MULTIPLICATIVE THINKING

Prepared by Dr Dianne Siemon, Associate Professor of Mathematics Education, RMIT
April, 2005

Multiplicative thinking is characterised by:

- a capacity to work flexibly and efficiently with an extended range of numbers (for example, larger whole numbers, decimals, common fractions, ratio, and per cent),
- an ability to recognise and solve a range of problems involving multiplication or division including direct and indirect proportion, and
- the means to communicate this effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).

In short, **multiplicative thinking** is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts.

For example, from this:

> 3 bags of sweets. 8 sweets in each bag. How many sweets altogether?

to this and beyond:

> Juli bought a dress in an end-of-season sale for $49.35. The original price was covered by a 30% off sticker but the sign on top of the rack said "Now an additional 15% off already reduced prices". How could she work out how much she had saved? What percentage of the original cost did she end up paying?

For example,

Lunch Orders

A school canteen offered 4 types of bread or rolls, 6 different sandwich fillings, 3 flavours of milk, 5 choices of health bars or snacks, and 4 choices of fruit? How many different lunch orders comprising at least a sandwich or roll, a drink, and a piece of fruit were possible?

Sand Pit

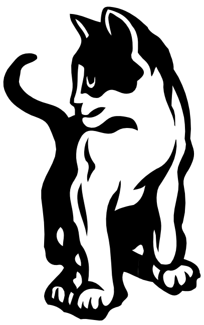
Sand is purchased in cubic metres. Exactly how many cubic metres of sand would be needed to fill a sand-pit that is 2.4 metres long, 160 cm wide and 33 cm tall?

Butterflies

2 drops of nectar are needed to feed 5 butterflies. How many butterflies could be fed with 12 drops of nectar?

Feral Cats

35 feral cats were found in a 146 hectare nature reserve. 27 feral cats were found in a 103 hectare reserve. Which reserve had the biggest feral cat problem?



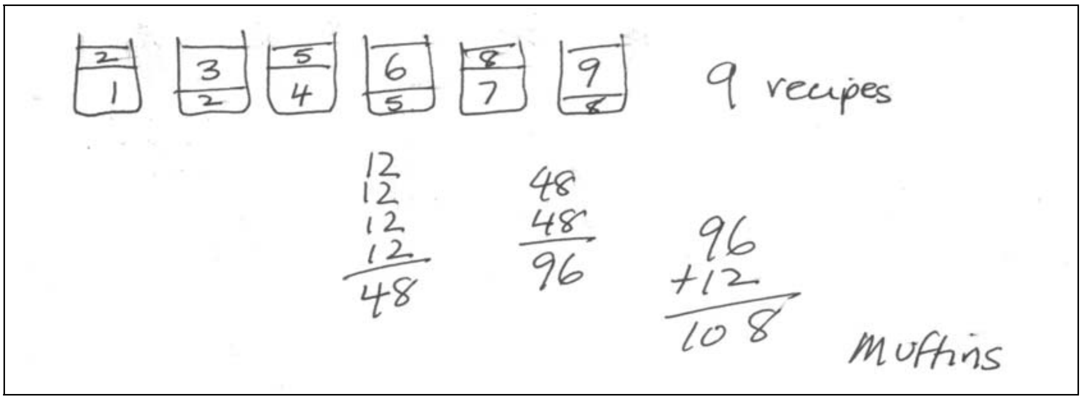
Enlargements

To fit a particular display space, a client wanted the width of an A4 image enlarged in the ratio 1:2.6 and the height enlarged in the ratio 1:3.7. The photographic shop based their charges on area. How much was the client asked to pay for the enlarged image if the cost of an A4 image was $12.45

