# Levels 9/10 MUSIC Activity

## Taking a Chance with Aleatoric Music

### Introduction to Numeracy in Music

The Music curriculum is structured around four interdependent strands, which include content descriptors and elaborations, each of which involves making and responding (VCAA, n.d.-a, n.d.-b):

* explore and express ideas
* music practices
* present and perform
* respond and interpret.

When making and responding, students develop musical skills and knowledge through activities involving the practices of listening, composing, performing and additional learning to support these musical practices.

Although the nature of music itself is abstract, it is a uniquely aural art form that requires both creative and critical thinking (Crawford, 2020a). Music students develop skills that are “transferable across learning contexts and support development of literacy and numeracy capabilities” (Australian Curriculum, Assessment and Reporting Authority [ACARA], n.d.). Numeracy skills relevant to students’ understandings in Levels 7 to 10 Music (as preparation for Levels 11 and 12 Music) include the following competencies:

* Identifying and analysing how the elements of music are used in different styles and genres (e.g., understanding and comparing patterns in the elements)
* Manipulating the elements of music and stylistic conventions to compose music (e.g., using critical thinking and logical reasoning with creative expression to experiment with different musical combinations and compositional devices)
* Using aural skills, music terminology, and symbols to recognise, memorise, and notate music performed or composed (e.g., investigating rhythmic and melodic patterns through sequencing numbers to identify patterns or rules, such as using number groupings to identify simple or compound time)
* Working with spatial thinking and number (e.g., through shapes, scale, proportion and ratio as applied to the basics of acoustics; units of measurement between musical intervals; identifying structures within musical works)
* Considering mathematical concepts such as rhythmic and interval patterns, alongside aesthetic aspects (e.g., beauty, style)
* Calculating, estimating, measuring, planning and creating musical works (e.g., using measurement to determine shape of a piece/song; melodic, harmonic, rhythmic and lyrical patterns; chord inversions)
* Positioning, locating, and sequencing of musical elements with and without technology (e.g., making a time-based or time-aligned work, such as film music, program music or sound effects)
* Using technical skills to develop mathematical thinking (e.g., using fractions as related to time signatures, aleatoric music to develop probability, algorithmic composition to apply formal sets of rules to determine musical outcomes)
* Problem-solving through visuospatial reasoning (e.g., understanding and recognising spatial relations among musical events, time and space. Playing a musical instrument requires musicians to translate musical symbols into specific motor actions while simultaneously monitoring the auditory signals produced by their instrument. This complex process allows musicians to develop strong associations between visual musical symbols, motor commands, auditory signals, and temporal patterns.

In Music, students continue to develop their numeracy skills in practical and conceptual ways, enabling them to apply this knowledge and skill set throughout their lifespan. The most relevant sub-elements of the numeracy learning progression for Music are: Number Patterns and Algebraic Thinking, Measurement and Geometry (basic acoustics), Comparing Units (ratios, rates, and proportion), Interpreting Fractions, and Interpreting and Representing Data (ACARA, n.d.). By incorporating mathematical concepts, students are able to address issues in music making and responding.

### Developing Numeracy Understanding in Music

Numeracy encompasses knowledge, skills, behaviours and dispositions (ACARA, n.d.). In Music, students engage with numeracy in multiple ways and contexts (e.g., using calculation, estimation, and measurement knowledge and skills to collect and make sense of information). When students learn to read and write music notation, they draw on their knowledge of fractions (e.g., halving, quartering, accumulating fractional parts, re-imagining the whole).

Through engagement with music, students can use and extend their numeracy capabilities when considering ‘the structure and form of music works, pitch (intervals, scales, octave identification), harmony, tuning systems, concepts relating to beat, pulse, rhythm, metre and sub-division and acoustics (ACARA, n.d.). Music making and responding involves learning to recognise and use patterns and sequences when composing, performing and listening. Reasoning and visuo-spatial skills can be developed through “singing, playing instruments and performing in ensembles” (ACARA, n.d.). Through analysing numerical data to research, interpret and evaluate evidence about musical works, students can determine how they are presented, performed, shared and appreciated.

The intimate relationship between music and mathematics predates the Pythagoreans, who deliberated over the connections between ratios and musical intervals (Pesic, 2013). Interdisciplinary contexts where content and concepts intersect and are made explicit in teaching and learning can result in powerful, authentic and meaningful learning (Crawford, 2020b). The impact and value of interdisciplinary learning lies in the potential for making connections not only across disciplines, but also within real-world contexts (Bazinet & Marshall, 2015; Law, 2018).

The diverse and rich links between music and mathematics include manipulating and combining the musical elements that relate to performing, composing and listening. Musical concepts (rhythm, melody, intervals, scales, harmony, tuning and temperaments) are directly related to mathematical concepts, such as proportions and numerical relations, integers, logarithms, arithmetical operations, and the content areas of algebra, probability, trigonometry and geometry (An et al., 2013; Harkleroad, 2006). Both mathematics and music involve the use of symbols to communicate and share ideas across times, locations, and cultures. In both disciplines, students can learn conventions for reading, interpreting, and writing these symbols. Students can discuss these ideas through performance and composition activities in which “different notation forms/systems such as graphic (semantic and non-semantic), spectrographic representation, culturally-specific notation, proportional notation, Western staff notation, interactive notation, historical and contemporary forms of tablature/TAB or types of chord notation” are explored (ACARA, n.d.).

Music can be harnessed by teachers to present mathematical problems in non-routine ways, providing students with opportunities to apply mathematical knowledge in authentic and meaningful contexts, connecting new knowledge to existing knowledge (Fllis & Fouts, 2001). In the past, educators have implemented several different instructional strategies to provide an interdisciplinary approach to teaching and learning music and mathematics, although the levels of integration have varied (An et al., 2011). Sellars (2018) proposed an integrated framework, designed to allow opportunities for reflection on social and cultural connections to the personal nature of numeracy. Goos et al. (2014) posited a numeracy development model that requires attention to real-life contexts; the application of mathematical knowledge; the use of representational, physical, and digital tools; and positive dispositions towards the application of mathematics.

Both Sellars’ (2008) and Goos et al.’s (2014) models closely relate to making and responding in Music, through the development of critical thinking and creative expression, to develop a unique musical voice within wider societies and cultures. Through explicit teaching and learning, teachers can highlight the interconnected nature of mathematical knowledge and skills with Music.

## Lesson Plan: Taking a Chance with Aleatoric Music

In this lesson, students become musicologists who apply their research to their own musical dice game compositions as they venture into the experimental world of aleatoric music. Students investigate the ideas, compositional devices and techniques that composers have used in creating aleatoric music in various historical contexts. Students will also evaluate and analyse the roles that aleatoric music has played in these diverse contexts and its evolution and influence on popular music genres and styles.

Aleatoric music, also called chance music (aleatory from Latin alea, meaning “dice”), is 20th century music in which chance or indeterminate elements are left for the performer to realise. The term was first coined by French composer Pierre Boulez to describe works where the performer was given certain liberties with some or all elements of a piece of music, such as the structure, sequence, and/or repetition of parts of a musical work. The term was intended, by Boulez, to distinguish his work from those composed through the application of chance operations by U.S. composer John Cage, who used strictly demarcated areas for improvisation according to specific directions and also unstructured pieces consisting of vague directives, such as playing a particular section for a specified period of time.

Musikalische Wurfelspiele (Musical Dice Games), popular in Western Europe in the late 18th and early 19th century could be considered an early precedent of aleatoric compositions. These games consisted of a sequence of musical measures/bars, for which each measure had several possible versions, and a procedure for selecting the precise sequence based on the throwing of a number of dice (Cope, 1996). Although unsubstantiated, the most well-known musical dice game is attributed to Mozart in 1792, where each dice roll results in randomly-selected small sections of music, which would be combined to create a musical piece (Hedges, 1978). This particular game is capable of producing 1116 = 45,949,729,863,572,161 different yet similar waltzes. Some measures have only one predetermined possibility, regardless of what the roll of the dice is (e.g., measure 8/16), whereas other measures have a different possibility for each roll (e.g., measure 1/16; Hedges, 1978). The premise was that from this already composed music, some aspects were left to chance, whereas others were carefully predetermined.

Students’ investigations and experimentations about how to create consonance and dissonance will reveal that the design of the game, including predetermined composed parts and limiting parameters and variables (e.g., using a particular mode or scale, or only certain notes contained within these modes/scales), is critical to the overall sound and tonal structure of the musical work.

Students will be required to problem solve how to convey musical instructions and can randomly generate various musical elements such as melody, rhythms, chords, dynamics, tempo/speed in bpm (beats per minute) and structure. Instrumentation can also be randomised; however, it is useful for students to utilise the instruments that their peers play in class and that they have available to them.

Considerations of using Western musical notation and/or graphic notation for musical scores and to convey instructions, and whether certain musical decisions will be left to performers, are also part of the decision-making process. Articulating a rationale for these aspects is an important part of making and responding in Music. Notable aleatoric works are *Music of Changes* (1951) for piano and *Concert for Piano and Orchestra* (1958) by John Cage, and *Klavierstück XI* (Keyboard Piece XI; 1956) by Karlheinz Stockhausen (German composer). Aleatoric technique is applied in many different popular musical styles and genres, especially electronic, rock and ambient music. Although the theme of this lesson, ‘Taking a Chance with Aleatoric Music,’ can relate to all four of the Music strands, the primary focus is on Explore and Express Ideas, Music Practices, and Respond and Interpret.

This lesson is informed by an experiential learning approach (Dewey, 1938) and Crawford’s (2014) multidimensional/non-linear teaching and learning model. Using a constructivist framework, students construct knowledge and meaning through engagement with learning that is authentic, experiential, student-centred, and holistic. Students become musicologists, composers and performing artists who share their music making and responding with the class.

By exploring the concepts of originality and probability, and challenging their preconceptions about what music is, students can develop multiple solutions through constructive imagination and creativity. Students will engage in an elaboration of ideas from their research of aleatoric music, drawing inspiration from other composers’ designs and ideas, to experiment with their own musical dice game designs and resulting compositions. As such, students will develop more divergent thinking, as well as have experience with the creative making process (Starko, 2005).

This creative expression will also be balanced with critical thinking (Crawford, 2020a). Students should be given intellectual control of their learning, including decisions such as working in groups or individually. Teachers can choose a range of collaborative learning approaches and different strategies for meaningful learning experiences (e.g., negotiation of roles and responsibilities in group work and using discussion techniques to acknowledge and validate students as expert researchers and performing artists in context).

### Prerequisite/Corequisite Knowledge: Music

Students need to have and/or develop the ability to:

* Research and document online resources (e.g., musical scores, audio recordings, journal articles, YouTube videos)
* Use Western written music notation and graphic notation to communicate musical instructions in creating compositions and when performing
* Analyse and evaluate music, and provide a rationale for a composer’s intentions
* Discuss the connections between the role of music in culture and society across diverse historical contexts and the influence that a particular musical genre can have on its evolution into popular genres and styles
* Write music for the instruments found in the class (including an understanding of transposing instruments and relative key signatures, where appropriate).

### Background Mathematical Skills and Understandings

Teachers of Music are not expected to teach the mathematical knowledge and skills that students will draw upon when engaging with this activity. The students will have learnt and should be adept with the required mathematical knowledge and skills to complete the activity. According to the Victorian Curriculum: Mathematics, the required mathematical knowledge and skills should have been developed in earlier years of schooling, that is, by the end of Level 8.

For this activity, the background mathematical skills and knowledge are:

* Knowledge of the concepts of chance, probability, and random samples
* Experience in conducting chance experiments that have random (but usually equally likely) outcomes
* Ability to calculate probabilities in which outcomes are equally likely (e.g., coin tossing, throwing a die)
* Knowledge of time and its relationship to composite variables such as speed [In the music context, students should be able to recognise that the speed at which music is played can be varied by changing the number of beats per minute (e.g., using a metronome).]

## Lesson Description

This lesson could be extended across 2–4 lessons depending on the level of detail in which musical concepts are explained and explored, and how the learning activities are unpacked. Extension ideas are provided for this purpose, as well as to cater for learning differentiation.

Teachers should familiarise themselves with the historical context of aleatoric music and its evolution to contemporary music genres and styles. Developing an understanding of the historical background will enable a rich discussion about the role that this music may have played in societal and cultural contexts. This discussion may include the evolution of experimental music and 20th century music, which have had a significant influence on not only popular genres and styles, but also on ways it is now used (e.g., digital games and film music).

Given that this lesson is designed to initiate explicit learning and thinking of mathematical concepts as related to music, the following resources may be useful in providing further background context:

* Mozart Dice Game by Konstantin Weixelbaum, available through the app store at a small cost: <https://apps.apple.com/us/app/mozart-dice-game/id946580946?l=de&ls=1>
* Berkowitz, A. (2010). *The improvising mind: Cognition and creativity in the musical moment*. Oxford University Press.
* Nierhaus, G. (2009). *Algorithmic composition: Paradigms of automated music generation*. Springer.
* Xenakis, I. (1992). *Formalized music: Thought and mathematics in composition* (6th ed). Pendragon Press.
* Zbikowski, L. M. (2002). *Conceptualizing music: Cognitive structure, theory, and analysis*. Oxford University Press.

Please note that the mathematics and its relationships to aleatoric music can be discussed in quite complex terms (e.g., algorithmic composition). As such, this lesson can easily be modified for Music elective classes in Levels 11 and 12. However, the focus of this lesson is Levels 9 and 10, and the content is taught more simplistically for this reason. You can also simplify the musical parameters and variations even further, and still use dice games to compose music with junior secondary students.

1. Play John Cage’s 1952 piece *4'33''*. The three-movement ‘silent piece’ is titled for its chance-determined total duration and marked “Tacet, for any instrument or combination of instruments.” A performance of the piece can be accessed at the following link: [https://m.youtube.com/watch?v=7wehyqv5tWc&t=139s](https://m.youtube.com/watch?v=7wehyqv5tWc&t=139s%20) (7:21).
2. At the conclusion of the piece, explain to students that this piece would confirm John Cage as one of the most controversial, yet significant, composers of the 20th century. Facilitate a class discussion using the following question prompts:

* What is music?
* Provide some background to Cage’s piece *4'33''* and ask: Could probability and randomisation be introduced into music in different ways?
* How might this probability and randomisation be achieved if considering the elements of music?

1. Provide students with the option to either work individually or in small groups of two or three to research the following terms: ‘music dice games,’ ‘aleatoric music,’ ‘chance music,’ ‘music probability and randomisation,’ ‘key compositional devices and techniques of aleatoric/chance music,’ and ‘European versus American aleatoric/chance music.’ Students should provide an example of at least two pieces to demonstrate key compositional techniques and devices used and describe the composers’ intentions.

Depending on time and learning differentiation, teachers can limit this research-based task by providing the musical examples and have students analyse these based on predetermined criteria. Through this approach, students will still be engaged in critical listening and analysis.

If required, teachers can also provide a brief introduction to aleatoric music and music dice games using video material before setting the research-based task, which may enable students to do a more focused investigation. Some examples of video material could be the following:

* Witold Lutosławski: Aleatoric Method [https://www.youtube.com/watch?v=v0H3HC12VG0](https://www.youtube.com/watch?v=v0H3HC12VG0%20) (11:30). This video is part of the London Philharmonia Orchestra’s series *Woven words: Music begins where words end* (Explore the series’ digital resources at <https://vimeo.com/channels/413147>). Series Advisor Steven Stucky talks with Philharmonia Orchestra members Samuel Coles (Principal Flute) and Mark van de Wiel (Joint Principal Clarinet) about Polish composer Witold Lutosławski's use of chance in his compositions, an approach referred to as “controlled aleatorism.” They work through a short passage of Lutosławski's 1961 piece *Jeux venitiens* (Venetian games), illustrating how each performance is unique as a result of the method used.
* Aleatoric Music: Live Looping & Chance - From Lutosławski to Video Game Music [https://www.youtube.com/watch?v=xabYn35ngaY](https://www.youtube.com/watch?v=xabYn35ngaY%20) (12:22). Aleatoric music is discussed in relation to the combination of looping and chance specifically. How the technique has evolved—including developments in modern composition as well as the use looping software and hardware—is briefly considered. An introduction to the use of aleatoric technique in video games is also explored.
* Making Music with Dice [https://www.youtube.com/watch?v=K6HcxxAWLoY](https://www.youtube.com/watch?v=K6HcxxAWLoY%20) (4:43). Randomly-composed music through the practice of letting dice (or other random elements) make musical decisions is explored, such as randomly-composed melodies and leaving important decisions up to performers.

1. Students then engage in a facilitated discussion as a whole class to evaluate the data found, as well as to explore and analyse aleatoric music and music dice games, using the specified research terms as prompts. You can extend the discussion to ignite students’ thinking about the influence of aleatoric music in societal and cultural contexts across different locations and times. Compositional techniques and devices should be a focus of this discussion, and links should be made to the original class discussion about how probability and randomisation could be achieved in music and through using the elements of music.
2. Students are to design their own music dice games based on their research about aleatoric music. Influences can be drawn from previous compositions and an accompanying rationale for the decisions made, such as musical instructions, compositional techniques and compositional devices used, should be made clear.

Students should write the music and game instructions for the instruments that their peers play in the class and that they have available to them. (If you have selected previous musical examples for students to use to limit the scope, then you can use these again as a stimulus for this musical dice game and compositional task.) Students can work individually or in groups. (This task can also be done as a whole class, where you can instead assign different groups to work on allocated musical elements that will combine to make the one class music dice game and resulting composition.)

A combination of Western musical notation and graphic music notation should be used, and a clear rationale should be provided for using the system designed. Musical bars/measures and instructions can be written by hand or using technology. It is useful to provide a range of dice for students to use in their games, as making them can take time. Therefore, students can focus on allocating the numbers or faces of each die to the various musical instructions.

1. Students should play the music dice games that they designed and perform the resulting pieces in class. These performances should be recorded, as it is anticipated that each performance will be different according to the musical elements that have been randomised via the dice.
2. It is valuable to play each dice music game a couple of times to demonstrate how changes in the randomised musical elements can alter the piece of music. Doing so could provide the basis for a rich discussion about whether there are certain variables that students would alter in the design to change the outcome, and why.
3. Peer assessment can be used to evaluate the planning and design elements, the critical and creative thinking applied to the compositions, the clarity with which the musical instructions are written and the authenticity of the compositions as examples of aleatoric music. In addition, the performance elements and ensemble work should also be assessed.

Further ideas for extension activities that involve aleatoric music ideas to explore probability and chance can be developed using the following suggestions:

* Birds on the Wires [https://www.youtube.com/watch?v=LoM4ZZJ2UrM](https://www.youtube.com/watch?v=LoM4ZZJ2UrM%20) (1:25). Brazilian composer Jarbas Agnelli used a photo (by Paulo Pinto) that he saw in a newspaper of birds sitting on five parallel wires as musical inspiration, and treated their positions as avian sheet music. He interpreted what he saw as music and orchestrated the tune in 2009. A live performance at the Guggenheim with Agnelli and students from the Julliard School and YouTube Symphony Orchestra is available here: [https://www.youtube.com/watch?v=gkRg\_FZdLgw](https://www.youtube.com/watch?v=gkRg_FZdLgw%20) (4:34).
* Application of Canadian composer Murray Schafer’s ideas regarding the natural environment as musical material creating soundscapes.
* Soundscape and landscape compositions to explore the Australian environment and diverse landscape through renowned Australian composer Peter Sculthorpe. Iconic music works such as Kakadu (1988) can be used to highlight his compositional style and approach. His significant and uniquely Australian sound is distinctively connected with the landscape, the Indigenous inhabitants of the land, and influences from South-East Asian music.

## Table 1: Links to the Victorian Curriculum – Music

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| Strand and Sub-Strand  (if applicable) | Content Description (Code) | Elaboration(s) |
| * Explore & Express Ideas | Improvise and arrange music, using aural awareness and technical skills to manipulate the elements of music to explore options for interpretation and developing music ideas.  (VCAMUE040) | Improvising and using technologies to develop the texture of a composition, for example, exploring instrumentation, dynamics and expressive techniques, using digital tools to create and manipulate tone colour.  Experimenting with layering of sound to develop a personal style in improvisation and composition.  Applying aural understanding of key and tonality when improvising and composing. |
| * Music Practices | Plan, develop and notate compositions with an understanding of style and convention.  (VCAMUM043) | [Adapted Elaborations]  Composing, improvising and arranging in different styles, exploring different instrumental combinations or combining acoustic and digital sounds.  Composing and arranging music using the elements of music to communicate style and genre.  Manipulating elements of music and performance techniques to elicit emotional responses |
| * Respond & Interpret | Evaluate a range of performances and compositions to inform and refine their own music making. (VCAMUR045) | [Adapted Elaborations]  Analysing how the use and combination of the elements of music defines their developing personal style and how their style is influenced by cultural and historical styles, for example, features of a piece that are typical of the social context in which it was created.  Listening to, analysing and comparing the performance practices of others to shape and refine their interpretation of a piece of music.  Comparing music from different styles to identify and describe stylistic, cultural and historical practices and inform their own composition and performance practice, for example, evaluating the use of the elements of music when listening to and interpreting music  Investigating why and how different traditions, styles and contexts affect the experience and interpretation of a piece of music and taking this into account when interpreting and composing music. |

## Table 2: Links to the 21st Century Numeracy Model

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| Aspect of the Model | How This Aspect is Addressed by the Lesson |
| **Attention to Real-Life Contexts**   * Citizenship * Work * Personal and Social Life | Students will develop an understanding of how patterns can be identified, evaluated and applied in authentic and real-life musical contexts through critical listening, performing, and composing.  Students will use mathematical processes to learn how to develop musical instructions that will form the basis or structure of a piece. Students will engage with chance, probability and randomisation in personally meaningful ways by developing interpretations of music and communicating the composer’s intentions in performances through designing and playing musical dice games.  Students are provided with opportunities to apply this knowledge to their own pieces, which are representative of an authentic musical genre and style. Students are challenged to question preconceived ideas, knowledge, and concepts about what music is and the influence that these ideas can have on contemporary popular genres and styles.  Considerations of the role of music in society and culture are developed with a focus on how and why experimental music that is dominated by mathematical concepts and processes is significant to the evolution of music. Collaboratively learning and performing in ensembles enable students to develop important personal and social development skills. |
| **Application of Mathematical Knowledge**   * Problem Solving * Estimation * Concepts * Skills | Students will develop skills in problem solving by using mathematical processes to identify issues and tensions with consonance and dissonance (e.g., using particular notes of scales/modes and key signatures) that contribute to an overall tonal structure.  Evaluation skills will be used to identify how the variables of the musical elements employed can combine to structure a musical piece and the degree of chance that can be applied to compositions.  Students will develop skills in comparing and creating different types of melodies, harmonies, and rhythmic patterns when designing their musical dice games. Students will practically apply the acquired mathematical knowledge and skills that performing artists and composers can employ in a particular musical genre and style (e.g., chance and probability within the aleatoric music context). |
| **Use of Tools**   * Physical * Representational * Digital | Students will manipulate and experiment with different combinations of musical elements. Students must notate their pieces using Western musical notation and graphic notation. A rationale should accompany the decisions made to support the design of musical instructions. Musical scores can be written by hand or using technology (e.g., Noteflight, MuseScore, Sibelius,). |
| **Promotion of Positive Dispositions**   * Confidence * Flexibility * Initiative * Risk | Students will develop the confidence and ability to interpret, analyse and compare musical patterns by using numeracy understandings and skills in a given context. Students make informed decisions through critical listening and analysis by engaging in making and responding in Music.  Students experiment as creative risk-takers in manipulating the elements of music and stylistic conventions to compose music (e.g., combining critical thinking and logical reasoning with creative expression to experiment with different musical combinations and compositional devices). |
| **Critical Orientation**   * Interpreting Mathematical Results * Making Evidence-Based Judgements | Students make and respond in Music using creative and critical thinking. Students’ ability to identify, compare and analyse musical patterns that are applied to the design of musical dice games will be developed throughout the lesson. The realisation of the decisions made (including variables left to chance, probability and randomisation) are authenticated in performance and composition contexts. An evidence base drives critical and creative decisions that contribute to new and innovative musical ideas, interpretations, compositions and performance practices. |

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