PARTITIONING - THE MISSING LINK IN BUILDING FRACTION KNOWLEDGE & CONFIDENCE

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The Middle Years Numeracy Research Project (MYNRP), conducted in Victoria from November 1999 to November 2000, used relatively open-ended, 'rich assessment' tasks to measure the numeracy performance of approximately 7000 students in Years 5 to 9. The tasks value mathematical content knowledge as well as strategic and contextual knowledge and generally allow all learners to make a start.

For the purposes of the MYNRP, numeracy in the middle years was seen to involve

- core mathematical knowledge (in this case, number sense, measurement and data sense and spatial sense as elaborated in the National Numeracy Benchmarks for Years 5 and 7 (1997));
- the capacity to critically apply what is known in a particular context to achieve a desired purpose; and the
- actual processes and strategies needed to communicate what was done and why.

Data from the final stage of the project indicates that teachers working in professional teams in a coordinated and purposeful way do make a difference to student numeracy outcomes, particularly where there was concerted focus on 'good' mathematics teaching. That is, the use of problem solving, extended discussion, student explanations, rich assessment and a range of materials, tasks and activities. However, the research also suggests that systems and schools still face a significant challenge in recognising and dealing with the issues involved in teaching and learning for numeracy at this level.

'Hotspots' identified by the research suggest that we need to pay careful attention to the 'big ideas' in mathematics and foster students' capacity to critically reflect on their learning. In particular, it would appear that we need to focus on the development of place-value, multiplicative thinking, rational number ideas, and what is needed to help students progress to the next 'big idea'. One of the objects of teaching and learning mathematics is to help the learner create meaningful mental objects that can be manipulated, considered, and used flexibly and creatively to achieve some purpose. This requires that teachers are knowledgeable of developmental pathways and key learning trajectories, so that all students at all levels have the opportunity to learn the mathematics they need to progress to further study and effective, rewarding citizenship.

WHY FRACTIONS?

It is no longer acceptable that students leave school without the foundation knowledge, skills and dispositions they need to be able to function effectively in modern society. This includes the ability to read, interpret and act upon a much larger range of texts than those encountered by previous generations. In an analysis of commonly encountered texts, that is, texts that at least one member of a household might need to, want to, or have to deal with on a daily, weekly, monthly or annual basis, approximately 90% were identified as requiring some degree of quantitative and/or spatial reasoning. Of these texts, the mathematical knowledge most commonly required was some understanding of rational number and proportional reasoning, that is, fractions, decimals, percent, ratio and proportion. An ability to deal with a wide range of texts requires more than literacy – it requires a genuine understanding of key underpinning ideas and a capacity to read, interpret and use a variety of symbolic, spatial and quantitative texts. This capacity is a core component of what it means to be numerate.

FORMALISING FRACTION IDEAS IN THE MIDDLE YEARS:

1. Review fraction language and ideas using continuous (eg, chocolate bars, pizzas, fruit) and discrete fraction models (eg, children in the grade, eggs in an egg-carton).

Continuous	Discrete
eg, 2 and 3 quarter pizzas	*eg, half the grade to art, half to the library*
eg. 2 thirds of the netball court	*eg. 2 out of 12 eggs are cracked*

2. Practice naming and recording every-day fractions using oral and written language, distinguishing between the count (how many) and the part (how much)

eg, 3 fifths, 3 out of 5 equal parts;
2 wholes and 3 quarters (fourths)

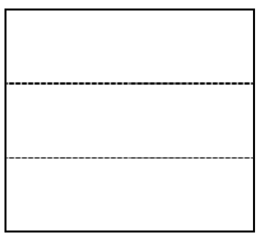
3. Use practical examples and non-examples to ensure foundation ideas are in place, that is,

- an appreciation of part-whole relationships and the requirement for equal parts,
- recognition of the relationship between the number of equal parts and the name of the parts (denominator idea), and
- an understanding of how fractions are counted (numerator idea)

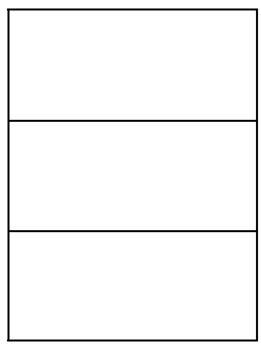
4. Introduce the 'missing link' - PARTITIONING - to support the making and naming of simple common fractions and an awareness that the larger the number of parts, the smaller they are

Use 'kindergarten squares' to investigate 'halving'. Explore and teach strategies for 'thirding' and 'fifthing' derived from paper folding/rope experiments and estimation based on reasoning about the size of the parts.

eg, Use Kindergarten Squares (coloured paper) to make and name fractions, construct diagrams



Think:
3 parts, smaller than 2 parts
1 third is less than 1 half
estimate
halve remaining part

