Teaching and Learning FOR, ABOUT and THROUGH problem solving

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"There are two principal reasons for teaching arithmetic. It is included in the curriculum because of (a) its utilitarian or practical value, and (b) its cultural or mental value". (Education Department of Victoria, 1944, p.2)

The usefulness of mathematics in solving problems in everyday life, work and further study has always been the justification for giving it a central place in the curriculum. As a consequence of these considerations, school mathematics programs in Years 1 - 8 have tended to devote most time and energy to the acquisition of number concepts and computational skills. Measurement and spatial ideas have been given a lesser place, although they too are not intuitively obvious to children and have a crucial role to play in analysing and solving problems, particularly those of a practical nature.

Another basis for the centrality of mathematics in the school curriculum, and for the importance given to number and computation in particular, can be traced back to Plato's belief that arithmetic trained the mind to deal with abstractions. The pervasiveness of this belief has meant that the teaching and learning of mathematics has tended to evolve with very little regard for the real and tangible.

Today, the unfortunate reality is that many students have not acquired the knowledge, skills and confidence that they need to solve practical, real-world problems, or to pursue further studies in mathematics.

> *"... although most students are reasonably proficient with computational skills, the majority do not understand many basic mathematical concepts and are unable to apply the skills they have learned in even simple problem situations" (Romberg, 1984, p.10)*
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> *"The impression that comes through is that a lot (of students) get through by rote learning methods to solve a particular problem and as soons as you ask a question that requires some reasonable understanding of mathematics, over 50% of candidates cannot do it" (Dr. Ken Sharpe, Chief Examiner HSC General Maths, 1985)*

There have been a number of significant responses to this widely observed phenomena: the Mathematics Curriculum and Teaching Program (Lovitt and Clarke, 1988), the new Mathematics Courses for the Victorian Certificate of Education, and "Exploring Mathematics in Classrooms" (EMIC), a professional development initiative of the Victorian Ministry of Education. Their message is clear: students must be given opportunities to construct their own knowledge based on understanding, and to develop the confidence and skills to apply that knowledge in unfamiliar situations. For example, three kinds of learning activity are seen to underpin the work requirements for the new VCE Mathematics Study:

- *"learning and practising skills and applying them in standard situations,*
- *solving problems of an unfamiliar and non-standard kind, and using mathematical modelling as a tool in applying mathematical knowledge to real world problems*
- *carrying out extended investigative projects using mathematical knowledge to investigate problems, situations or issues." (Jane, 1989)*

The foundations for and requirements of such an expectation can be seen schematically in Figure 1.

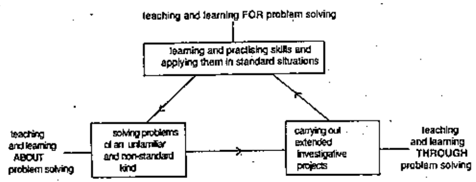

Figure 1

In the past, most classrooms have tended to concentrate on only **ONE** aspect of this model, namely, learning and practising skills and applying them in standard situations, or teaching and learning **FOR** problem solving. The remaining outcomes were simply assumed to be the

natural and inevitable consequence of such a focus. In other words, it was assumed that if students were trained rigourously in "the basics", they would be able to solve problems in adult life and develop appropriate habits of mind.

Obviously, such a singular and exclusive focus is no longer justified. Having the knowledge, skills, processes and confidence to solve unfamiliar problems and carry out extended investigative projects cannot be acquired overnight. All three aspects, teaching and learning **FOR**, **ABOUT** and **THROUGH** problem solving, must be included simultaneously in the curriculum in a manner and proportion that is appropriate to the background, interests and needs of the students concerned. Proposed forms of alternative assessment make such an approach a matter of urgency.

Before examining the implications of this assertion for the teaching of mathematics in Years 1 to 8, we will look at what problem solving entails.

Two problems

A black bag contains a collection of squares, pentagons and hexagons. Altogether there are 16 shapes in the bag. If there are 77 vertices and 76 diagonals, how many of each shape are in the bag?

If you feel that you would like to attempt this task, but you are not sure about what you might do, then you have a problem. If you have no interest in this task, or if you already have access to a procedure or strategy which you are confident will produce the correct solution, then you do not have a problem at all. Notice that, *having a problem* is a very personal phenomena. It depends on who you are, what you know and how you feel about the task in question. For example,

Three bags of jelly-snakes, ten snakes in each bag, how many jelly-snakes are there altogether?