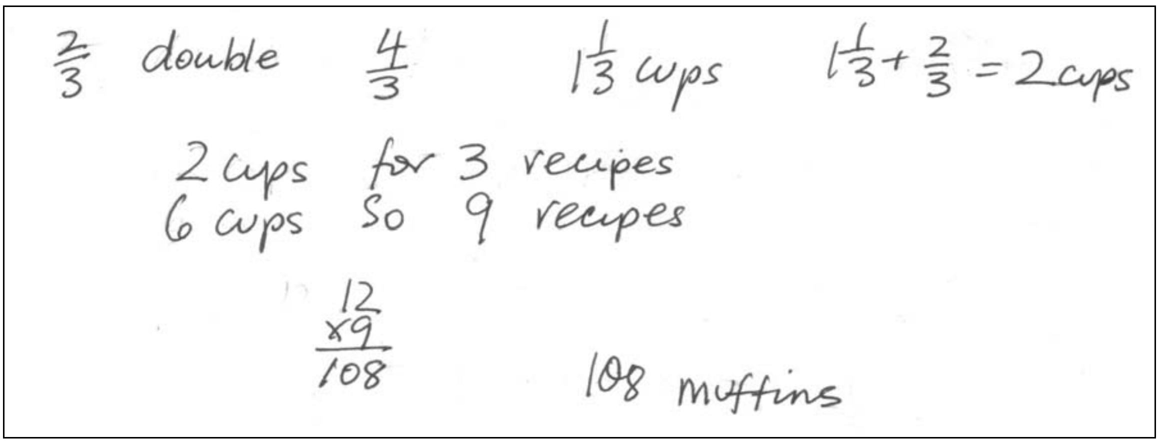
Multiplicative thinking is evident in non-additive solutions to problems like the following:

> A muffin recipe requires 2/3 of a cup of milk. Each recipe makes 12 muffins. How many muffins can be made using 6 cups of milk?

A solution which added 2/3 repeatedly to find that this can be done nine times to get 6 cups and then added 12 nine times is indicative of ***additive thinking***.

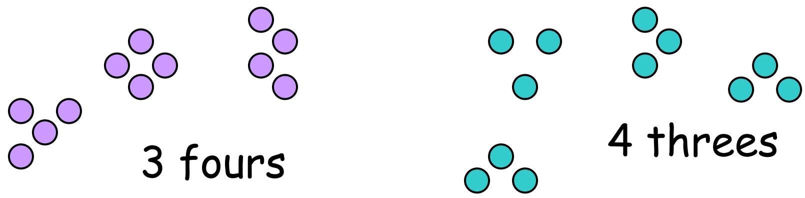


A solution which determined that 9 recipes could be made on the basis that 3 quantities could be made from 2 cups of milk and then multiplied 9 by 12 to get 108 muffins is indicative of ***multiplicative thinking***.



MULTIPLICATIVE CONCEPTS

Groups of:



Has led to:

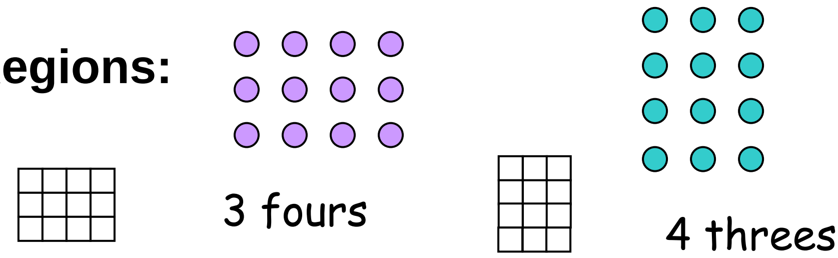
1 x 4
2 x 4
3 x 4
4 x 4
5 x 4
...

and:

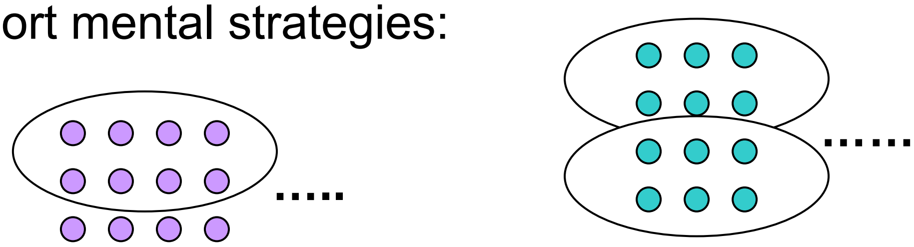
1 x 3
2 x 3
3 x 3
4 x 3
5 x 3
...

That is, to **counting groups** of equal size, repeat addition and quotation ('guzinta') division

Arrays and Regions:



Support mental strategies:



3 of anything:
Double the group and 1 more group

4 of anything:
Double the group and double again (double double)

Note the shift to an **equal number of groups of different size**, eg,

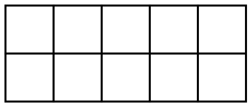
3 ones, 3 twos, 3 threes, 3 fours, 3 fives ….

No longer counting groups, but thinking instead of the number of groups as a **factor.**
Supports partition (sharing) division.

3 x 1
3 x 2
3 x 3
3 x 4
3 x 5
...

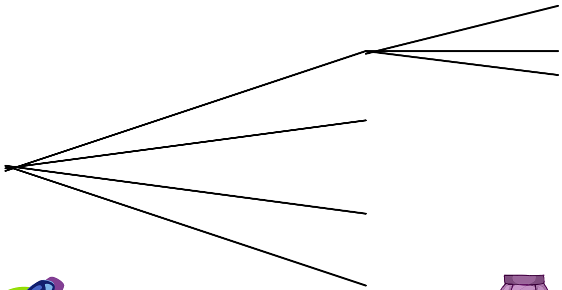
The region idea also supports **fraction renaming,** eg,

2 parts
halves



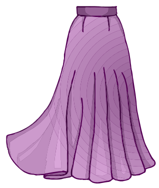
By 5 parts *fifths* → **tenths**

Cartesian product or 'for each' idea:



How many combinations of tops and skirts?

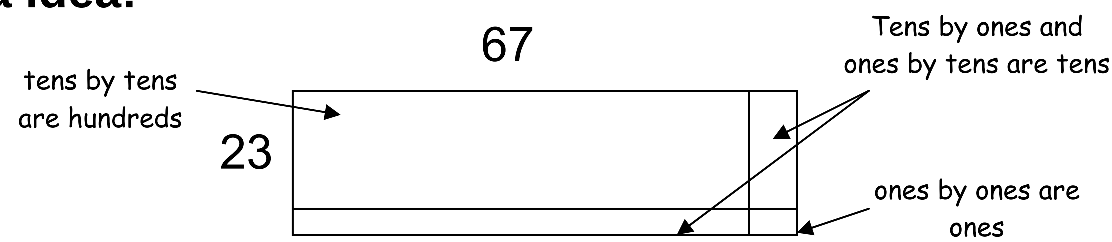
4 tops



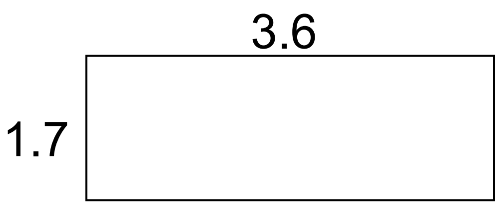
3 skirts

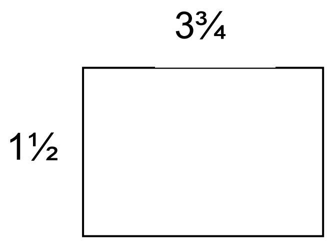
Supports multiplication involving fractions and decimals, proportion, ratio and per cent

Area idea:



Supports multiplication of 2-digit by 2-digit numbers





and multiplication involving decimals and fractions.

BUILDING FRACTION KNOWLEDGE AND CONFIDENCE

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1. Review fraction language and ideas using ‘real-world’ CONTINUOUS and DISCRETE materials.

Continuous	*Discrete*
eg, chocolate bars, pizzas, sandwiches, string, paper streamers, plasticene, water	*eg, children in the grade eggs in an egg-carton, smarties, marbles,*

2. Practice naming and recording every-day parts and wholes using oral and written language (NOT SYMBOLS), for example,

2 thirds of the netball court
half the class to art,half to the library
2 and 3 quarter pies
1 and half dozen eggs
5 eighths of the pizza

Discuss difference between ‘how many’ and ‘how much’

3. Use examples and non-examples to ensure students understand that EQUAL PARTS are required, for example,

cut up oranges or pizzas and share discrete quantities such as ‘sweets’ to demonstrate fair/unfair shares

4. **Explore paper folding and HALVING** to re-affirm equal parts and consider what happens as the number of parts increases:

AS THE NUMBER OF PARTS INCREASES, THE SIZE OF THE PARTS DECREASES

And how these parts might be named, this ultimately leads to another generalization:

THE NUMBER OF PARTS NAMES THE PART

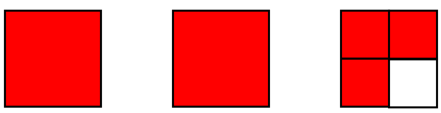
Prepare a chart to record the names of fractions in the Halving family:

No. of parts	NAME
1	whole
2	halves
4	quarters
8	eighths
16	sixteenths
32	thirty-seconds
64	sixty-fourths
128	…

Pattern?

Note link to ordinal number names and use of plurals to avoid confusion, eg, “eighths” as opposed to “eighth in line”.

Use paper streamers, squares and rectangles to **make and name** proper and mixed fractions in the halving family.



5. Introduce PARTITIONING strategies more formally.

> On the basis that counting and colouring parts of someone else's fraction diagram is next to useless, I believe we need to involve students in making and naming their own fraction models and representations and that this is best achieved by explicitly teaching a small range of partitioning strategies that I refer to as *halving*, *thirding* and *fifthing*.

(a) Explore strategies for THIRDING AND FIFTHING using paper streamers, squares and rectangles

(b) Describe and justify folding techniques

(c) Use paper streamers, squares and rectangles to **make and name** proper and mixed fractions from the THIRDING and FIFTHING families.

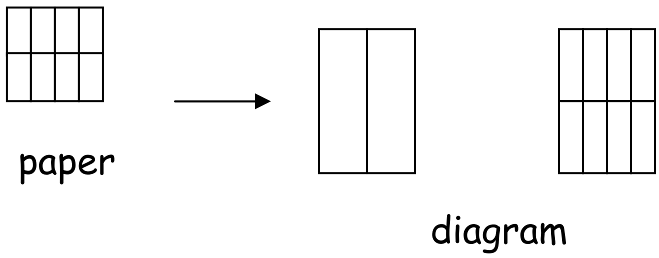
Find out everything you can about 2 and 2 thirds

(d) Extend the fraction naming chart:

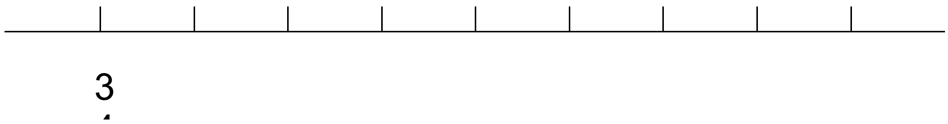
No. of parts	NAME
1	whole
2	halves
3	thirds
4	quarters
5	fifths
…	…

(e) Explore what happens when strategies are combined, eg, *halving* and *thirding,* or *thirding* and *fifthing.*

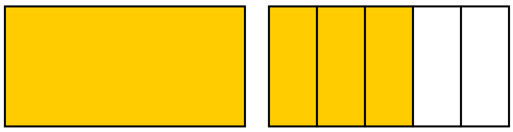
(f) Use paper models to generate **fraction diagrams and number line representations** without measuring.



What did the first fold do?
… the second? … the third?
… and so on



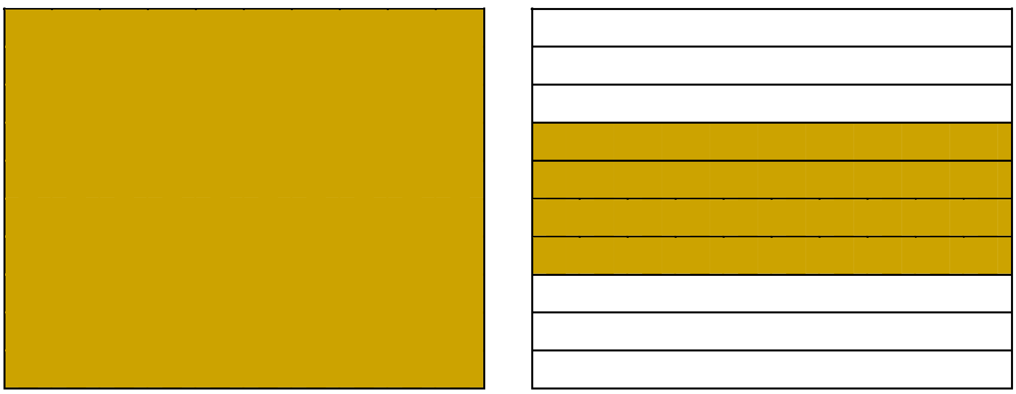
6. Introduce the fraction symbol.



1 whole and 3 out of 5 equal parts … 1 and $^{3}/_{5}$ … $1^{3}/_{5}$

Encourage students to draw and label their own fraction diagrams.