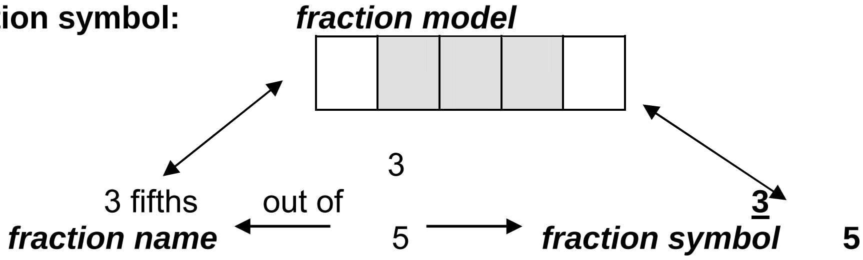


'bank fold' - three equal parts derived from paper-folding → derived diagram

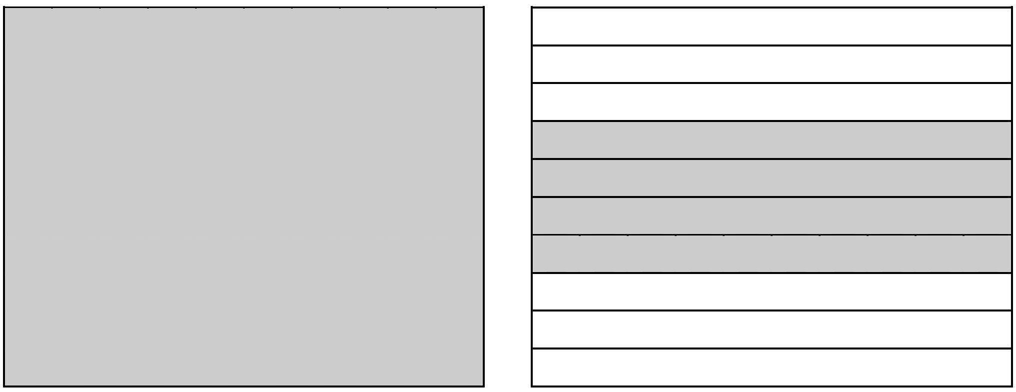
Develop 'fifthing' strategy by folding, observing and similar reasoning:

3 and 2 fifths

5. Introduce the fraction symbol:



6. Introduce tenths via diagrams. Make and name ones and tenths, introduce decimal recording as a new place-value part



1 and 4 tenths

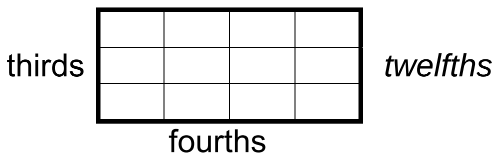
14 tenths

1 and $\frac{4}{10}$

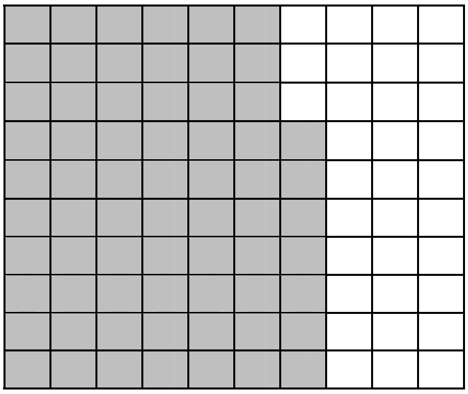
1.4

Treat as new place-value part. That is,
(a) establish new unit, 10 tenths is 1 whole, 1 whole is 10 tenths;
(b) make, name and record ones and tenths, pointing out the need for a marker to show where the ones begin; and
(c) consolidate understanding through comparing, ordering, counting forwards and backwards in place-value parts, and renaming.

7. Extend partitioning techniques to develop understanding that thirds by fourths produce twelfths, tenths by tenths give hundredths and so on.



8. Extend decimal fraction knowledge to hundredths using diagrams and metric relationships, introduce percentage.



67 hundredths

0 ones
6 tenths 7 hundredths

0.67

67 per cent, 67%

Need to treat as new place-value part

9. Explore fraction renaming (equivalent fractions) using paper-folding, diagrams, fraction trays (fraction wall) and games.

Establish the generalisation that if the number of parts (denominator) increases by a certain factor then the number of parts required (numerator) increases by the same factor

10. Link thousandths to metric relationships. Rename measures (grams to kilograms etc). Use partitioning strategies to show where decimals live.

Eg, 4.376 lives between 4 and 5…partition into tenths …it lives between 4.3 and 4.4 …apply metaphor of a magnifying glass to 'stretch' out line between 4.3 and 4.4, partition into ten parts to show hundredths …it lives between 4.37 and 4.38 …repeat process to show thousandths and identify where 4.376 lives.

11. Introduce addition and subtraction of decimals and simple fractions to support place-value ideas, extend to multiplication and division by a whole number.

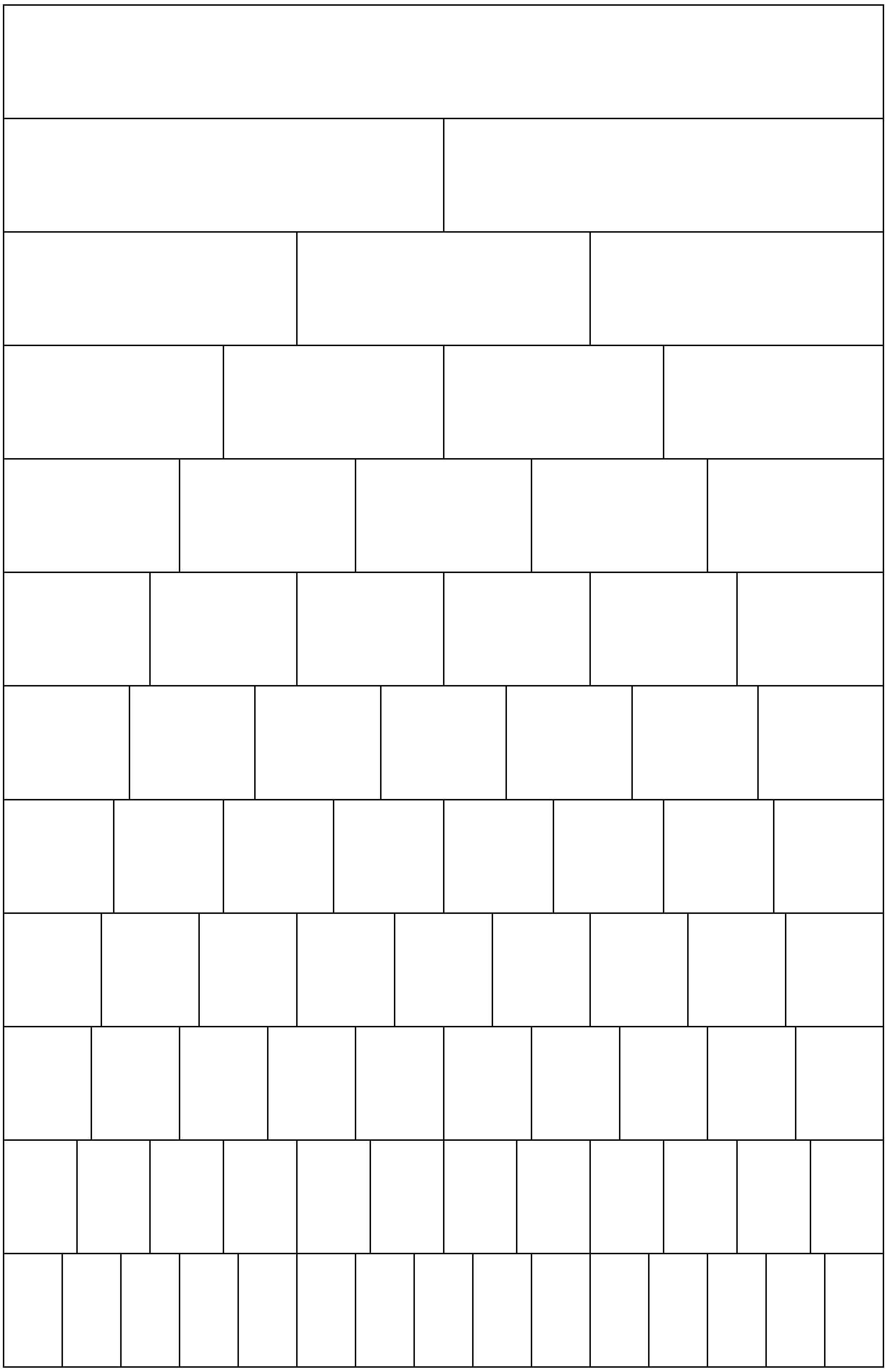
$$\begin{array}{r} 4.26 \\ \underline{+\ 7.38} \end{array} \qquad \begin{array}{r} 5\frac{2}{3} \\ \underline{-3\frac{5}{8}} \end{array}$$

References:

- Booker, G, Bond, Briggs, J. & D, Davey, G. (1997) *Teaching Primary Mathematics*. Melbourne: Pearson
- Cahn, R (1999) Maths Fun Pack – Computer games for Years 5 – 9 (available from AAMT, www.aamt.edu.au)
- Curriculum Corporation (2001*) Maths Project 300*. Web-based resource available from Curriculum Corporation (see www.curriculum.com.au) – eg*, Fraction Estimation*
- Graham, N (1998) *Dice and Spinners*, Claire Publications: UK (also available from OLM)
- Lovitt, C. & Clarke, D. (1985*) Mathematics Curriculum and Teaching Programme (MCTP) Activity Banks Volumes I and II*, Curriculum corporation: Melbourne
- Lovitt, C. & Lowe, I. (1993) Chance & Data Investigations – MCTP, Curriculum Corporation: Melbourne

How many wholes can you make?

Choose a card that says 'how many' and a card that says 'how much'. Colour in or cover the amount. The winner is the person with the greatest number of wholes.



1 whole
2 halves
3 thirds
4 fourths
5 fifths
6 sixths
7 sevenths
8 eighths
9 ninths
10 tenths
12 twelfths
15 fifteenths

If you draw a blank you can choose your part.

Card Master:

1	1		
2	2	halves	halves
3	3	thirds	thirds
4	4	fourths	fourths
5	5	fifths	sixths
1	1	fifths	sixths
2	2	eighths	ninths
3	3	eighths	ninths
4	4	tenths	halves
5	5	thirds	fourths

MIXING FRUIT JUICE

Every year the seventh grade students at Langston Hughes School go to an outdoor education camp. During the week-long trip, the students study nature and participate in recreational activities. Everyone pitches in to help with the cooking and cleanup.

Ari and Maria are in charge of making orange juice for all the campers. They make the juice by mixing water and orange juice concentrate. To find the mix that tastes best, Ari and Maria decided to test some recipes on a few of their friends.

Ari and Maria tested four juice mixes.

Mix A
2 cups concentrate
3 cups cold water

Mix B
1 cup concentrate
4 cups cold water

Mix C
4 cups concentrate
8 cups cold water

Mix D
3 cups concentrate
5 cups cold water

1. Which recipe will make the juice that is most "orangey"? Explain your answer
2. Which recipe will make the juice that is the least "orangey"? Explain your answer.
3. Assume that each camper will get half a cup of juice. For each recipe, how much concentrate and how much water are needed to make juice for 240 campers? Explain your answer.

Source: Lappin et al, Connected Mathematics Project, 1998

TARGET PRACTICE

You need: 3 or 4 ten-sided dice (different colours are useful but not essential) and a copy of the game sheet for each student.

To play:

1. Take turns to throw the dice.
2. Choose 2 or 3 of the numbers thrown to make a number as close to the target number as possible.
3. Mentally calculate 'how close' and record in the space provided.
4. The winner is the player with the smallest total.

Numbers thrown	TARGET	Number Made	How close?

TOTAL	

PARTITIONING – THE MISSING LINK IN DEVELOPING FRACTION IDEAS

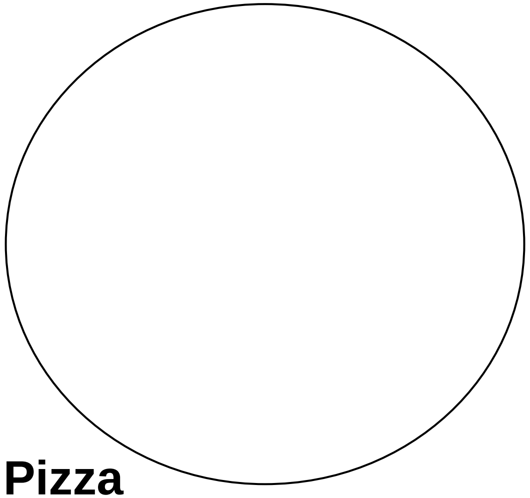
Shade and partition to show 3 and 2 thirds

Partition to show 3 eighths

Show 6 and 3 tenths

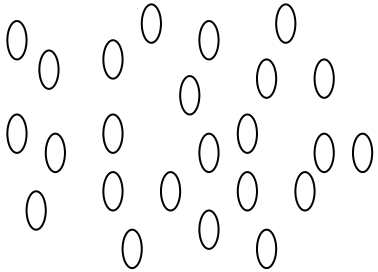
Partition to show 2 fifths

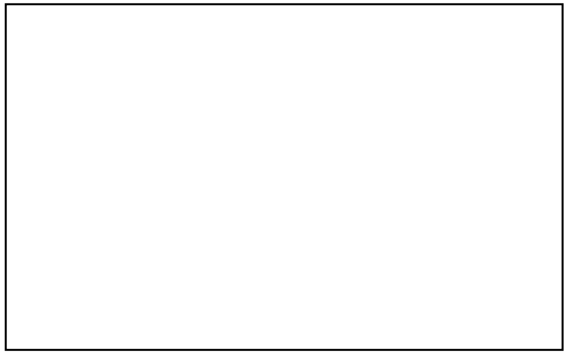
Partition to show 1 sixth



Pizza

24 eggs





Tray of Lasagna

Partition to show $3\frac{3}{4}$, 4.8, $5\frac{1}{3}$, and 6.2