This is unlikely to be a problem for anyone reading this article, but it may well be a problem for a large number of children in their first year of school, who might well have an interest in solving this problem, but lack the means to do it directly.

A problem then, is a task or situation

- that you want to or need to solve,
- that you believe you have some reasonable chance of solving, either individually or in a group, but
- for which, you, or the group, have no immediately available solution strategy.

Because a problem is such a personal experience it is impossible to say that one task is a problem while another is an exercise (a task for which a solution procedure is immediately available). For example, there is evidence to suggest that 6)354 is a problem for many grade four children, while,

How many handshakes are possible amongst a group of 5 people?

is not (Siemon, in preparation). Nor does it make much sense to talk about *good* problems. Any problem may be rendered *good*, *bad* or *not a problem*, depending on how and when it is used, to whom it is addressed and for what purpose it is being used.

According to the definition above, all tasks which are problems necessitate the solver's engagement in some sort of process. Like *process* or *creative* writing, the **problem solving process** is directed at achieving a goal and usually has associated with it, recognisable beginning, middle and phases. For example, in the shapes problem, the solver has to retrieve what he or she knows about the properties of each shape, decide on a plan of action (for example, guess and check, construct a table, use equations or draw a diagram), implement the chosen strategy, evaluate its success, modify subsequent behaviour if necessary, and finally, communicate findings in a form that is meaningful to others.

Obviously, the success or otherwise of such a process depends on the solver's ability to access and monitor relevant knowledge and strategies. If a five year old does not know that "three" names a group of three, cannot make a group of ten, count reliably and efficiently, or recognise the "3 tens" is another name for "thirty", then it is highly unlikely that the jelly-snakes problem would be solved.

Problem solving occurs when an individual or group engages in a process, which directs and monitors what is known and how it is applied in order to achieve a solution to a problem. Problem solving can be improved by ensuring solvers.

- have access to relevant knowledge, skills and strategies.
- recognise and accept responsibility for the problem solving process, and
- have the confidence necessary to criticaly reflect, monitor and direct what they know and do during a problem solving episode.

Teaching and learning **FOR** problem solving ensures access to the first of these. Teaching and learning **ABOUT** problem solving ensures access to the second, while teaching and learning **THROUGH** problem solving ensures access to the third. The implications of each of these three approaches for the teaching and learning of mathematics in years 1 to 8, will now be considered in more detail.

Teaching and learning FOR problem solving

Access to relevant knowledge, skills and strategies, can only be assured if these are established on the basis of understanding. The knowledge and strategies used by children and adults in out-of-school settings (Carraher, Carraher and Schliemann, 1985; Carraher, 1988) often reveal more about what children have **not** learnt and understood at school than they do about the power and generalisability of such strategies to a range of other problem types. Clearly an understanding of numbers will always be fundamental to learning and using mathematics, as it is a major topic in its own right and underlies all of the computational procedures. Indeed, if mathematical ideas are to be communicated effectively and if concepts and skills are to be taught without confusion, it is important that the numeration ideas which give meaning to numbers be well established. Place value, regrouping and renaming are the fundamental concerns of numeration, and misconceptions with these have proven to represent the largest source of children's difficulties in arithmetic (Booker, 1987). Even when a child's errors appear in addition, subtraction, multiplication or division or with decimal fractions, the real source of the difficulty often lies in an inadequate understanding of numeration.

Recently, a more general and more pervasive view of problem solving has become the focus of the mathematics syllabus. The use and interpretation of number and computational procedures that this demands means that the need for a properly established background of skill and understanding has become even more essential.

At the same time, the inclusion of particular problem-solving considerations has diminished the time available to develop purely arithmetic knowledge. The integration of calculator use from the earliest years might appear to offset this loss of time by requiring less highly developed skills but, concurrently, the need for well-understood processes is greatly increased. Not only the understanding which can direct the use of calculators and computers but also an ability to estimate results, to make approximate calculations and to check answers.

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Organising this development demands an analysis of the requirements for each concept and procedure. This has always been recognised but, in the past, attention was largely give solely to the mathematical needs of the task. The basic components of each task were organised and presented in such a way that before a new level was encountered, each sub-skill had already been met and learning proceeded from the simple to the more complex. However, while this type of analysis and organisation is necessary, it is not sufficient. Children do not simply receive knowledge as it is given and organised; they construct their own meanings for what is presented. If the order of presentation does not accord with their needs as novices, they will easily form misconceptions related to their individual prior knowledge and understanding. Thus, it is necessary in planning the introduction of number ideas and computational skills to consider the underlying mathematical understandings and how these are best developed by children. Materials are needed to bring out the base ten nature of the number system; a meaningful language is needed to name the numbers, direct the computational procedures and lead to a system of recording numbers and the algorithms that is understandable both in terms of the mathematical ideas and the children's developing mathematical world.

Such an analysis shows that most of the understanding needed to process and structure the algorithms is drawn from numeration, that is, the skills and understandings needed to name, write, process and interpret numbers. It is important that children are able to consider numbers in a variety of ways: not just in terms of the counting with which number meaning begins, nor even with the system of place value which governs the construction and computation of numbers, but also with the various ways in which a number can be renamed. For instance.

537 is 5 hundreds 3 tens 7 ones; 5 hundreds 2 tens 17 ones 53 tens 7 ones; and 537 ones.

Since a lot of learning of number ideas occurs informally, even before a child reaches school, it is necessary to consider the order and timing of the introduction of numeration concepts. Most children will arrive at school with a substantial grasp of counting and the numbers to ten. If follows that a program limited only to numbers up to ten will not be very stimulating and motivating. Nor will it restrict the child's number development to these numbers. Many children will develop their own understanding of unumbers to one hundred, but few are likely to build this on a basis of place value rather than counting.

Research (Brooker, 1985) suggests that such an inability to see place value as an integral part of numbers larger than ten is a major factor in children's later computational errors. Thus, it is important to bring the full aspects of the two-digit numbers to children's attention soon after they begin school, once the numbers to ten have been mastered. In contrast, a lot of the first year of school has traditionally been given to activities which are concerned with logical activities such as ordering, sorting and classifying because of a belief that these are necessary prequisites for the development of number. Recent research (Clements and Callahan, 1983) has thrown doubt on this supposition, highlighting the importance of counting based cognitive tasks instead and revealing the strong link between logical activities and the development of general problem solving abilities.

While other aspects of number have also been introduced at a later than optimal time, more frequently number ideas have been introduced too soon, too closely together or even not at all. It is for these reasons that children have experienced difficulties with teen numbers, numbers with internal zeros, large numbers, the

process of rounding and the use of inequality symbol. The size of the numbers and their order within the number sequence might seem an adequate basis for organising their teaching, but it is even more important to consider the development from one new idea to the next, the regularity of the patterns in naming the numbers and the difficulties which could arise in recording and reading them. This has produced a different sequence to that traditionally used but one which experience has shown to be particularly smoothly effected with young children (Barry, Booker, Perry and Siemon, 1983-89). This specific teaching of numeration ideas is a very necessary aspect of the development of computational skills, the consolidation of these skills also plays a crucial part in bringing about a full realisation of numeration.

Often, too little emphasis has been given to the concept for each operation, restricting the development to only one aspect or expression of the meaning, and focussing too early on the mastery of basic number combinations and written recording. While an ability to provide immediate, correct basic facts is crucial to completing computations, an understanding of what each operation means, how it is to be interpreted and symbolised and which sort of action it represents is even more essential to using mathematics. Children need to be able to make the links between situations from their own experimental world to a language that describes the situations and symbols which express it in mathematical terms. The language used needs to be extended over time to all of the expressions met in real situations that imply a particular operation as well as the more formal expressions met solely in mathematical contexts.

When learning basic facts, attention needs to be paid to the strategies by which answers can be obtained and time needs to be allocated to those facts which require most attention. This means that pairs of facts such as '9 and 4' and '4 and 9' or '3 fives' and '5 threes' should be learnt together while subtraction facts should build on known addtition facts. Division facts should build on known multiplication facts rather than have two sets of related facts learnt in isolation. Only those facts that need to be learnt should be stressed; thus the tens facts should be recognized as knowledge acquired in numeration rather than treated again as if the earlier learning did not exist, and the twos facts in multiplication should be seen as a different expression of the doubles facts learnt in addition. Most importantly, not until children know how to obtain answers should a stress on obtaining them quickly be made. If children do not know how to obtain the answers that the teacher or learning situation (including games) requires then they must be reduced to guessing or learning by rote.

Only when children have acquired the concept and basic facts for an operation is it possible to develop the algorithm for larger numbers. This development will be meaningful if the manner in which materials are used to introduce the procedures, the language that is formulated to describe their use and the way in which these lead to the final recorded forms are consistent from the initial concepts, through the basic facts to the symbolic procedures (see Fig. 2).