7. Introduce tenths as a new place-value part via diagrams. Make, name and record ones and tenths.

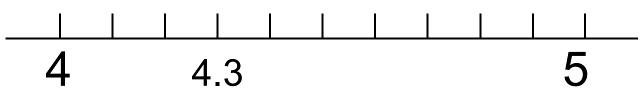


1 and 4 tenths

$1^{4}/_{10}$

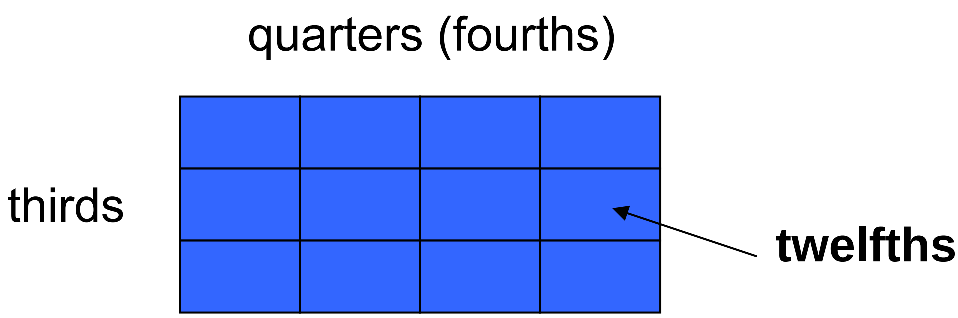
1.4

Use partitioning strategies and open number lines to show ones and tenths, for example,

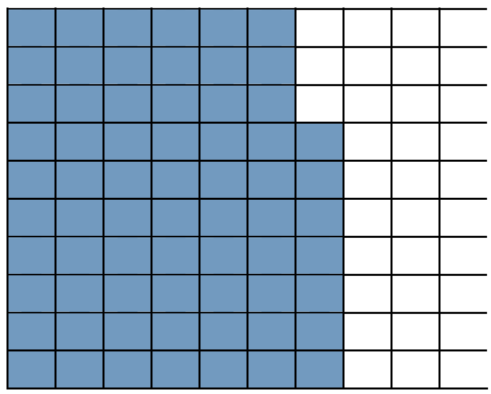


Consolidate by comparing, ordering/sequencing, counting forwards and backwards in ones and tenths, and renaming.

8. Extend partitioning techniques to develop an understanding that thirds by fourths give twelfths, tenths by tenths give hundredths and so on.



9. Introduce hundredths as a new place-value part via diagrams and metric relationships. Introduce percentage.



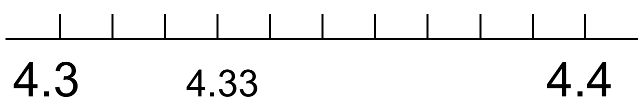
67 hundredths

0 ones 6 tenths 7 hundredths

0.67

67 per cent, 67%

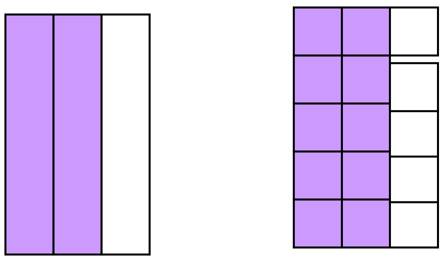
Use partitioning strategies and open number lines to show ones, tenths and hundredths, for example,

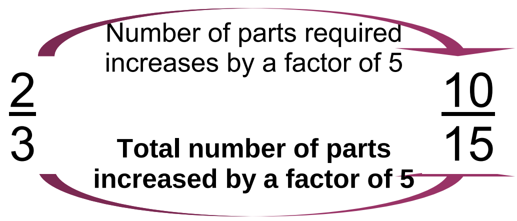


Consolidate by comparing, ordering/sequencing, counting forwards and backwards in ones and tenths, and renaming.

10. Explore fraction renaming (equivalent fractions) using paper-folding, diagrams and fraction kits to arrive at the generalization:

IF THE NUMBER OF PARTS IS INCREASED BY A CERTAIN FACTOR THEN THE NUMBER OF PARTS REQUIRED IS INCREASED BY THE SAME FACTOR





11. Introduce addition and subtraction of decimals and simple fractions to support place-value ideas, extend to multiplication and division by a whole number.

$$\begin{array}{r} 3.8 \\ +\ 4.7 \\ \hline \end{array} \qquad \begin{array}{r} 7\frac{3}{4} \\ +\ 5\frac{2}{3} \\ \hline \end{array} \qquad \begin{array}{r} 8\frac{1}{3} \\ -\ 3\frac{5}{8} \\ \hline \end{array}$$