contexts.
Students use estimates for computations and apply criteria to determine if estimates are reasonable or not.

Mathematical focus:

- Proportional reasoning.

Related interview task:

- 17: Pod Tunes or New Tunes?

Materials:

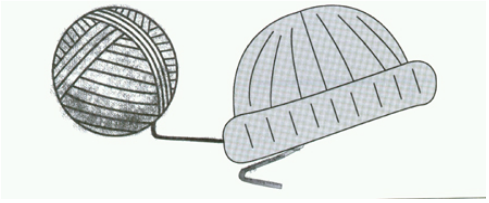
- Activity sheets.

Instructions:

The following two tasks focus on the appropriateness of multiplicative thinking in proportional situations, where many students will want to use additive thinking.

Knitting

It takes 10 balls of wool to make 15 beanies. How many balls of wool does it take to make 6 beanies?



10 balls	15 beanies
? balls	6 beanies

Students record their thinking then share their strategies with the class.

These numbers work well in exposing the inadequacy of additive thinking, as thinking of this kind will probably give one ball as an answer (five less than the six beanies or 9 less than the 10 balls), and students can see that the one ball for six beanies seems very much out of proportion to the 10 balls for 15 beanies.

Cooking

A recipe needs four cups of sugar and 6 cups of flour. If you decide to make a larger amount of the recipe, you'll need six cups of sugar. Then, how many cups of flour will you need for the recipe to work?

4 cups sugar	6 cups flour
6 cups sugar	? cups flour

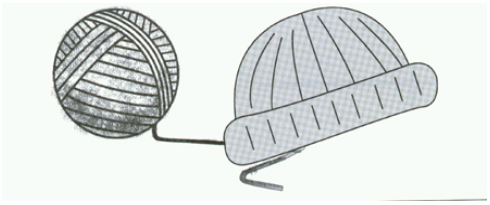
Students record their thinking then share their strategies with the class.

Similarly, (very common) additive thinking will yield eight as an answer (two more cups of sugar, so two more cups of flour). Sometimes extreme cases can help to show the inadequacy of additive thinking. Using the same logic, it would be zero cups of sugar and two cups of flour (would that work?) or 100 cups of sugar and 102 cups of flour (do you think that would work?).

29.1 Activity sheet: Cooking and Knitting

Knitting

It takes 10 balls of wool to make 15 beanies.
How many balls of wool does it take to make 6 beanies?



10 balls	15 beanies
? balls	6 beanies

Explain your reasoning.

Cooking

A recipe needs 4 cups of sugar and 6 cups of flour
You decide to make a larger amount of the recipe, and have 6 cups of sugar.
How many cups of flour will you need for the recipe to work?

4 cups sugar	6 cups flour
6 cups sugar	? cups flour

Explain your reasoning.

30. Washing Powder

Links to Victorian Essential Learning Standards:

Level 4 Number
Students apply operations in practical contexts, including the use of money.

Mathematical focus:

- Proportional reasoning.

Related interview task:

- 17: Pod Tunes or New Tunes?

Materials:

- An activity sheet per student.

Instructions:

Provide a task sheet for each student or have the students work in groups.

Pose the following problem:

You are shopping for washing powder and your favourite brand is on special. As shown, there are two different possibilities. For one kilogram of this powder you would pay $4 but for 1.5 kilograms you would pay $6.79. Which is the better buy?

Have the students record their thinking and share their strategies with the class.

30.1 Activity Sheet: Washing Powder

"Which is the better buy?"

$4.00
1 kg
20 Loads

$6.79
1.5 kg
30 Loads

Explain your reasoning: