Numeracy Improvement Guide for School Leaders

Victoria State Government | Education and Training

Contents

Introduction

This Guide is designed to help school leaders catalyse change in numeracy and mathematics outcomes in your school, framed around the core elements and dimensions of the Framework for Improving Student Outcomes 2.0.

This change is needed to ensure that every Victorian student leaves school strongly numerate and with the mathematics knowledge, skills, capabilities, and dispositions they need to support their chosen pathways and to make complex decisions in a mathematics-rich world.

The Guide brings together new and previously published knowledge and research about excellence – and issues – in numeracy and mathematics education. In bringing these things together into one place, the Guide aims to provide coherence, as well as an agreed way forward for the system – about what works in numeracy improvement.

A suggested FISO 2.0 Improvement Cycle accompanies the Guide (Section 1). The Cycle has been included to help school leaders identify specific priorities and actions for mathematics and numeracy improvement as part of annual implementation planning.

By using the Guide these priorities and actions should have a sharp focus, addressing the underlying causes of low achievement and growth in student learning.

Structure

This Guide is divided into six parts:

- Section 1 outlines an approach **school leaders** can take to identify specific priorities and actions for mathematics and numeracy improvement in your school. This approach is framed around a **FISO 2.0 Improvement Cycle** and will direct you to appropriate sections of the Guide.
- Section 2 covers the foundations and should be read by **school leaders** and **leaders of numeracy and mathematics**. The foundations highlight the need for all teachers to be supported to develop the numeracy capability of their students. The foundations also include a need to ensure that mathematics teaching is proficiency-rich, helping develop independent maths learners. All teachers should be engaged in dialogue around the content of Section 1.
- Section 3 should also be read by **school leaders** and **leaders of numeracy and mathematics**. It outlines the key workforce and student considerations that need to be attended to in order to improve numeracy and mathematics outcomes. It also describes the characteristics of quality numeracy and mathematics teaching, learning and assessment. Where relevant, leaders should engage teaching teams in dialogue around the content of Section 3, using the adult learning architecture of the school (e.g., Professional Learning Communities, Professional Learning Teams).
- Sections 4, 5 and 6 identify key priorities for each stage of student learning and development in numeracy and mathematics: Foundation to Level 2, Levels 3 and 4, Levels 5 to 9 and Levels 10 to 12. Advice has been written primarily for **leaders of numeracy and mathematics**, to support them to engage teaching teams and individual teachers in dialogue about planning and practices. **School leaders** will also benefit from engaging with the content, particularly if analysis of school data and other evidence of learning is revealing areas for improvement in specific stages of schooling.
- A References section which may be useful for further reading and exploration.

Features

Each section of this Guide has the following key features to aid readability:

- **Key messages** that summarise each section, for those looking for an entry point to the content
- **Key resources** that will help schools enact the guidance provided
- **Reflective prompts** for school leaders and for use with teachers to help them engage with and reflect on the content and plan further action
- **Links to FISO 2.0** core elements and dimensions
- **Link summaries** to make it easier for readers to access what may be helpful to them in their context.

Where should I start?

A recommended approach for engaging with this Guide is for school leaders and leaders of numeracy and mathematics to gather and step through a FISO 2.0 Improvement Cycle. Section 1 outlines an approach schools may like to take.

What additional support can I access to unpack and use this Guide?

Senior Education Improvement Leaders (SEILs) and Education Improvement Leaders (EILs) are familiar with the contents of this Guide and can provide additional support. Your SEIL and EIL also have access to several short videos that may be useful in introducing the Guide to your staff.

A large suite of professional learning in numeracy and mathematics is available from the department and the Victorian Academy of Teaching and Leadership. This suite is aligned to the contents of this Guide. If you are unfamiliar with what is on offer, speak to your SEIL and EIL.

1: Suggested FISO 2.0 Improvement Cycle

This section takes you through the steps in the FISO 2.0 Improvement Cycle to help you identify specific priorities and actions for mathematics and numeracy improvement as part of annual implementation planning.

Begin with **Evaluate and Diagnose**. This could involve the following steps:

1. Gather data/evidence, and discuss and agree on what you already know about:
    a. your *students* (individuals, cohorts and across the school) and their performance, progress and dispositions in numeracy and mathematics, in 2022, 2021 and over time (e.g., 5-year trends); look specifically for:
        i. priority cohorts that are not tracking as well in system-level data, including Koorie students, girls, students from low socio-economic backgrounds, students who are refugees, students with disability, students who are in out-of-home care, and students from rural/regional areas (note: that these may not be represented in your school's data) – see 3.1.3 Priority cohorts for further information
        ii. students who are making less than expected growth in learning (i.e., less than one level of learning in one calendar year)
        iii. learning gaps/misconceptions, which are often apparent in disaggregated data (e.g., item level) – see 3.3.1 What students know and can do for further information/tools to use
        iv. areas of variance within the school
        v. other related data (e.g., attendance).
    b. your *teachers and support staff* and their confidence and capability in leading or supporting the numeracy and mathematics learning of their students – see 3.1.1 Teacher belief orientations and dispositions for further information
    c. *you and other leaders* in the school and your confidence and capability in leading and sustaining whole school improvement in numeracy and mathematics education
    d. *the broader school community* and their aspirations for their children as mathematics learners, as well as their past experiences and dispositions towards numeracy and mathematics.

2. Across *students*, *teachers, and support staff*, *yourself and other leaders* and *the broader school community*, identify and note:
    a. strengths in numeracy and mathematics
    b. what needs strengthening
    c. gaps in knowledge and understanding that might need further investigation.

3. Review the contents page of this Guide, and identify and note:
    a. sections/content that may help you with your school with its strengths, stretching thinking/practice
    b. sections/content that may help you with what your school needs to strengthen, offering actions for consideration.

4. Read relevant sections and discuss them as a group using an agreed protocol (e.g. The Final Word protocol or The Making Meaning protocol, from the School Reform Initiative). Note key insights and implications that could inform next steps.

Move to **Prioritise and Set Goals**. This could involve the following steps:

1. Considering learnings across Steps 1-4, and the priorities identified in the Guide in Section 4 (Foundation to Level 4), Section 5 (Levels 5 to 9) and Section 6 (Levels 10 to 12), what priorities have emerged for further action for:

    a. *students*
    b. *teachers and support staff*
    c. *you and other leaders*
    d. *the broader school community*.

2. Considering these priorities:

    a. what expectations/goals will you set for improvement?
    b. how will you communicate these to relevant stakeholders?
    c. how will you ensure that these stakeholders:
        i. know and understand these expectations?
        ii. commit to the further learning and action required for improvement to occur?

3. Considering these priorities and expectations, and sections/content from the Guide:

    a. what are the approaches/key improvement strategies you want to implement to address the challenges your school is facing in numeracy and mathematics?
    b. how will this impact on students' learning growth?
    c. what resources are required in the short, medium, and long-term to ensure effective and sustainable implementation?
    d. what are realistic timeframes for this improvement work to be started, embedded, *and* extended, noting that this may take several months or even years?

Progress to **Develop and Plan**. This could involve the following steps:

1. Regarding the approaches/key improvement strategies identified at Prioritise and Set Goals, above, decide and document in your Annual Implementation Plan **who** will lead each approach/action, **what** they will be leading (in detail), for **whom**, **how** and **by when**.

2. Augment this plan with a professional learning approach which is documented in School Planning Online Tool (SPOT) which identifies professional learning needs that will need to be addressed for improvement occur, for:

    a. *students*
    b. *teachers and support staff*
    c. *you and other leaders.*

3. Document how will you address these needs:

    a. through existing structures and opportunities, the school is engaged in?
    b. through new opportunities, for example, those offered by the department and the Victorian Academy of Teaching and Leadership?
4. How will you ensure that you and other leaders engage actively as instructional leaders in professional learning?

5. What challenges and barriers do you anticipate, and how will you respond/adapt?

Finish with **Implement and Monitor**. This could involve the following steps:

1. How will you monitor your new approaches/strategies for numeracy and mathematics improvement, for example, through your Professional Learning Community/ies? Consider:

    a. what you will look at, when and how?
    b. the use of simple audit tools and checklists, work samples and learning walks to regularly assess whether practices have changed as agreed.

2. How will you ensure that a focus on students and their learning is maintained?
3. How will you manage expectations of staff at the initial stage of implementation?
4. How will you communicate to relevant stakeholders as you progress?
5. How will you know if and when you need to adjust your approach?

2: Understanding mathematics and numeracy education

Key messages

- Numeracy and mathematics are not synonymous.
- In Victorian schooling, mathematics is defined as the Victorian Curriculum: Mathematics, one of the eight learning areas of the Victorian Curriculum.
- Numeracy involves:
  - recognising and understanding the role of mathematics in the world
  - having the dispositions and capacities to use mathematical and statistical knowledge and skills purposefully.
- Mathematics programs need to be numeracy-rich: with a real-life context, application of mathematical knowledge, use of tools, promotion of positive dispositions, and involving a critical orientation.
- Being numerate requires experience in the use of mathematics beyond the mathematics classroom.
- Teachers of all school subjects have an important role to play in developing the numeracy capabilities of students.

2.1 What is mathematics?

"We do not know who first started applying mathematics to scientific study, but it is plausible that it was the Babylonians, who used it to discover the pattern underlying eclipses, nearly 3,000 years ago. But it took 2,500 years and the invention of calculus and Newtonian physics to explain the patterns" *(Watson, 2022, p. 1).*

Questions about the nature of mathematics have been the subject of often heated debate for some 2,300 years.

Number, measurement and geometry, statistics and probability are common aspects of most people's mathematical experience in everyday personal, study and work situations. Equally important are the essential roles that algebra, functions and relations, logic, mathematical structure and working mathematically play in people's understanding of the natural and human worlds, and the interaction between them (VCAA 2017).

In Victorian schooling, mathematics is defined as the Victorian Curriculum: Mathematics, one of the eight learning areas of the Victorian Curriculum.

The Victorian Curriculum: Mathematics is organised by the three strands of Number and Algebra, Measurement and Geometry, and Statistics and Probability, with each strand organised by sub--strands which provide a focus and a sequence for the development of related concepts and skills within strands and across levels (VCAA 2017).

The proficiencies of Understanding, Fluency, Problem Solving and Reasoning are fundamental to learning mathematics and working mathematically and are applied across all three strands (VCAA 2017).

Mathematics is at the foundation of numeracy capability. You can't have numeracy without mathematics.

2.2 What is numeracy?

The Australian Association of Mathematics Teachers (AAMT) states that:

> **"To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life"** (DEETYA, 1997, p. 15).

This definition has become widely accepted in Australia and has formed the basis for much numeracy-related research and curriculum development.

The Victorian Curriculum and Assessment Authority has expanded on the AAMT definition:

> **"Numeracy is the knowledge, skills, behaviours and dispositions that students need in order to use mathematics in a wide range of situations"** (VCAA, 2020).

In the department's Literacy and Numeracy Strategy, numeracy is defined as:

> **"…the knowledge, skills, behaviours and dispositions that students need in order to use mathematics in a wide range of situations. It involves recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully"** (DET, 2018, p. 6).

Numeracy involves recognising and understanding the role of mathematics in the world and having the dispositions and capacities to use mathematical knowledge and skills purposefully.

Numeracy is not 'mathematics light.' It is mathematics that is challenged by being applied beyond the classroom.

There is now strong evidence that other areas of development – such as motivation and resilience, perseverance and reasoning – support achievement in numeracy as well.

2.2.1 Connecting numeracy and mathematics

Numeracy and mathematics are not synonymous (DEETYA, 1997) even though the terms *numeracy*, *mathematics* and *mathematical literacy* are often used interchangeably (Groves et al., 2006). A

Key resource

Mathematics Monograph: Critical connections between numeracy and mathematics

by Dave Tout, Senior Research Fellow, Numeracy and Mathematics, Tertiary and Vocational Assessment Services, ACER

This monograph explores research that increasingly shows that life and work in the 21st century requires higher levels of mathematics and numeracy of its citizens. It posits that numeracy and mathematics are intrinsically connected and both are needed in an ever changing, globalised and technological world. The paper also examines the numeracy skills required of students to develop and leave school with, and how we can better address these in our teaching and learning.

numerate person can use and apply mathematics in a range of contexts outside the mathematics classroom.

For Perso, mathematics is learned as a body of knowledge, which can then be applied to a context through determining that “some mathematics will help here” (Perso, 2011, p. 32). Perso offered the following example of a task that assesses a mathematical skill, rather than an application of mathematics, since most of the decisions are made for the student:

> *Use the Pythagorean Theorem to find the height of a tree which casts a shadow 2.35 metres long and has an angle of elevation with the ground of 53 degrees (give your answer to two decimal places).*

Similarly, a typical primary textbook problem might ask students to:

> *Measure these angles using a protractor.*

In this example, while the students may be applying a skill to the task, a context is not provided.

Numeracy-rich learning activities can be recognised by drawing upon the conceptual framework of Goos, Geiger and Dole (2012; also discussed in Goos et al, 2019). In this framework, numeracy is conceptualised as comprising four elements and an orientation:

- **Element 1:** Attention to real-life contexts (citizenship, work, and personal and social life)
- **Element 2:** Application of mathematical knowledge (problem solving, estimation, concepts, and skills)
- **Element 3:** Use of tools (representational, physical, and digital)
- **Element 4:** The promotion of positive dispositions towards the use of mathematics to solve problems encountered in day-to-day life (confidence, flexibility, initiative, and rewarding risk)
- **Orientation:** A critical orientation to interpreting mathematical results and making evidence-based judgements.

2.2.2 Numeracy across the curriculum

In Australia, it has been widely recognised that teachers of all school subjects have an important role to play in developing the numeracy capabilities of students (Bennison, 2014).

While mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom (Goos et al, 2019).

Strengthening numeracy across the curriculum does not mean that every teacher must become a teacher of the Victorian Curriculum: Mathematics. It means that every teacher must be an expert teacher of their own subject (e.g., geography, science, music) which will necessarily mean taking special care to utilise, scaffold and strengthen students’ numeracy skills in that subject.

2.2.2.1 Numeracy Learning Progressions

Teachers who need extra support to scaffold their students’ numeracy capability can look to the Numeracy Learning Progressions for extra support.

The Numeracy Learning Progressions have been designed to help teachers develop a detailed and nuanced understanding of student numeracy development, especially in the early years, by outlining a set of steps within areas of numeracy learning (VCAA, 2022).

Teachers can use the Numeracy Learning Progressions to:

- support the development of understanding of the numeracy demands and opportunities within their learning areas
- provide support for teaching subject specific numeracy.

At the level of the whole school, they can be used to:

- enable collaboration in building a shared understanding of numeracy and numeracy development
- promote and support professional learning in numeracy
- provide a common language for discussing students' numeracy progress
- focus explicit numeracy teaching and learning planning and implementation across the curriculum.

Key resource

Numeracy Across the Curriculum Resources: Levels 7 to 10 (navigate to 'Numeracy across the curriculum')

Developed in partnership with the Victorian Curriculum and Assessment Authority and Monash University, this set of resources helps teachers of learning areas other than mathematics across Levels 7 to 10:

- *recognise numeracy in their learning area/s*
- *embed numeracy in their learning area*
- *assess numeracy learning*
- *deal with challenges and dilemmas using strategies recommended by experts.*

Resources include lesson plans and teaching ideas and make explicit links to the Numeracy Learning Progressions, numeracy skills required of students to develop and leave school with, and how we can better address these in our teaching and learning.

Reflective prompts	
School leaders	To what extent is there observable evidence (planned and enacted) of an attention to a numeracy focus: i) in mathematics, and ii) in other curriculum areas at your school? How do teachers and support staff in your school understand the relationship between numeracy and mathematics? How might the Mathematics Monograph: Critical connections between numeracy and mathematics be utilised to affirm or challenge thinking? How might the Numeracy Across the Curriculum Resources: Levels 7 to 10 be utilised to plan incremental change? In reflecting on this chapter, what are some initial steps your school might take to strengthen numeracy education? (note: you may wish to add to these steps as you consider other sections and questions in this document) How is your school currently using the Numeracy Learning Progressions? How might they be used to support whole student improvement in numeracy?

For leaders to use with teachers	How numeracy-rich are the learning and teaching programs you design, plan, teach and assess? Considering your planning documents, what are some explicit opportunities for numeracy capability to be developed by your students? What support might you need and who can provide it? Considering the numeracy framework from Goos, Geiger and Dole, what element could you explore to strengthen the learning experiences for your students?

FISO 2.0

Core element	**Dimension**
Leadership	*The strategic direction and deployment of resources to create shared goals and values; high expectations; and a positive, safe and orderly learning environment*
Teaching and learning	*Documented teaching and learning program based on the Victorian Curriculum and senior secondary pathways, incorporating extra-curricular programs*

Link summary

- Mathematics Monograph: Critical connections between numeracy and mathematics
- Numeracy Across the Curriculum Resources: Levels 7 to 10 (navigate to 'Numeracy across the curriculum')
- Numeracy Learning Progressions

2.3 Numeracy and the proficiencies

Key messages

- The proficiencies are the actions students engage in when learning mathematics and working mathematically. They are embedded practices.
- Learning programs need to support students to develop relational understanding (knowing both what to do and why) rather than instrumental knowledge (rules without reasons)
- Students also need to develop skills in:
  - choosing procedures and in recalling facts and concepts
  - applying mathematical knowledge to solving real-life problems, drawing of a range of competencies (communication; mathematising; representation; reasoning and argument; devising strategies, and using symbolic formal and technical language
  - generalising from results of calculations, and using generalisations to derive strategies for unfamiliar problems
- These skills are the "engine room" of high performance as they equip students to tackle unfamiliar problems.
- Students also need to be supported to develop every day and specialist mathematics language.

The mathematics used to support numeracy is learned through a body of knowledge and the proficiencies.

The Australian Curriculum Assessment and Reporting Authority (2017) has outlined four proficiencies as Understanding, Fluency, Problem-solving and Reasoning and the Victorian Curriculum: Mathematics, Foundation to Level 10 (VCAA, 2017) has further defined these.

These proficiencies are the actions students engage in when learning mathematics and working mathematically. They are dispositions that support students as they engage with challenges that extend from the familiar to the unfamiliar. They allow students to reflect and evaluate effectiveness and efficiency.

The proficiencies are assessed by the Trends in International Mathematics and Science Study) TIMSS and the PISA. This is because the Organisation for Economic Cooperation and Development (OECD) deem them to be essential to the diverse array of maths-related situations that learners will encounter throughout their lives.

Key resource

FUSE Proficiencies Repository

This repository, published on the FUSE platform, provides guidance and resources for incorporating the proficiencies into teaching and learning programs. It includes videos, readings, discussion protocols, suggested learning activities, 'deep dives' into each proficiency, and school examples.

2.3.1 Numeracy and understanding

Understanding is defined as making connections between representations of numbers and having concepts of partitioning and combining numbers (VCAA, 2017).

Richard Skemp's seminal writing from the 1970s explained the difference between instrumental knowledge (rules without reasons) and relational understanding (knowing both what to do and why).

Gould (2016) explained how issues arise when learning rules without reason. Rules can appear to change depending on the mathematics involved, particularly when the mathematics becomes more complex.

Take, for example, the errors students make in subtraction; the most common error is to subtract the smallest number from the largest, such as in the problem 96 – 58 to get the answer 42. These types of errors show students' lack of understanding of subtraction.

If students hold a relational understanding of mathematics when carrying out computational procedures, they are less likely to make errors when applied to other numeracy contexts. For example, Hurst (2018) suggested using examples in context to encourage estimation like "I have $53, and I spend $27. How much money do I have left? Is the answer more than $30 or less than $30?"

> ***"It is important to determine how students solve mathematics problems. Not just whether they have relevant skills but also if they understand what they are doing. When students practise procedures, they do not understand there is a clear danger they will practise incorrect procedures"*** *(Hurst, 2018, p.19).*

Hurst argued that the predominance of teaching computational procedures at the expense of concepts and connections limits students' understanding and that there needs to be a focus on both the concepts and the procedures. It is not that learning one will necessarily develop the other, but that teaching focuses on both together.

2.3.2 Numeracy and fluency

Fluency is defined as developing skills in choosing procedures and in recalling facts and concepts (VCAA, 2017). Research into fluency has helped us to question the extent to which the knowledge of facts are necessary for computation and, in particular, mental calculations.

For example, Heirdsfield, Dole, and Beswick's (2007) study showed how students often use derived facts strategies when they are unable to recall known facts. For example, if a student knows that 6 + 6 = 12, then they can quickly see that 6 + 7 = 13. They do not know the fact for 6 + 7 but can use their known fact of 6 + 6 to derive a new one.

The teaching experiment used in this study, showed that direct teaching, using visual models such as ten frames and number lines in an instructional program, helped to focus on developing the facts that would be useful for derived facts strategies.

Key facts were facts to 10, facts through 10, and doubles. The study noted the importance of tailoring instruction to the needs of learners and the delicate balance between explicit instruction and encouraging students' own thinking.

The use of procedural fluency in numeracy is not dealt with explicitly in research. However, becoming fluent in the recall of fundamental facts and a range of flexible procedures helps children apply mathematics to contexts outside of the mathematics educational setting.

2.3.3 Numeracy and problem-solving

> ***"In the real world… we have to make the judgments and decisions about what mathematical knowledge might be relevant, and how to apply that knowledge"*** *(Turner, 2011, p. 58).*

Problem-solving is defined as formulating, modelling, simulating, and recording authentic situations involving operations, comparing numbers, and using properties of numbers to continue patterns (VCAA, 2017).

Turner (2011) suggested six competencies that are needed to apply mathematical knowledge to solving real-life problems. These were:

- **communication** – reading and interpreting statements, then forming a mental model of the situation
- **mathematising** – transforming the problem to a mathematical form using a known or given mathematical model or by creating a model
- **representation** – translating between different representations (e.g., equations, formulas, graphs, diagrams, and concrete materials) to understand the situation, find a solution process and present the solution
- **reasoning and argument** – being able to explore how different pieces of information are connected in a problem and to provide justifications
- **devising strategies** – finding a mathematical strategy to solve a problem
- **using symbolic formal and technical language** – understanding and using symbolic expressions (e.g., arithmetic expressions and operations) and the formal, technical language needed to interpret the mathematics.

2.3.4 Numeracy and reasoning

Reasoning plays a "critical role in developing students' understanding and promoting creative thinking in mathematics" (Vale, Bragg, Widjaja, Herbert and Loong, 2017, p. 3).

Reasoning is defined as generalising from results of calculations and using generalisations to derive strategies for unfamiliar problems, adapting the known to the unknown, transferring learning from one context to another (VCAA, 2017).

Research has suggested that primary school students can use mathematical reasoning but that they often experience difficulties in working this way due to limited opportunities to justify or generalise (Clarke, Clarke, and Sullivan, 2012).

A stumbling block in the teaching and learning of reasoning has been the limitation in understanding what reasoning is. Some Victorian teachers are unable to define reasoning and are unsure how to support their students in developing mathematical reasoning (Loong, Vale, Bragg and Herbert, 2013).

2.3.5 Language proficiency and mathematics

> ***"…mathematics content is delivered through language and so all mathematics teachers are teachers of the language of mathematics"*** *(Adoniou and Qing, 2014, p. 12).*

We often think that mathematics is 'language free,' but the demands of the Victorian Curriculum: Mathematics often mean that students are working in two registers: everyday language and specialist mathematics language. Some terms have specific technical meanings in mathematics (e.g., 'volume' or 'prime') and others have a precise meaning such as 'half.'

Working with these two language registers can create ambiguities and challenges for many students, particularly for students learning English as an Additional Language. These challenges increase when mathematics is embedded in word problems which contain both language and numerical complexities.

Adoniou and Qing suggest that "…mathematics content is delivered through language and so all mathematics teachers are teachers of the language of mathematics" (2014, p. 12).

Key resource

Literacy in Mathematics (part of the Literacy Teaching Toolkit)

The department's Literacy Teaching Toolkit has a section dedicated to helping teachers of mathematics across Years 7 to 10. This section covers:

- *an introduction to literacy in mathematics*
- *developing understanding in mathematics*
- *communicating understanding in mathematics.*

Reflective prompts	
School leaders	How independent are the maths learners at your school? How well do they understand what they are being asked to do and why? How might you build the capacity of your teachers and support staff to strengthen their commitment to and focus on the development of mathematics proficiencies? What might the FUSE Proficiencies Repository offer to support these efforts? How well do your teachers scaffold the development of the language of mathematics? What is already strong? What might need to be strengthened? How might the Literacy in Mathematics section of the Literacy Teaching Toolkit support these efforts?
For leaders to use with teachers	Considering your planning documents, how strongly do the mathematics proficiencies feature in the learning experiences offered to your students? What might need to be strengthened, and how will you go about this? Select a problem-solving task that you currently offer students. Use Turner's six competencies to analyse the task and identify how it might be strengthened. What other uses might there be for Turner's framework, in your own practice and in collaborative work with colleagues? How well are your students using the language of the mathematics proficiencies, as well as the language of mathematics, when describing their learning? What could you adjust to enhance their use? How might the FUSE Proficiencies Repository and the Literacy in Mathematics section of the Literacy Teaching Toolkit support these efforts?

FISO 2.0

Core element	Dimension
Teaching and learning	*Documented teaching and learning program based on the Victorian Curriculum and senior secondary pathways, incorporating extra-curricular programs* *Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships*
Engagement	*Activation of student voice, agency, leadership and learning to strengthen students' participation and engagement in school*

Link summary

- FUSE Proficiencies Repository
- Literacy in Mathematics section of the Literacy Teaching Toolkit
- Numeracy Glossary in the Birth to Level 10 Numeracy Guide
- Mathematics Glossary for the Victorian Curriculum: Mathematics

3: Teaching and assessing mathematics and numeracy

3.1 Developing a capacity to teach mathematics and numeracy

Key messages

- Highly effective teachers of numeracy have a connectionist belief orientation. They:
  - hold coherent beliefs and understandings about mathematics and numeracy and the relationship between the two
  - use teaching approaches which connect different areas of mathematics and different ideas in the same area of mathematics
  - use students' descriptions of their methods and their reasoning to establish connections and address misconceptions
  - emphasise using mental, written, part-written or electronic methods of calculation
  - ensure that all students are challenged
  - challenge students to think
  - encourage purposeful discussion
  - use a variety of assessment and recording methods
  - display positive attitudes to mathematics
- Mathematics anxiety is common in Australian mathematics classrooms, and impacts performance. It is important for schools to identify mathematics anxiety in students and teachers and employ responses that focus on reducing and regulating it.
- Student background characteristics, such as gender, socioeconomic background and Indigenous status have been shown to impact on student performance in mathematics. Where they exist, they require specific actions from school leaders and teachers to address them, including addressing bias.
- Approximately eight percent of people worldwide have dyscalculia, rates comparable to those of dyslexia.
- Students may be experiencing difficulty in mathematics for a range of reasons. Teachers need to understand why students are experiencing difficulty before devising and delivering interventions.

3.1.1 Teacher belief orientations and dispositions

In their seminal study 'Effective Teachers of Numeracy,' Askew et al. (1997) identified that what distinguished highly effective teachers from other teachers was a particular set of coherent beliefs which underpinned their teaching of numeracy.

They identified three sets of belief orientations which were:

- **connectionist**: beliefs based around both valuing pupils' methods and teaching strategies with an emphasis on establishing connections within mathematics and developing the language of mathematics
- **transmission**: beliefs based around the primacy of teaching and a view of mathematics as a collection of separate routines and procedures
- **discovery**: beliefs clustered around the primacy of learning and a view of mathematics as being discovered by pupils.

"…it was clear that those teachers with a strongly connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strongly discovery or transmission orientations" *(Askew, et al., 1997, p. 28).*

Teachers within each orientation held a particular set of beliefs about what it is to be a numerate pupil, how pupils learn to become numerate and how best to teach a pupil to become numerate.

In summary, Askew et al. (1997) found that highly effective teachers:

- held a particular set of coherent beliefs and understandings which underpinned their teaching of numeracy, including what it means to be numerate, the relationship between teaching and pupils' learning of numeracy, presentation, and intervention strategies
- used teaching approaches which connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols, and diagrams
- used pupils' descriptions of their methods and their reasoning to help establish connections and to address misconceptions
- emphasised the importance of using mental, written, part-written or electronic methods of calculation and particularly emphasised the development of mental skills
- ensured that all pupils were being challenged, not just those who were considered more able
- challenged pupils to think through explaining, listening and problem-solving
- encouraged purposeful discussion
- used a variety of different assessment and recording methods to monitor pupils' progress
- had knowledge and awareness of conceptual connections between the areas that they taught in the primary mathematics curriculum
- were more likely to have undertaken mathematics-specific continuing professional development over an extended period
- displayed very positive attitudes to mathematics.

3.1.2 Mathematics anxiety

"Many people think of mathematics as one of the most logical, most impersonal branches of knowledge, yet it inspires more emotion than any other school subject" *(Zaslavsky, 1994, p.5).*

Mathematics anxiety is common in Australian mathematics classrooms. For instance, in PISA 2012, 25 per cent of Australian 15-year-old students reported feeling helpless when doing a mathematics problem (Thomson, DeBortoli and Buckley, 2013).

Research shows that mathematics anxiety can interrupt working memory, leading to more error-making, and reducing the capacity to successfully participate in mathematics (Ashcraft and Kirk, 2001; Eden, Heien and Jacobs, 2013; Ma, 1999). Students who consistently experience mathematics anxiety when engaging with mathematics are also more likely to avoid mathematics subjects, courses, and careers.

The causes of mathematics anxiety revolve around a fixed mindset or belief about mathematics performance and learning (e.g., "I am no good at maths"). These beliefs are usually long-standing and formed through experiences with parents, teachers, and peers. They are also influenced by previous experiences struggling with mathematics (Buckley, Reid, Good, Lipp and Thomson, 2016).

It is important to identify mathematics anxiety in students and teachers.

Research shows that successful responses to mathematics anxiety focus on reducing and regulating mathematics anxiety (Buckley, 2020). They include:

- highlighting role models in the community or in fiction that challenge negative stereotypes about mathematics
- reframing mathematics anxiety to acknowledge its positive and constructive qualities (mathematics anxiety can only be experienced by someone who values mathematics; moderate levels of anxiety are associated with optimal levels of performance)
- employing direct or psychological strategies when students are experiencing anxiety (e.g., deep breathing, expressive writing)
- using pedagogical practices that:
    - foster a growth mindset
    - challenge negative thinking about mathematics
    - provide students with opportunities for personal mathematics accomplishment.

Key resource

Mathematics Monograph: Mathematics anxiety

by Dr Sarah Buckley, Senior Research Fellow, ACER

This monograph explores the often negative and extreme emotional response of many Australian students to learning mathematics. It helps leaders and teachers understand:

- *how mathematics anxiety negatively impacts learning and teaching*
- *the symptoms, causes and companions of mathematics anxiety*
- *the difference between reducing mathematics anxiety directly and indirectly*
- *strategies that can identify and address mathematics anxiety in students and teachers.*

Key resource

Maths Anxiety webinars: Session 1 and Session 2

by Dr Catherine Pearn, Senior Lecturer, Melbourne Graduate School of Education and Sarah Buckley, Senior Research Fellow, ACER

In these recorded webinars, teachers can deepen their understanding of mathematics anxiety, explore the science, education, and psychology behind it and learn how to alleviate it in classrooms.

3.1.3 Priority cohorts

3.1.3.1 Gender, socioeconomic background and Indigenous status

Student background characteristics, such as gender, socioeconomic background, Indigenous status, immigrant background and language spoken at home, have been shown to impact on student performance in mathematics.

For example, in PISA 2018 (OECD, 2019) boys outperformed girls, students from higher socioeconomic backgrounds performed better than students from lower socioeconomic backgrounds, and non-Indigenous students performed better than Indigenous students. The mathematics performance of first-generation and foreign-born students was similar and above that of Australian-born students, and there was no difference in performance between students who spoke English at home versus those that spoke a language other than English.

Research suggests that gender differences in beliefs and attitudes are subtly affecting students' performance and motivation in mathematics (Forgasz and Leder, 2020). They include:

- more boys than girls say they like doing mathematics
- more boys than girls are confident they can do well in mathematics
- more boys than girls indicate that their parents and teachers expect them to do well in mathematics
- more boys than girls, when shown a mathematics question, state that they can solve it correctly
- more boys than girls expect to use mathematics in their work
- more boys than girls volunteer to answer or ask a question in class
- more girls than boys say they like to work with others when doing mathematics.

Key resource

Mathematics Monograph: Gender and mathematics

by Helen Forgasz and Gilah Leder, Monash University

This monograph explores why gender remains a factor the negatively impacts girls':

- *achievement in mathematics*
- *attitudes about mathematics and towards themselves as a learner of mathematics*
- *participation in senior mathematics studies.*

It also describes the challenges and barriers that teachers may face when addressing gender inequities in mathematics learning, and what to do about them.

Gender differences were revealed in a study by Berry and Picker, who asked students to draw a mathematician:

"In the 306 surveys returned from schools in England and the USA, the images of mathematicians were primarily male, all were white, the majority with glasses and/or a beard, balding or with weird hair, invariably at a blackboard or computer. When the drawing included a blackboard, one of two types of writings was generally on it: trivial arithmetic, such as 1+1=2; or a meaningless gibberish of mathematical symbols and formulas" *(Berry and Picker, 2000, p. 25).*

Forgasz and Leder recommend the school leaders and teachers begin with gathering and analysing school-level data (e.g., NAPLAN numeracy results, VCE subject enrolments and past results) to determine if there is a problem and where it is showing up. Additional Steps are recommended in their Mathematics Monograph: Gender and mathematics, and include working directly with students to explore their thoughts and feelings about mathematics (Forgasz and Leder, 2020).

Research also suggests that schools need to be more aware of the impact of cultural bias on the mathematics performance of Aboriginal and Torres Strait Islander students. For example, in the area of money and financial mathematics, learning activities that encourage students to aspire to the accumulation of personal wealth may put Aboriginal and Torres Strait Islander students at odds with what their family and community(ies) consider important.

For Matthews, mathematics teaching and learning must be meaningful to the social and cultural contexts of Aboriginal and Torres Strait Islander students. He offers a model for how to do this in his Mathematics Monograph: Teaching mathematics from a cultural perspective (Matthews, 2021). Such a model offers an opportunity for mathematics classrooms to become more inclusive for all students, not just Aboriginal and Torres Strait Islander students. This is because it supports teachers to draw on the way mathematics has been created, translated, and interpreted by many cultures around the world.

Key resource

Mathematics Monograph: Teaching mathematics from a cultural perspective

by Professor Chris Matthews, University Technology of Sydney

This monograph argues that mathematics can and should be taught in a culturally inclusive way, to better support and empower students from culturally and linguistically diverse backgrounds. It reflects on the unique perspectives Aboriginal and Torres Strait Islander peoples bring to understanding the world mathematically, which can benefit all students by connecting them with the meaning and purpose of mathematics.

3.1.3.2 Disability and learning difficulty in mathematics

While numeracy learning difficulties and dyscalculia have received less attention than other developmental learning disorders in Australia, international prevalence rates suggest that approximately eight percent of people worldwide have dyscalculia – rates comparable to those of dyslexia (Reeve and Waldecker, 2019).

In the past, learning difficulties in numeracy have been characterised as general cognitive problems (for example, low IQ). It is now understood that a student may be experiencing difficulty in mathematics for a range of reasons, including:

Key resource

Learning Difficulties Information Guide: Numeracy

This Guide provides a foundation for understanding and responding to learning difficulties in numeracy, including dyscalculia. It aims to help build the confidence and practical capability of teachers to identify, assess and implement interventions to support students with learning difficulties in day-to-day practice.

- Student related factors: chronic absenteeism; sensory impairment(s); delayed acquisition of language; social-emotional difficulties; specific cognitive problems (for example, issues with working memory); overlap with a literacy learning difficulty
- Social and/or environmental factors: limited early language and literacy exposure; low socio-economic status; family history of learning disabilities; disrupted learning (for example, school refusal); if English is not a student's first language, or the primary language spoken at home, this may contribute to initial difficulties while the child masters the language.

Dyscalculia is regarded as a specific learning disability, resulting from differences in the brain that affects the typical acquisition of numerical skills (Butterworth, 2019).

The American Psychiatric Association DSM V (2013) defines dyscalculia as a learning deficit associated with difficulties processing numerical information, learning arithmetic facts, and performing calculations. In approximately 25 percent of cases, dyscalculia overlaps with dyslexia (Butterworth, 2005).

The department's Learning Difficulties Information Guide: Numeracy provides practical advice, links to tools and resources and case studies to help teachers identify and assess numeracy learning difficulties and dyscalculia, and intervene effectively.

Reflective prompts	
School leaders	What beliefs and understandings do your teachers hold about mathematics and numeracy? How well matched are they to Askew's Effective Teachers of Numeracy research? How might the school leadership team, and other leaders in the school, work together to respectfully challenge problematic beliefs and misunderstandings? To what extent is maths anxiety impacting performance in mathematics in your school? If you are not sure, how might you find out more? What insights and ideas does the Mathematics Monograph: Mathematics anxiety offer that might be important for your school to take forward? What steps will you take and how will you monitor progress? To what extent are student background characteristics impacting performance in mathematics in your school? What specific actions might the school leadership team, and other leaders in the school, take to address them? (note: you may wish to read the Mathematics Monograph: Gender and mathematics and the Mathematics Monograph: Teaching mathematics from a cultural perspective to insights and ideas) How will you monitor progress? How well equipped is your school to identify, understand and respond to students with dyscalculia and/or difficulty in mathematics? What insights, ideas and supports does the Learning Difficulties Information Guide: Numeracy offer that might be important for your school to take forward? What steps will you take and how will you monitor progress? How might existing structures (e.g., PLCs) be harnessed to sharpen your school's focus on priority cohorts and mathematics performance? How might a focus on a selected priority cohort have wider benefits for students and the school?

For leaders to use with teachers	What beliefs and understandings do you hold about mathematics and numeracy – for example, about who can succeed as a maths learner, what helps them succeed, and who you need to keep a special eye out for to ensure that optimal progress is made? How well matched are your beliefs and understandings to Askew's Effective Teachers of Numeracy research? What are 2-3 focus areas for your further development as an effective teacher of numeracy? How might you go about this? How anxious are your maths learners? What specific strategies do you have in place – before, during and after learning – to manage and reduce maths anxiety? How well are they working? What insights and ideas does the Mathematics Monograph: Mathematics anxiety and Maths Anxiety webinars (Session 1 and Session 2) offer that might be important for you to take forward in your teaching? How well equipped are you to cater for the diverse backgrounds and needs of your maths learners, including those with disability and/or learning difficulty? What are 2-3 focus areas for your further development as a teacher? How might you go about this?

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Core element	Dimension
Leadership	*The strategic direction and deployment of resources to create shared goals and values; high expectations; and a positive, safe and orderly learning environment*
Teaching and learning	*Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships*
Assessment	*Systematic use of data and evidence to drive the prioritisation, development, and implementation of actions in schools and classrooms*
Support and resources	*Responsive, tiered and contextualised approaches and strong relationships to support student learning, wellbeing and inclusion*

Link summary

- Mathematics Monograph: Mathematics anxiety
- Maths Anxiety webinars: Session 1 and Session 2
- Mathematics Monograph: Gender and mathematics
- Mathematics Monograph: Teaching mathematics from a cultural perspective
- Learning Difficulties Information Guide: Numeracy

3.2 Developing high-quality mathematics and numeracy programs

Key messages

- High-quality mathematics and numeracy programs are challenging and balanced.
- Challenging programs:

  - prioritise cognitive activation strategies (problem solving, reflection and guided discovery), where students are supported to summarise, question, clarify predict and justify their thinking and solutions
  - encourage students to concentrate on methods used to arrive at a solution rather than focussing just on the answer.
- Balanced means utilising:
  - a mix of teacher-directed and student-centred strategies, as well as different types of strategies overall
  - varied lesson structures, selected based on the mathematics content and proficiency focus of the lesson.

3.2.1 A challenging mathematics program

Hattie (2009) describes quality teachers as those who *challenge* their students. Specifically, instructional processes that teach pupils cognitive strategies and require them to summarise, question, clarify and predict appear to be particularly important for learning.

While certain teacher-directed strategies, such as setting clear learning goals, summarising previous learning, and asking questions to check student understanding has shown to help when solving basic problems, research also shows that student-centred strategies, such as effective differentiation and enabling collaboration supports students to solve more complex problems and positively impacts motivation (OECD, 2016).

In fact, *all* teaching strategies work best when teachers also challenge their students and encourage them to focus on the process rather than just the answer.

The OECD uses the term ‘cognitive activation’ to describe this notion of challenge, describing it as the:

> **“…practices capable of challenging students in order to motivate them and stimulate higher-order skills, such as critical thinking, problem solving and decision making. This strategy not only encourages students to find creative and alternative ways to solve problems but enables them to communicate their thinking processes and results with their peers and teachers”** (OECD, 2016, p. 6).

Key resource

[Middle Years Maths Challenges]

A new set of e-books have been published, providing schools, teachers, and students with opportunities to engage in authentic, interesting, and creative mathematics experiences across Levels 5 to 9 of the Victorian Curriculum: Mathematics. There are 75 Middle Years Maths Challenges – 13 challenges per level as well as 10 challenges designed specifically for students who need extra support. Each Challenge has a mathematics and a proficiency focus. Each Challenge has an accompanying assessment rubric. 7 challenges focus on exploring Aboriginal and Torres Strait Islander histories, cultures, and perspectives.

In 2012, PISA asked students to indicate how often their teacher used cognitive activation strategies in their mathematics classes (OECD, 2016, Figure 2.2). Students said that their teachers:

- encouraged them to reflect on problems
- crafted problems that required them to think for an extended time

- asked them to use their own procedures for solving complex problems
- presented problems with no immediately obvious method of solution
- presented problems in different contexts
- helped them to learn from mistakes they had made
- asked them to explain how they had solved a problem
- asked them to apply what they had learned to new contexts
- crafted problems with multiple solutions.

This study showed that cognitive-activation instruction was associated with a 19-point increase in mathematics scores across OECD countries, after accounting for other teaching strategies (OECD, 2016, Figure 2.2).

Key resource

[Birth to Level 10 Numeracy Guide Resources tab]

The online Birth to Level 10 Numeracy Guide has a tab which links teachers to over 100 quality-assured resources to support planning, teaching, and assessment in mathematics and to scaffold numeracy across the curriculum. Teachers can browse the whole collection or filter by developmental stage, Numeracy Focus Area, HITS, or proficiency.

3.2.2 A balanced mathematics program

A balanced mathematics program develops increasingly sophisticated and refined mathematical proficiency and must aim to build understanding, foster fluency, encourage problem solving, and develop reasoning skills.

To create a rich and balanced mathematics program, students will need to experience a range of lesson structures, task types and pedagogies in any sequence of learning.

The need for a variety of teaching strategies and lesson structures is reinforced by research that recommends:

> **"A single teaching strategy will not work for all mathematics problems …. teaching complex mathematics skills might require different instructional strategies than those used to teach basic mathematics skills…. teachers need a diverse set of tools to teach the breadth of their mathematics curriculum and to help students advance from the most rudimentary to the most complex mathematics problems"** (OECD, 2016, p. 15).

Sullivan (2020) offers four different lesson types that are useful when designing a balanced numeracy program:

1: Active teaching

Active teaching is a form of teacher-led instruction. However, rather than the teacher modelling a process to be followed and practised by students, active teaching involves a focus on developing "comprehension of specific aspects of mathematics or to foster conceptual understanding of a procedure or technique" (Sullivan, 2020, p. 33). The teacher is active at all stages of the lesson and students have opportunities to suggest alternative methods and form generalisations.

2: Imagined representations

A key aspect of problem solving is "imagining possibilities" (Sullivan, 2020, p. 38). These investigative type lessons often use a realistic or practical context to foster strategic competence. Lessons are usually less structured where there is no immediate solution pathway. Such tasks are sometimes known as Fermi Problems, named after the Italian physicist Enrico Fermi, that emphasise logical thinking, questioning, metacognition, and estimation.

3: Purposeful games and puzzles

One way of building mathematical proficiency is through purposeful games and puzzles (Sullivan, 2020). However, mathematics learning does not occur simply because of engaging with the game or puzzle (Russo, Russo and Bragg, 2018). The teacher is active during the lesson and engages in dialogue with students to draw attention to the mathematics as students explore the game or puzzle. This enables the mathematical connections to be explicit.

In designing a balanced numeracy and mathematics program, games also offer the advantage of increasing student motivation and positively engaging them in their learning (Russo, Russo and Bragg, 2018). To select suitable games, Russo, Russo and Bragg (2018) identify five recommend five things to look for:

- Mathematical games should be engaging, enjoyable and generate mathematical discussion
- Mathematical games should appropriately balance skill and luck
- Exploring important mathematical concepts and practising important skills should be central to game strategy and gameplay
- Mathematical games should be easily differentiated to cater for a variety of learners, and modifiable to cater to a variety of concepts
- Mathematical games should provide opportunities for fostering home-school connections.

4: 'What if?'

'What if' lessons are often referred to as open-ended questions or structured mathematical inquiry. These lessons tend to focus on adaptive reasoning as well as strategic competence and conceptual understanding (Sullivan, 2020). 'What if?' tasks usually have multiple solution pathways and modes of representation. They foster student collaboration, communication, reasoning, and justification and can be differentiated to suit a range of learning needs. It supports students to convince themselves, others and sceptics of their pathway and solution (Sexton, 2019).

3.2.3 High Impact Teaching Strategies

The way educators plan and teach mathematics and numeracy will be different to literacy and other learning areas. This is important when considering how the department's High Impact Teaching Strategies (HITS; DET, 2020) are enacted in a numeracy context.

Sullivan and Russo (2021) suggest the following approach to use of the HITS:

- **Setting goals**, especially articulating clear learning intentions ("Today we are learning about …") guides teachers in their support for, and interactions with, students.
- Consistent **Structuring** (of) **lessons** allows students time to engage with and be confident about what is to come in the lesson, noting that lesson structure can vary depending on the particular experiences (see 3.2.2 A balanced mathematics program, above; see breakout box, below, on a way to structure a lesson in order to scaffold student thinking).
- **Explicit teaching** is about ensuring students know what to do and why. The challenge is to do this without detracting from opportunities for students to think for themselves.
- **Worked examples** can guide learning; the teacher should ensure that examples come from students (because of their own insights and creativity) as well as from themselves, third-party resources.
- **Collaborative learning** can be built into every mathematics lesson noting that students need time to think for themselves prior to the collaboration.
- **Multiple exposures** emphasise the planning of further experiences like the initial problem as a means of consolidating learning.
- **Questioning and feedback** can help teachers ensure that the predominant mode of interaction is not "telling" but "asking" and "responding."
- **Metacognitive strategies** develop as the result of thinking, where students become aware of the connection between their own thinking and the ways they learn and use mathematics.
- **Differentiated teaching** implies that teachers foster collaborative classroom communities where the problems are effectively adapted to the needs of individuals.

An approach to structuring lessons to scaffold student thinking

Adapted from Russo, Bobis and Sullivan, 2021

- **Anticipate** advises teachers to plan experiences carefully, including considering the intended mathematics and associated pedagogies, and how students might respond
- **Launch** involves addressing language and representational issues and posing problems, allowing time for students to engage for themselves
- During **Explore**, teachers interact with students, supporting those who need it and extending those who are ready
- In **Summarise**, teachers facilitate dialogue and discussions to facilitate students to articulate their strategies and solutions, using work samples as evidence of learning.
- **Review** is for the teacher to make explicit the intended learning

(re) Launch prompts teachers to consolidate the initial learning by posing problems that are "a bit the same and a bit different" from the original. The original problem *activates student thinking*. Further problems *consolidate the learning*.

How many minutes should we be teaching mathematics each week?

The department does not set minimum delivery times for mathematics. This is in part because time allocations are not a measure of the quality of the teaching and learning program. However, time *is* important – students need to be given time to learn, and how much time they are given influences the knowledge and skills that can be addressed.

In 2012 the Australian Curriculum, Assessment and Reporting Authority (ACARA) provided curriculum writers with indicative time allocations for each learning area and subject of the Australian Curriculum, with Australian Curriculum content to be 'teachable' within these allocations (ACARA, 2012, p. 8). The allocation for the Australian Curriculum: Mathematics was:

- 18 per cent of teaching time across Levels F to 4 (equivalent to around 4.5 hours per week)
- 16 percent of teaching time across Levels 5 and 6 (equivalent to around 4 hours per week)
- 12 percent of teaching time across Levels 7 to 10 (equivalent to around 3 hours per week).

Schools may wish to use the ACARA indicative time allocations to inform local decisions about the number of minutes they teach mathematics each week, but should note that:

- at some levels, there are a small number of additional content descriptors in the Victorian Curriculum: Mathematics than in the Australian Curriculum: Mathematics
- schools with specific priorities to support and improve student achievement in numeracy and mathematics should consider time allocations that exceed those provided by ACARA.

Delivery time examples

Glen Waverley Secondary College

- Years 7, 9 and 10: five 50-minute sessions per week
- Year 8: four 50-minute sessions per week

Gisborne Primary School

- Years Prep to 6: minimum of 5 hours per week

Castlemaine North Primary School

- Years Prep to 4: minimum of 5 hours per week
- Year 5 and 6: minimum of 5 hours per week, with up to 7 hours per week when weekly STEAM challenges are included

Should we integrate the teaching of the Victorian Curriculum: Mathematics with other learning areas and capabilities?

The department's Curriculum Programs Foundation to 10 policy advises schools to provide a "structured mathematics program in mathematics at each year level from Foundation to Level 10". This is ordinarily understood to be a 'standalone' program, recognising the importance of mathematics as a discipline in and of itself, but also as an enabling discipline for learning in other areas like science, geography, and music.

There are circumstances where it is appropriate for schools to integrate other learning areas into their mathematics program. For example, schools often adopt a Science, Technology, Engineering and Mathematics (STEM) approach to teaching and learning, focussing on a 'real world' problem. However, research tells us that this approach can be risky if programs fail to address the need for students to develop deep understanding of important mathematical ideas and the connections between them (Becker & Park, 2011; English, 2016; Larson, 2017; Shaughnessy, 2013).

3.2.4 Scaffolding numeracy in non-mathematics programs

Highly skilled teachers recognise that a wide range of knowledge, skills, behaviours, and dispositions relevant to a discipline can be enhanced through developing numeracy capability.

However, scaffolding numeracy in non-mathematics programs can sometimes be tricky. This is because it requires teachers to know and understand the mathematics that naturally resides in a discipline (e.g., geography), as well as how to teach it in the context of the discipline (e.g., in mapping, exploring, and using scale, position and location).

For example, for Level 7 English teachers to support students to engage with an English activity on rhetorical devices that involves the use of statistics (e.g., to persuade readers of a news article), such teachers are not expected to teach students how to calculate mean, median, mode and range, and construct and compare stem-and-leaf plots and dot plots. Instead, English teachers should focus on supporting students to develop and apply critical literacy skills to understand and interrogate the ways in which statistics are used within a range of texts. This may involve teaching text processing strategies that help students interpret a variety of data representations typically found in news articles.

> **Key resource**
>
> **Birth to Level 10 Numeracy Guide HITS tab**
>
> *The online Birth to Level 10 Numeracy Guide has a tab which links teachers to the N-HITS (Numeracy High Impact Teaching Strategies). Key elements of each N-HIT are included, as are practical examples from schools.*

Further information and links to support resources for scaffolding numeracy in non-mathematics programs can be found at 2.2.2 Numeracy across the curriculum and 6.1 Numeracy for all learners.

Reflective prompts	
School leaders	How widespread are the use of cognitive activation strategies in your school's mathematics programs? If they are not widespread, what might sit behind this that school leaders, and other leaders in the school, need to address? If they are widespread, how might they be further strengthened? How balanced (i.e., including a variety of teaching strategies and lesson structures) are your school's mathematics programs? If they are not balanced, what might sit behind this that school leaders, and other leaders in the school, need to address? If they are balanced, how might they be further strengthened? How widespread, varied and discerning are your school's use of the HITS in mathematics and teaching and learning? How much time is allocated to teaching mathematics at your school? To what extent is this sufficient, not just for teaching but for learning? Drawing on evidence and insights from this chapter, what are 2-3 targeted, specific, and achievable steps your school can take to ensure that all students at your school experience the highest-quality mathematics teaching and learning?
For leaders to use	How widespread are the use of cognitive activation strategies in your teaching? Which ones might you introduce? If you need support to do so, who might be able to help?

with teachers	What are the typical lesson types in your classroom? How do Sullivan's four different lesson types affirm, challenge, or extend your current thinking and practice? What might they offer you in terms of next steps? What tweaks might you make to your current use of the HITS to ensure all types are used and use is discerning?

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Core element	Dimension
Teaching and learning	*Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships*
Engagement	*Activation of student voice, agency, leadership and learning to strengthen students' participation and engagement in school*

Link summary

- Middle Years Maths Challenges
- Birth to Level 10 Numeracy Guide Resources tab
- Birth to Level 10 Numeracy Guide HITS tab
- HITS catalogue on FUSE (contains several mathematics and numeracy examples)

3.3 Assessing mathematics and numeracy

Key messages

- Excellent mathematics teachers:
  - regularly assess and report student learning outcomes, both cognitive and affective, including skills, content, processes, and attitudes
  - use assessment strategies that are fair, inclusive, and appropriate to both the students and the learning context
  - maintain ongoing, informative records of student learning outcomes that are used to map progress and to plan future learning experiences
  - provide constructive, purposeful, and timely feedback to students and their families.
- Teachers should:
  - use research informed and quality assured assessment tools, including those provided by the department and VCAA
  - avoid only or overly relying on external, standardised assessment tools to inform their understanding of student learning
  - use tools that provide opportunities for students to explain their thinking and reasoning
  - collect, value, and use evidence:

    - on what students know and can do from what they see and hear in the classroom
    - on student attitudes and dispositions
  - use a range of assessment strategies, including rich tasks, feedback and reflection, student self-assessments, student portfolios, validated tools, anecdotal evidence, teacher moderated student assessment tasks, student self-reflections, interests, and surveys.

In the same way that there are many ways to teach numeracy and mathematics well, there are also many ways to assess what students know and can do. The Australian Association of Mathematics Teachers (AAMT, 2006) suggest that excellent mathematics teachers:

- regularly assess and report student learning outcomes, both cognitive and affective, including skills, content, processes, and attitudes
- use assessment strategies that are fair, inclusive, and appropriate to both the students and the learning context
- maintain ongoing, informative records of student learning outcomes that are used to map progress and to plan future learning experiences
- provide constructive, purposeful, and timely feedback to students and their families.

This requires teachers to take an active role in assessment to ensure a range of assessment tools and strategies are used. This is particularly important given the limitations of traditional pen and paper tests, multiple choice items, or online mathematics assessments where it is often not possible to determine how students worked out the answer.

Key resource

Mathematics Assessment Tools

The department and VCAA offers a range of free assessment tools that are research informed and quality assured to assist teachers make consistent and balanced assessment of students' understanding of mathematics. Tools include:

- **Mathematics Online Interview**
- **Fractions and Decimals Online Interview**
- **Scaffolding Numeracy in the Middle Years**
- **Assessment for Common Misunderstandings**
- **Abilities Based Learning and Education Support (ABLES)** for mathematics (students with mild to moderate cognitive impairment)
- **Digital Assessment Library – Mathematics**

3.3.1 What students know and can do

Assessment in mathematics and numeracy is more than forming judgements about a student's ability and performance. It monitors the learner's understanding of the mathematical language, concepts, and skills and what they need to do to succeed. This requires:

- an understanding of how learning develops
- what skills and knowledge learners need to progress
- the common misunderstandings that can delay learning.

Clarke (2021) argues that "assessments which provide opportunities for students to explain their thinking are more valid, reliable and powerful". This requires teachers to understand what different student responses might mean, as well as practical ideas to address the learning needs identified.

Clarke (2021) also argues that "our greatest source of evidence on what students know and can do comes from what teachers see and hear in the classroom". This highlights the importance of teacher knowledge to extend understanding of what is already known. Copur-Gencturk Tolar (2022) define mathematics teaching expertise as:

> **"The content-specific knowledge for teaching, which includes teachers' own understanding of the mathematics, their pedagogical content knowledge (e.g., knowledge of students' mathematical thinking, their instructional practices, and the representations they use), and their ability to notice content-related issues in the moment of mathematics teaching"** (p.2).

It is therefore important that teachers avoid only or overly relying on external, standardised assessment tools to inform their understanding of student learning.

To support teachers to use and act on meaningful assessments, the department offers a range of assessment tools that show student thinking. Two tools of note are Scaffolding Numeracy in the Middle Years and Assessment for Common Misunderstandings (see breakout box for further information).

Key resource

Mathematical Concepts Series

- Counting and place value
- Fractions and decimals
- Addition and subtraction
- Multiplication and division
- Problem solving in mathematics

Each of the webinars explains a key concept in mathematics and how to recognise content-related issues in moments of mathematics teaching.

Key resource

Scaffolding Numeracy in the Middle Years

The Scaffolding Numeracy in the Middle Years Project was a partnership between RMIT, the department and the Tasmanian Education Department and ran from 2003 to 2006. The Project investigated the efficacy of a new assessment-guided approach to improving student numeracy outcomes in Years 4 to 8. Outputs from the Project are available on the department website and include:

- a user guide
- a Learning and Assessment Framework for multiplicative thinking
- assessment materials
- authentic tasks
- a resource library.

Assessment for Common Misunderstandings

Assessment for Common Misunderstandings provides a set of easy-to-use diagnostic tools that expose critical aspects of student thinking in relation to key aspects of number, an area of mathematics that research has shown to be most responsible for large differences in student performance by the middle years of schooling. Advice on interpretation of results and targeted teaching responses is also provided.

The department and VCAA also offer formal assessment tools, including the:

- **Mathematics Online Interview**, a tool for assessing the mathematical understanding of students in the early years of schooling; the Interview assesses students' knowledge, skills, and strategies in relation to key 'growth points' in the strands of number and algebra, measurement and geometry and was developed from the Early Numeracy Research Project (ENRP)
- **Fractions and Decimals Online Interview**, a tool for assessing students' mathematical understandings and strategies around fractions, decimals, ratio, and percentage across Years 5 to 8.
- **ABLES** for mathematics for students with mild to moderate cognitive impairment, which covers the development of numeracy skills needed to notice, describe, understand, and use numeracy information, including number and its operations, shape, and pattern.
- **Digital Assessment Library – Mathematics**, a tool designed to provide meaningful and timely information about student skills, knowledge and understanding across the three strands of mathematics for Levels 2-10.

School based assessment may also contribute to building a clear picture of the learner. Examples of assessment in mathematics and numeracy include feedback and reflection, student self-assessments, student portfolios, validated tools, anecdotal evidence, teacher moderated student assessment tasks, student self-reflections, interests, and surveys.

In shifting towards more holistic judgments of student work, rich tasks are another example of school-based assessment. Clarke (2021) characterises rich tasks as those that:

- connect naturally with what has been taught
- address a range of outcomes within the one task
- provide a measure of choice or 'openness'
- encourage students to disclose their own understanding of what they have learned
- allow student to show connections between concepts they have learned
- provide an opportunity for students to transfer knowledge from a known context to a less familiar one
- draw the attention of teachers and students to important aspects of mathematical activity
- help teachers to decide what specific help students may require in the relevant content areas.

In this way, rich tasks can be used as a meaningful and time efficient formative assessment tool as they form part of teachers' regular planning and teaching routines.

3.3.2 Student disposition

As you would have learned in Chapter 4, teacher dispositions to mathematics matter. This is also the case for students.

Student disposition is often considered the fifth proficiency in mathematics education and is a major determinant of educational outcomes (Kilpatrick, Swafford and Findell, 2001). Kiplatrick and colleagues define students who have a 'productive disposition' as having an:

> **"Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy"** (p. 116).

The aim of assessing disposition is to provide students with opportunities to express what they believe helps them learn mathematics; this provides teachers with insights that can inform their teaching.

Reflective prompts

School leaders	How consistent is your school's approach with the AAMT's characteristics of excellence in mathematics assessment? What could be strengthened? What standardised mathematics assessment tools does your school use? How well is the purpose of these assessments understood? To what extent does the information provided by these tools guide the actions of school and middle leaders, and classroom teachers (e.g., targeted teaching; identification of adjustments)? How broad are the range of mathematics assessment strategies used in your school? How might they be broadened? What shared principles do your teachers draw on to develop mathematics tasks for students? Which of Clarke's characteristics of richness are included? What could be strengthened?
For leaders to use with teachers	What kind of assessment strategies do you typically use? How does this compare to colleagues? Which of the following might you use more of to enhance the assessment of mathematics: rich tasks, feedback and reflection, student self-assessments, student portfolios, validated tools, anecdotal evidence, teacher moderated student assessment tasks, student self-reflections, interests, and surveys? How frequently do you assess students' cognitive and affective mathematics learning, including skills, content, processes, and attitudes? What is most and least assessed? Why do you think that is? To what extent is the assessment of thinking and reasoning a feature of your mathematics assessment? What benefits might more assessment of thinking and reasoning provide you and your students? How well is assessment information used to map student progress in mathematics, and to plan future learning experiences?

FISO 2.0

Core element	Dimension
Assessment	*Systematic use of assessment strategies and measurement practices to obtain and provide feedback on student learning growth, attainment and wellbeing capabilities* *Systematic use of data and evidence to drive the prioritisation, development, and implementation of actions in schools and classrooms*

Link summary

- Mathematics Online Interview
- Fractions and Decimals Online Interview
- Scaffolding Numeracy in the Middle Years
- Assessment for Common Misunderstandings
- ABLES for mathematics (students with disability and/or additional needs)
- Digital Assessment Library – Mathematics

4: Priorities for Foundation to Level 4

4.1 Foundation to Level 2

Key messages

- Numeracy development is critical in the early years of schooling
- Effective teachers of early numeracy
  - focus on important mathematical ideas
  - structure purposeful tasks
  - use a range of materials/representations/contexts
  - use teachable moments as they occur
  - make connections to previous mathematical experiences
  - use individual and group structures within lessons
  - use a range of question types to probe and challenge thinking and reasoning
  - have high but realistic expectations of all children
  - use reflection to draw out key mathematical ideas
  - use a variety of assessment methods
  - display confidence in their own knowledge of mathematics and a belief that mathematics can and should be enjoyable.
- Teachers need to recognise and build on the prior to school numeracy experiences of children, particularly cultural home experiences
- Play needs to be a significant feature of effective early years' numeracy pedagogy, balancing teacher-led, teacher-initiated, and child-initiated activity
- Parents/carers have an important role to play in helping their child develop a positive attitude, motivation to learn, and academic achievement in mathematics.

A comprehensive study undertaken in Victoria (Clarke et al., 2002) identified several characteristics common to effective teaching of early numeracy.

As part of the Early Numeracy Research Project, they found that effective teachers of numeracy in the early years:

- focus on important mathematical ideas and make the mathematical focus clear to children
- structure purposeful tasks that engage children and enable different possibilities, strategies, and products to emerge
- use a range of materials/representations/contexts for the same concept
- use teachable moments as they occur and make connections to previous mathematical experiences
- use an introductory whole group activity to engage children and then a variety of individual and group structures within the lesson
- use a range of question types to probe and challenge children's thinking and reasoning and encourage children to explain their mathematical thinking/ideas

- have high but realistic expectations of all children
- use reflection to draw out key mathematical ideas during and/or after the lesson
- use a variety of assessment methods and modify planning because of assessment
- display confidence in their own knowledge of mathematics at the level they are teaching and a belief that mathematics can and should be enjoyable.

The following three priorities have been identified as key areas that impact students' disposition towards mathematics as well as their understanding of key concepts in the early years of primary education.

4.1.1 Prior to school experiences

As part of a wider project investigating how mathematics learning might be supported as children start school, Goff, et al. (2013) found that "numeracy might not be a key priority for schools as children make the transition from preschool to primary school" (p. 367). An extensive body of literature, however, stresses the importance of recognising the prior to school numeracy experiences many young children bring to formal schooling (e.g., Gervasoni and Perry, 2013; Knaus, 2013; MacDonald, 2018; Worthington, 2018).

Young children often bring a great deal of mathematical knowledge and dispositions for learning mathematics to formal schooling, with much of it having been learned from everyday experiences (Perry, Dockett and Harley, 2012). A review of the research noted the predictive power of mathematical knowledge at school entry for later mathematical achievement (MacDonald, 2018).

Worthington (2018) noted that teachers are often not aware of children's mathematical 'funds of knowledge' (Moll, Amanti, Neff and Gonzalez, 1992) or the rich cultural knowledge they have developed. She describes several examples which demonstrate how children's home cultural experiences are manifested in pretend-play and other child-initiated activities in early educational settings.

For example, Worthington documented the case of 'Shereen' whose funds of knowledge and cultural home experiences were linked with buying, preparing, ordering, and eating food, which were highly regarded as significant social experiences. All Shereen's pretend-play narratives were centred on food, either shopping for food or playing cafes where she took orders for food, informed customers of items available and their cost and even admonished those who failed to eat everything.

4.1.2 The importance of play

Play is regarded as a significant feature of effective early years' pedagogy and has a focus on how children learn rather than what children learn (Tucker, 2014).

Play can take many forms, and may include 'free-flow play' (Bruce, 1991), where play is freely chosen by the child and is characterised by:

- child initiated activity in a meaningful context
- child controlled activity and ownership of the activity by imagining, making decisions and predictions
- experimenting with strategies and taking risks in a safe context
- showing curiosity
- seeking pleasure from the essence of the activity.

According to Fisher (2010), to fully support mathematical and/or numeracy development, playful activity requires adult involvement at some level; a balance of practitioner-led, practitioner-initiated, and child-initiated activity is desirable.

Others (e.g., Worthington, 2018) warn against the "pedagogization of play" that is planned or structured to meet curriculum targets (Worthington, 2018, p. 254), resulting in play being marginalised and misunderstood. She argues that spontaneous and sustained social pretend-play can create a rich social-ecocultural niche in early childhood. It also provides opportunities to improve students' critical thinking, problem-solving, and collaborative skills, which are important to their development as maths learners.

This spontaneous play often occurs in contexts that are meaningful to young children, thus providing an ideal opportunity to promote numerate behaviour.

The Victorian Early Years Learning and Development Framework (VEYLDF) (DET, 2016, p. 14) establishes that play is essential to stimulate and integrate a wide range of children's intellectual, physical, social, and creative abilities. The framework distinguishes between integrated child-directed play and learning, guided play and learning, and adult-led learning, but acknowledges that all are effective in advancing children's knowledge.

Play is explicitly referred to in the VEYLDF Practice Principle 7: Integrated Teaching and Learning approaches where it is named as an essential element in early childhood learning and development. Furthermore, play has been shown to:

- Allow for the expression of personality and uniqueness
- Enhance dispositions such as curiosity and creativity
- Enable children to make connections between prior experiences and new learning
- Assist children to develop relationships and concepts
- Stimulate a sense of wellbeing. (DEEWR, 2009)

According to MacDonald (2018), play contexts provide opportunities for children's spontaneous invention of their own mathematical strategies where they will often use their lived experiences to inform the play, resulting in solving problems presented in the play.

4.1.3 At home with numeracy

"Even though it looked like you were just playing a game, you could see the benefit of the numeracy and the maths skills in it" *(Muir, 2018, p. 173).*

There is general consensus in the literature that parental involvement affects student achievement (Sheldon and Epstein, 2005), and that students' learning is maximised when strong educational partnerships between school, community and home are developed (Groves, Mousley and Forgasz, 2006).

In terms of the relevance for early numeracy development, the literature reviewed showed that parents remain largely uninformed about contemporary mathematics practices and how they can assist in developing their child's numeracy at home (e.g., Muir, 2009; Muir, 2012; Pritchard, 2004).

There are several initiatives and programs that have been documented in the literature that have been shown to build an increased understanding of contemporary mathematics classroom practices (Muir, 2018). Sheldon and Epstein (2005) found, for example, that several activities that were effective included evening workshops, involving both parents and children, and provision of teacher-designed interactive homework and mathematics materials for families to use at home.

Other research has reported on the use of 'take home' packs (Goos and Jolly, 2004), numeracy bags (Muir, 2009) and mathematics backpacks (Orman, 2000).

Key resource

Numeracy @ Home Suite

The department has published a suite of resources to support families with young children to explore everyday mathematics and numeracy in fun and interactive ways. The suite includes:

Everyday Maths Animations

- *Three animations (each approximately three minutes long) to support families to engage in conversations about mathematics and numeracy in everyday activities. Available in several community languages.*

The Mathscots and Teacher Guide

- *An animation series, comprising 10 stand-alone episodes (each about 60-90 seconds long), featuring four dogs who have an interest in mathematics. Initially designed to encourage families of children aged 5 to 8 to explore maths and numeracy at home. A Mathscots Teacher Guide is now available which provides a range of class-based activities aligned with the Victorian Curriculum: Mathematics at Foundation to Level 2.*

Flip, Make, Play Cardsets and Teacher Guide

- *A pack of eight cardsets that focus on early algebra, geometric visualisation, properties of shapes, position and location, measurement, data reasoning, and the likelihood of events. Cards support families to play games, create patterns, make shapes, and spend time together exploring numbers and shapes. A Parent Information Booklet is available. A new Flip Make Play Teacher Guide provides four class-based activities for each of the eight card sets: activities comprise child-centered pedagogies focussed on:*
    - *developing and extending children's understanding of the underpinning mathematics*
    - *promoting mathematics discourse and reasoning.*
- *Activities are aligned with the Victorian Curriculum: Mathematics at Foundation.*

Key resource

Mathematics Monograph: Engaging families in mathematics education

by Leicha A. Bragg, Sandra Herbert and Jill P. Brown, Senior Lecturers in Mathematics Teacher Education, Deakin University

This monograph explores how important families are to a child's attitude, motivation to learn, and academic achievement in mathematics. It provides recommended actions and helpful tips to show how teachers and schools can encourage families to engage more with mathematics at home.

Reflective prompts	
School leaders	Drawing on Early Numeracy Research Project findings, how effective is your school's teaching of numeracy/mathematics in the early years? What is already strong? What needs to be strengthened? To what extent are *Prior to school experiences*, *The importance of play* and *At home with numeracy* priorities at your school? Which one might most need to be strengthened? What professional learning opportunities might you need to provide to teachers and support staff to enact this new focus for action? How could the Numeracy @ Home Suite of resources be utilised by your school to build the numeracy capability of children alongside their families?
For leaders to use with teachers	Select 2-3 of the Early Numeracy Research Project findings that resonate with you most. Why do they matter to you? Which other findings might you explore through your practice in the coming weeks and months? What do you currently do to identify and build from your students' prior knowledge and experiences in F-2? What might you add to this repertoire of practices? To what extent do you build play into your numeracy/mathematics program in F-2? What are 2-3 key opportunities to enhance play for the benefit of your students? How well do you currently facilitate family engagement in your students' numeracy/maths learning? What opportunities are there for expanding this focus in coming planning?

FISO 2.0

Core element	Dimension
Teaching and learning	*Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships*
Engagement	*Activation of student voice, agency, leadership and learning to strengthen students' participation and engagement in school* *Strong relationships and active partnerships between schools and families/carers, communities, and organisations to strengthen students' participation and engagement in school*

Link summary

- Mathematics Monograph: Engaging families in mathematics education
- Everyday Maths Animations
- The Mathscots and Teacher Guide
- Flip, Make, Play Cardsets and Teacher Guide
- Other practical examples of the teaching of numeracy through play: Birth to Level 2 sections of each of the Numeracy Focus Areas in the Birth to Level 10 Numeracy Guide

4.2 Levels 3 and 4

Key messages

- Number sense is critical to student mathematical development as they progress through primary school. It includes:
  - understanding of number concepts and operations
  - application of numbers and operations be able to work flexibly with two-digit and three-digit numbers
  - the ability and inclination to:
    - use understanding in flexible ways to make mathematical judgements
    - develop useful strategies for handling numbers and operations
- A 'Big Ideas' approach to teaching enables students to develop a deeper understanding of mathematics and its interconnectedness, both within the world of mathematics and between the world of mathematics and the real world.
- Programs need to be designed to ensure that students ask questions, engage in dialogue, and use efficient mental and written strategies with increasing fluency to solve a range of problems.

The following three priorities have been identified as key areas that impact students' disposition towards mathematics as well as their understanding of key concepts in the middle years of primary education.

4.2.1 Building number sense

Number sense is critical to student mathematical development as they progress through the years of primary school. McIntosh, Reys and Reys (1997) define number sense as:

> **"a person's understanding of number concepts, operations, and applications of numbers and operations. It includes the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful strategies for handling numbers and operations"** (p. 10).

McIntosh, Reys, Reys, Bana and Farrell (1997) emphasise three major components of number sense that all students need to learn (p. 64):

- **Number concepts:** understanding the meaning, size, orderliness, and representations of numbers; this includes relative size, that is, developing a sense of the size of a number in relation to other numbers, including benchmarks
- **Effects of operations**: understanding mathematical properties and the relationship between operations
- **Computational strategies:** awareness of multiple strategies, and being alert to reasonableness of numbers, solutions, and strategies.

Livy (2019) describes these measurement concepts as essential numeracy, emphasising the importance of the following **measurement** key understandings:

- **comparison** of attributes: which includes paying attention to where the object starts (e.g., having the same origin) and the size of the unit being used as a measurement tool (e.g., minutes or hours)
- **estimation** of measures and using benchmarks to know which is bigger, heavier, or longer
- **iteration**: that is, repeating the same unit and the **use of instruments** (e.g., rulers)
- **connections**: linking formal and informal units, and understanding how length, area and volume are connected, but not the same.

In a number sense classroom Clarke (2022) encourages teachers to ask:

- How did you get your answer?
- Can you explain it another way?
- Did anyone think about it differently?
- What is the next question I'm going to ask you?
- What do you 'see' when I say …?

Key resource

[Mathematics Curriculum Companion]

The Mathematics Curriculum Companion is a website which unpacks the Victorian: Curriculum Mathematics from Level A to Level 10A. It:

- *is organised by strand, sub-strand, and level*
- *incorporates the proficiencies*
- *unpacks each content description, providing teachers with relevant content knowledge*
- *provides suggested teaching and learning ideas, which are numeracy-rich*
- *provides links to quality-assured interactive tools and videos, including from ClickView.*

The Companion has a new feature which supports teachers to differentiate their programs. This feature allows them to "follow" concepts up and down the learning continuum, supporting them to design and plan, teach, and assess at the point of student need.

4.2.2 The role and use of algorithms

***Usiskin defines an algorithm as "a finite, step-by-step procedure for accomplishing a task that we wish to complete"** (1998, p.7).*

In their large scale study of the development of addition and subtraction strategies, Gervasoni, Giumelli and McHugh (2017) analysed growth point data from the department's Mathematics Online Interview tool for nearly 22,000 Australian primary school students. Findings highlighted:

- a broad range of strategies used by students in each grade level
- that few students (only 21%), including those in Grade 6, reach Growth Point 6, which involves the mental calculation of two-digit and three-digit numbers.

The researchers argue that until students reach Growth Point 6, they are unable to deal with complex algorithms. As such, they recommend that primary school teachers spend more time on developing number sense and flexible mental strategies, rather than focusing on approaching calculations in a procedural way.

4.2.3 Building mathematical connections

Askew (1999) describes effective teachers as those that pay attention to building connections. This includes connections between:

- different aspects of mathematics (e.g., between trusting the count, addition, subtraction and place value or between fractions, decimals and percentages and multiplicative thinking)
- different representations of mathematics, moving flexibly between words, objects, diagrams, and symbols, and multiple representations
- students' methods – valuing these and being interested in students' thinking but also sharing and building on their methods.

Rather than thinking of curriculum descriptors as lists of dot points to be checked off, the proficiencies can serve as organising framework that fosters a connected view of the curriculum. Moreover, adopting a 'Big Ideas' (Siemon, 2022) approach can help students build connections and transfer knowledge across a range of contexts (see breakout box, above, [Teaching with the Big Ideas in Mathematics]).

Key resource

Mathematics Monograph: [Teaching with the Big Ideas in Mathematics]

by Dianne Siemon Emeritus Professor, RMIT University

This paper explores the 'Big Ideas' in mathematics, which provide a basis for a more coherent approach to the teaching and learning of mathematics. 'Big Ideas':

- *are ideas, strategies, or ways of thinking about a part of mathematics without which students' progress will be seriously impacted*
- *encompass and connect with other ideas and strategies.*

A 'Big Ideas' approach enables students to develop a deeper understanding of mathematics and its interconnectedness, both within the world of mathematics and between the world of mathematics and the real world. Research shows that highly effective teachers of numeracy are those that prioritise making such connections. See *[Teacher belief orientations and dispositions] in Chapter 4 for further information.*

Reflective prompts	
School leaders	How well is the development of number sense embedded in classroom practice across Levels 3-4 at your school? What classroom dialogue and/or learning artefacts demonstrate this? To what extent are your mathematics programs across Levels 3-4 prioritising the development of flexible mental strategies? Which big idea do you your teachers focus on in their teaching across Levels 3-4? How are these decisions connected to the decisions at Levels F-2 and 5-6 (i.e., so that

	big ideas are developed across the years of primary schooling)? How knowledgeable are your middle leaders and teachers about number sense, and about the big ideas of mathematics across Levels 3-4? If professional learning needs are apparent, what steps will you take to address this?
For leaders to use with teachers	How well is the development of number sense embedded in your classroom practice across Levels 3-4? What classroom dialogue and/or learning artefacts demonstrate this? To what extent does your mathematics program prioritise the development of flexible mental strategies? What might be strengthened? Which big idea do you focus on in your teaching? What about other teachers across Levels 3-4? What 2-3 opportunities might there be to strengthen your focus on the big ideas, individually and with colleagues?

FISO 2.0

Core element	Dimension
Teaching and learning	Documented teaching and learning program based on the Victorian Curriculum and senior secondary pathways, incorporating extra-curricular programs
	Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships
Engagement	Activation of student voice, agency, leadership and learning to strengthen students' participation and engagement in school

Links summary

- Mathematics Curriculum Companion
- Mathematics Monograph: Teaching with the Big Ideas in mathematics

5: Priorities for Levels 5 to 9

Key messages

- Middle years mathematics programs should be designed to foster students' behavioural, cognitive, and emotional engagement. This means that learning experiences should be designed to be relevant to students, now and into the future.
- Programs should also include an increasing variety of task types, lesson structures and activities that cater for diverse learners.
- Across Levels 5-9, students need to extend their number sense knowledge and skills to understand rational numbers. It is also essential that learning experiences explicitly link to key constructs, ideas, representations, and models of number.

- There are high rates of out-of-field teaching of mathematics in Australia. It is important for school leaders to:
    - understand out-of-field teaching and its impact on student performance, particularly across Years 7 to 10
    - employ responses that focus on reducing and managing out-of-field teaching.
- All students should have opportunities to learn mathematics in mixed-ability classes.

The following four priorities have been identified as key areas that impact students' disposition towards mathematics as well as their understanding of key concepts in the middle years of primary and secondary education.

5.1.1 Behavioural, cognitive, and emotional engagement

Engagement in mathematics is more than just 'having fun.' Engagement in mathematics operates at three levels: cognitive, affective, and behavioural (Fredricks, Blumenfeld and Paris, 2004).

When viewed through a mathematical lens, engagement occurs when:

- mathematics is a subject students enjoy learning
- students value their mathematics learning and see its relevance in their own lives, now and in the future
- students see connections between the mathematics they learn at school and the mathematics they use outside school (Attard, 2012, p. 10).

Key resource

Levels 5 to 8 Learning Sequences

A new website provides teachers with access to rich and engaging learning sequences for English and mathematics covering Levels 5 to 8 of the Victorian Curriculum. Sequences are comprehensively mapped to the Victorian Teaching and Learning Model, the Literacy Teaching Toolkit, and the Mathematics Teaching Toolkit. Sequences have been developed by seconded teachers and were comprehensively reviewed. They include assessment ideas and are customisable.

Key resource

Middle Years Maths Challenges

A new set of e-books have been published, providing schools, teachers, and students with opportunities to engage in authentic, interesting, and creative mathematics experiences across Levels 5 to 9 of the Victorian Curriculum: Mathematics. There are 75 Middle Years Maths Challenges – 13 challenges per level as well as 10 challenges designed specifically for students who need extra support. Each Challenge has a mathematics and a proficiency focus. Each Challenge has an accompanying assessment rubric. 7 challenges focus on exploring Aboriginal and Torres Strait Islander histories, cultures, and perspectives.

Framework for Engagement with Mathematics

Reproduced with permission from Catherine Attard's "I don't like it, I don't love it, but I do it and I don't mind": Introducing a framework for engagement with mathematics (2014).

In the mathematics classroom engaged students are:

- actively participating (e.g., in group discussions, practical, relevant activities and homework tasks
- genuinely valuing (e.g., "this learning will be useful to me in my life outside the classroom")
- reflectively involved in deep understanding of mathematical concepts and applications, and expertise.

In an engaging mathematics classroom, positive pedagogical relationships exist where:

- students' backgrounds and pre-existing knowledge are acknowledged and contribute to the learning of others
- interaction amongst students and between teacher and students is continuous
- the teacher models enthusiasm and an enjoyment of mathematics and has a strong pedagogical content knowledge
- the teacher is aware of each student's abilities and learning needs
- feedback to students is constructive, purposeful, and timely

Pedagogical repertoires mean:

- there is substantive conversation about mathematical concepts and their applications to life
- tasks are positive, provide opportunities for all students to achieve a level of success, and are challenging for all
- students are provided with an element of choice
- technology is embedded and used to enhance mathematical understanding through a student-centred approach to learning
- the relevance of the mathematics curriculum is explicitly linked to students' lives outside the classroom and empowers students with the capacity to transform and reform their lives
- mathematics lessons regularly include a variety of tasks that cater to the diverse needs of learners

Key resource

Career Education Resources: Mathematics

The VCAA has published a set of career education resources for use by teachers of mathematics. The resources provide practical examples of how an existing activity can be changed, adapted, or extended to enrich the career-related learning embedded within the activity. Such examples provide rich, relevant, and real contexts for mathematics learning.

5.1.2 Extending number sense

Across Levels 5 to 9, students need to extend their number sense knowledge and skills to understand rational numbers (e.g., fractions, decimal fractions, and percentages). It is essential that sequenced learning experiences also explicitly link to the key constructs, ideas, and models (Pearn, 2019).

The **key constructs** for rational number are:

- part-whole (e.g., part of a length, shading of a region, part of a group)
- measurement (i.e., focus is on how much, rather than how many parts)
- division (e.g., fairly share $10 with 4 people)
- operator (e.g., ⅔ of a class are football fans)
- ratio (i.e., can be representing part:whole or part:part, and therefore essential to understand the context).

The **key ideas** are quality, equivalence, benchmarking, and partitioning.

Multiple representations help students embed learning, generalise, and make connections with various **models** (e.g., area, length, and set/collection). These representations should include visual, language and symbolic (VLS). Through these VLS representations students are then able to create evidence of their mathematical reasoning (Sexton, 2019) and create deep understanding and connections within and between concepts.

Key resource

[Mathematics Curriculum Companion]

The Mathematics Curriculum Companion is a website which unpacks the Victorian: Curriculum Mathematics from Level A to Level 10A. It:

- *is organised by strand, sub-strand, and level*
- *incorporates the proficiencies*
- *unpacks each content description, providing teachers with relevant content knowledge*
- *provides suggested teaching and learning ideas, which are numeracy-rich*
- *provides links to quality-assured interactive tools and videos, including from ClickView.*

The Companion has a new feature which supports teachers to differentiate their programs. This feature allows them to "follow" concepts up and down the learning continuum, supporting them to design and plan, teach, and assess at the point of student need.

5.1.3 Out-of-field teaching

There are high rates of out-of-field teaching of mathematics in Australian schooling.

A 2020 report by Deakin University (Hobbs et al, 2020) showed that out-of-field teaching:

- is at least twice as high in lower secondary (Years 7 to 10) as in upper secondary (Years 11 and 12)
- is more likely in rural and regional settings than metropolitan settings
- is more common for teachers in their early career when they are least equipped to be effective and is likely to negatively impact retention of early career teachers

Additional data collected by the department in late 2021 shows that the proportion of out-of-field teachers in a school is correlated to the school's Student Family Occupation and Education (SFOE) score.

The 2020 report also showed that:

- teachers are less likely to undertake professional learning in out-of-field subjects
- experienced 'in-field' teachers can provide highly effective subject-related support for out-of-field teachers.

Out-of-field mathematics teaching can be managed locally by:

- supporting teachers to undertake a qualification (e.g., graduate certificate) to qualify as 'in-field' as a mathematics teacher
- avoiding or limiting out-of-field teaching loads for beginning and early career teachers
- avoiding or limiting changes to out-of-field teaching loads year-on-year (i.e., if you give a teacher and out-of-field mathematics load in one year, avoid adding out-of-field science teaching the next year)
- providing out-of-field teachers with access to experienced in-field mentors who are willing and able to support them
- facilitating (e.g., through timetabling; time release) collaborative planning, teaching, and assessment, where out-of-field teachers work alongside experienced in-field colleagues
- encouraging, facilitating, and prioritising out-of-field teachers' access to:
    - high-quality learning and teaching resources (see breakout box above: Levels 5 to 8 Learning Sequences)
    - professional learning
    - professional networks

Key resource

Levels 5 to 8 Learning Sequences

A new website provides teachers with access to rich and engaging learning sequences for English and mathematics covering Levels 5 to 8 of the Victorian Curriculum. Sequences are comprehensively mapped to the Victorian Teaching and Learning Model, the Literacy Teaching Toolkit, and the Mathematics Teaching Toolkit. Sequences have been developed by seconded teachers and were comprehensively reviewed. They include assessment ideas and are customisable.

5.1.4 Mixed ability grouping

There is no evidence from research syntheses and meta-analyses to suggest that between class ability grouping (i.e., streaming/tracking) is associated with improved mathematics achievement.

While small positive achievement benefits have been reported for gifted students undertaking specialist programs and for within class ability grouping under certain conditions, the research that underpins these two claims tends not to consider the impact of ability grouping on students' self-confidence, efficacy, or long-term social standing (Siemon, 2022; Clarke and Clarke (2008)).

This view found support in the National Numeracy Review Report (Council of Australian Governments, 2009), which recommended that the use of same ability grouping in primary and junior secondary schooling be discouraged given the evidence that it contributes to negative learning and attitudinal outcomes for less well achieving students and yields little positive benefit for others (p. 4).

The preferred approach is, instead, opportunities to learn mathematics in mixed-ability classes as they have been shown to lead to better cognitive and social outcomes for all students (Siemon, 2022). For example, in a US study, Boaler and Staples (2008) found that where mathematics was taught in mixed-ability classes using a student centred approach (i.e., rich tasks, collaboration, and a focus on understanding and reasoning), the students not only outperformed their peers at the other two schools that streamed their classes and taught mathematics traditionally, they also “learned more, enjoyed mathematics more and progressed to higher mathematics levels” (p. 609).

Key resource

Mathematics Monograph: Ability grouping in mathematics

by Dianne Siemon Emeritus Professor, RMIT University

This paper explores the issue of ability grouping in mathematics, focussing on within class ability groupings and between class groupings. It invites school leaders and teachers to consider the issue of ability grouping in school mathematics around the following questions:

1. What is the problem that ‘ability’ grouping, in its various forms, seeks to address?

2. How can we organise the teaching and learning of mathematics to support all learners?

Reflective prompts	
School leaders	How engaged are your Levels 5-6/7-9 students: behaviourally, cognitively, and emotionally? What classroom dialogue and/or learning artefacts demonstrate this? How well do your Levels 5-6/7-9 mathematics programs cater for diverse learners, including having a variety of task types, lesson structures and activities? What might be strengthened? Across your programs and the years of schooling, how well is number sense continuing to be built? What might need to be strengthened? To what extent is out-of-field teaching in mathematics occurring in your context? What is working well in your efforts to manage it locally? What actions might you take to further reduce its impact on student learning outcomes? How and why do you group students in your mathematics classes? What works well in the current approach? Where are the challenges, and what steps might you take to ensure equitable outcomes for all students?
For leaders to use with teachers	How engaged are your Levels 5-9 students: behaviourally, cognitively, and emotionally? What classroom dialogue and/or learning artefacts demonstrate this? How well do your mathematics programs cater for diverse learners, including having a variety of task types, lesson structures and activities? What might be strengthened? Across your program, how well is number sense continuing to be built? What might need to be strengthened?

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Core element **Dimension**

Leadership	*The strategic direction and deployment of resources to create shared goals and values; high expectations; and a positive, safe and orderly learning environment* *Shared development of a culture of respect and collaboration with positive and supportive relationships between students and staff at the core*
Teaching and learning	*Documented teaching and learning program based on the Victorian Curriculum and senior secondary pathways, incorporating extra-curricular programs* *Use of common and subject-specific high impact teaching and learning strategies as part of a shared and responsive teaching and learning model implemented through positive and supportive student-staff relationships*
Engagement	*Activation of student voice, agency, leadership and learning to strengthen students' participation and engagement in school*

Link summary

- Levels 5 to 8 Learning Sequences
- Mathematics Curriculum Companion
- Middle Years Maths Challenges
- Career Education Resources: Mathematics
- Mathematics Monograph: Ability grouping in mathematics

6: Priorities for Levels 10 to 12

Key messages

- School-based curriculum programs at Year 10 need to provide every student with the opportunity to reach the Level 10 achievement standard in the Victorian Curriculum: Mathematics
- Teachers of all school subjects should continue to focus on developing the numeracy capabilities of their students across Year 10, 11 and 12.
- Students should be discouraged from 'downgrading' their mathematics studies in the senior secondary years. Instead, they should be encouraged to choose the mathematics subject/s that interest them and offer the best preparation for destinations beyond school.

6.1 Numeracy for all learners

6.1.1 Year 10

The Pathways stage (Years 9 and 10) of the Victorian Curriculum Foundation to Level 10 is structured to ensure that students engage in a broad education *and* begin to plan their senior secondary program of study. This can create a challenge for secondary schools, who need to manage and balance this dual focus.

It is important that school-based curriculum programs at Year 10 provide every student with the opportunity to reach the Level 10 achievement standard in the Victorian Curriculum: Mathematics, and that teachers of all school subjects continue to focus on developing the numeracy capabilities of their students (as per 2.2.2 Numeracy across the curriculum, above).

This is because “fundamental mathematical knowledge, skills, understandings and dispositions [are essential for students] to solve problems in real life contexts for a range of workplace, personal, further learning and community settings relevant to contemporary society” (VCAA, 2021).

For example, in their 2018 study of the numeracy practices and skills of 250,000 adults across 33 countries/economies, the OECD found that “people with very strong skills in numeracy, and those who make frequent use of numeracy in a wide range of situations, benefit from a considerable comparative health advantage over other adults, all things being equal” (Jonas, 2018, p. 1).

They also found that adults are increasingly responsible for making complex health decisions, and that higher numeracy skills are linked with adults making effective medical decisions through analysis of risk related to treatment or non-treatment choices.

Higher numeracy proficiency has also been strongly linked with higher rates of employment and, where employed, higher wages (OECD, 2016a). In contrast, lower numeracy proficiency has been linked with an increased likelihood as a young person to be involved with the criminal justice system, including community and custodial sentences (NSW Department of Attorney General and Justice, 2012).

6.1.2 Years 11 and 12

Senior secondary education is usually undertaken by students in Years 11 and 12. Senior secondary education is important as it prepares students for initial entry into the workforce, for vocational education and training and higher education studies (VRQA, 2022). Yet it can be a time when a focus on building the numeracy capability of students can weaken.

Secondary schools need to ensure that senior secondary programs continue to support numeracy development in all learners. This is because mathematics underpins success in many studies, particularly the mathematics proficiencies of reasoning and problem solving. Below are two examples of questions from recent Victorian Certificate of Education (VCE) written examinations, which illustrate this importance:

Example 1: Business Management (2018; © VCAA, reproduced by permission)

Question 3 (16 marks)

The manager of Wilkinson’s Window Tinting was disappointed with the performance of the business after one year of trading. As a result, she decided to purchase new machinery and automate the tinting process. At the end of the second year of trading, the manager examined a range of key performance indicators in order to assess the extent to which this change had been successful.

Key performance indicator	Year 1	Year 2
net profit figure	$47 000	$23 000
rate of productivity growth	2%	8%
number of customer complaints	112	28
rate of staff absenteeism	average of four days per year per staff member	average of 12 days per year per staff member

b. With reference to the data above, analyse the extent to which the purchase of the new machinery has assisted Wilkinson’s Window Tinting in improving the performance of the business. 6 marks

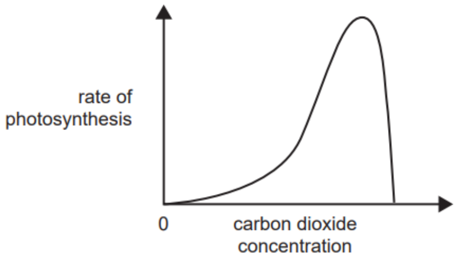
Example 2: Biology (2021; © VCAA, reproduced by permission)

Question 6

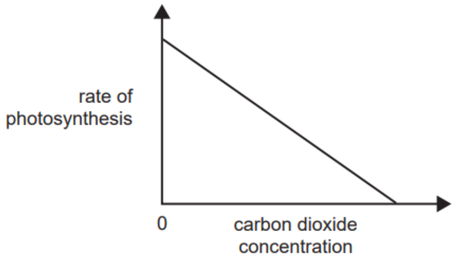
In a series of experiments, the rate of photosynthesis in plant cells was measured in environments with different concentrations of carbon dioxide. All other variables were kept constant.

Which one of the following graphs reflects the trend that would be shown by the results of these experiments?

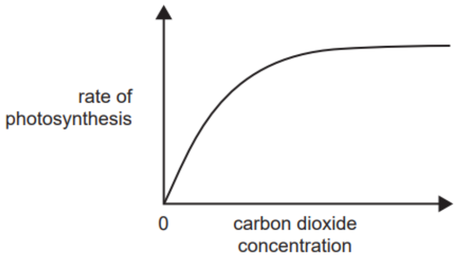
A.



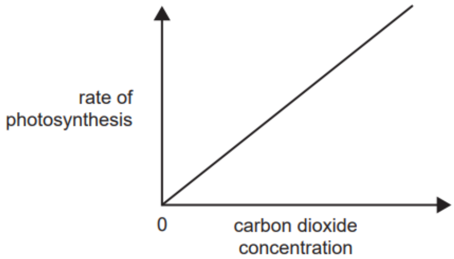
B.



C.



D.



Numeracy and mathematics in senior or foundation secondary qualifications from 2023

From 2023 there are 3 VCAA certificates that may be awarded:

- the Victorian Certificate of Education (VCE), which includes the VCE Vocational Major (VCE VM)
- the Victorian Pathways Certificate (VPC)
- the Intermediate Victorian Certificate of Applied Learning (VCAL), for students who meet enrolment guidelines (note: only being awarded in 2023)

VCE mathematics studies from 2023 are:

- Vocational Major Numeracy Units 1 to 4
- Foundation Mathematics Unit 1 to 4
- General mathematics Unit 1 to 4
- Mathematical Methods Unit 1 to 4
- Specialist Mathematics Unit 1 to 4

Visit the VCAA website for the study designs of these studies.

The VCE VM and the VPC include compulsory numeracy units. See VCE Vocational Major Numeracy for further information.

6.1.3 The GAT in 2022

As part of the reform of the VCE and VCAL system, Victorian senior secondary students will this year have their literacy and numeracy skills measured against new standards in a reformed GAT.

The new standards will indicate whether students have demonstrated the literacy and numeracy skills typically expected of someone completing their secondary schooling – giving another indication of their readiness to move onto further education, training, or employment.

The reformed GAT underscores the importance of Years 10 to 12 programs continuing to support numeracy development in all learners.

For more information on the reformed GAT, see the VCAA's The GAT in 2022 webpage.

6.2 Mathematics for pathways

There is variation in the range, focus and difficulty of maths subjects offered at VCE. However, all VCE mathematics studies have similarities when it comes to the knowledge and skills students will develop. They also teach students how to think, reason and communicate mathematically, describe, and analyse data and evidence, and use digital technologies.

Mathematics is essential to a range of study and career choices — including vocational trades, nursing, teaching, and mathematical sciences.

It is therefore important that students are discouraged from 'downgrading' their mathematics studies in the senior secondary years (e.g., choosing what may be a less appropriate mathematics study for their pathway due to a perception that it will be easier). Instead, students should be encouraged to choose the mathematics subject/s that interest them and offer the best preparation for destinations beyond school.

Level 10A of the Victorian Curriculum: Mathematics provides optional, additional content for students to be extended in their mathematical studies in preparation for senior secondary studies.

To learn more about initiatives to improve career education in Victorian government schools, see Transforming career education.

Reflective prompts	
School leaders	To what extent are your Year 10 programs: i) providing every student with the opportunity to reach the Level 10 achievement standard in the Victorian Curriculum: Mathematics, and ii) continuing to focus on developing the numeracy capabilities of your students across the curriculum? How well do your VCE teachers understand the numeracy demands of the subjects they teach? How might VCE study designs or examinations be used to establish and strengthen a numeracy focus? How well informed are your staff and students on the range of study and career choices available to them for which mathematics is essential? What might be strengthened? (Initiatives to improve career education in Victorian government schools are listed at Transforming career education). To what extent are students choosing mathematics subject/s that interest them and offer the best preparation for destinations beyond school? If they aren't, what's

	stopping them and what might the school do in response?
For leaders to use with teachers	As a Year 10, 11 or 12 teacher, how well do you understand the numeracy demands of the subjects you teach? In your own practice, what might be strengthened to establish and strengthen a numeracy focus in your classes?

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Core element	**Dimension**
Teaching and learning	*Documented teaching and learning program based on the Victorian Curriculum and senior secondary pathways, incorporating extra-curricular programs*
Assessment	*Systematic use of data and evidence to drive the prioritisation, development, and implementation of actions in schools and classrooms*
Support and resources	*Responsive, tiered and contextualised approaches and strong relationships to support student learning, wellbeing and inclusion*

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