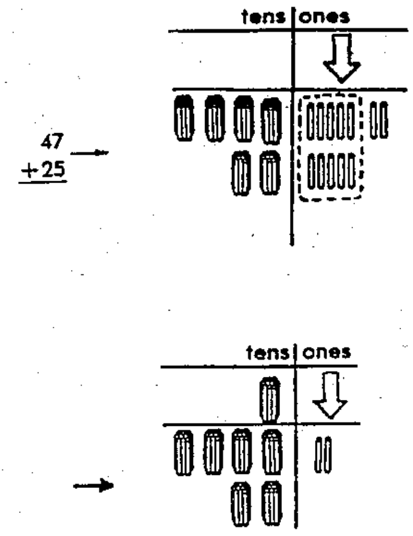
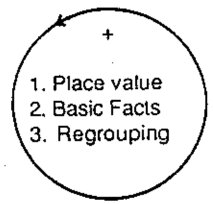


Figure 2

Later, this same processing can be extended to decimal fractions and common fractions so that we can summarise the addition process as a cycle of steps involving:



Subtraction, Mulitiplication and Division have a similar basis. An analysis of the understanding needed to develop computational abilities shows that there are fundamentally only two forms of thinking involved in all of the operations of arithmetic.

Finally, if should not be forgotten that a stress on calculator usage, and the approximations to answers that planning problem solving strategies often demands, will require a good ability to estimate answers. The mathematics that is required today to prepare children for problem solving requires that they are competent in pencil and paper, calculator, approximate and mental computation and, more importantly, feel confident in choosing which is most appropriate to their particular needs in any given situation.

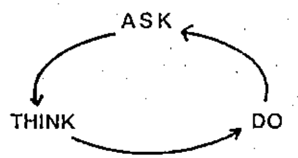
Teaching and learning ABOUT problem solving

While the underlying skills and meanings associated with number, computation, measurement and geometry are a necessary basis on which to build problem solving abilities, there are also more general strategies and understandings needed which relate to the problem solving process per se. Teaching and learning **ABOUT** problem solving is needed

- to create an awareness of the act of problem solving,
- to develop mechanisms to access appropriate concepts, skills and strategies, and
- to monitor and direct their use in the course of problem solving.

Simply providing open-ended tasks, investigations, co-operative group work and opportunities for problem solving, while necessary, is not sufficient to ensure that learning occurs or that appropriate, powerful problem solving behaviour will emerge. Approaches which enable students to recognise and accept responsibility for their learning and for the problem solving process are also needed.

An important assumption underpinning the HBJ Mathematics Series (Barry, Booker, Perry and Siemon, 1983-9) is that mathematical knowledge cannot be simply transmitted; it must be constructed by the learner in relation to previous knowledge and experience. While the importance of this constructive linking activity has already been emphasised in relation to the concepts, language and skills associated with numeration and computation, it is also important in relation to the acquisition of procedural knowledge, that is, knowledge about what to do, when and how. The ASK-THINK-DO model of the problem solving process developed for the HBJ Mathematics Series can be represented similarly.



and is intended to prompt the same sort of activity, that is, to encourage students to consciously link their conceptual and procedural knowledge by focussing on what is required, what is needed, how it can be obtained and how it relates to what is already known. In doing this, the model draws students attention to the problem solving process and encourages them to accept responsibility for directing and monitoring their own problem solving behaviour. Recent research (Siemon, 1986, 1988) has established that overt attention to this in the classroom improves students problem solving performance and confidence.

Applying the ASK-THINK-DO model in the mathematics classroom

Ultimately, one of the most important aspects of problem solving is the ability to represent real-world problems in the form which facilitates the use of the most powerful strategy available to the solver. Consider the following problem.

A farmer had sheep and emus together in a paddock. If he counted 34 heads and 88 legs, how many sheep and how many emus were in the paddock? (HBJ 4/123)

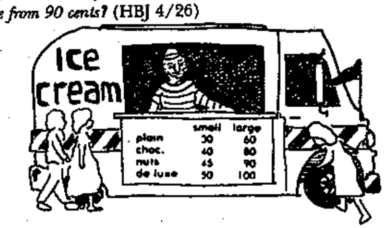
A fourth grade child can solve this problem using a variety of strategies (trial and error, draw a diagram, make a model, if ... then reasoning and so on), but we would expect a student at year 11 to realise that this is one example of a whole class of problems which can be solved most powerfully by the use of simultaneous equations. While it is important to identify and accept that children will bring a range of processing, representing and solution strategies to any problem and may well be able to "get the right answer" by "their preferred method", it is the function of schooling to demonstrate the limitations and boundaries of such strategies and to challenge, modify and extend those strategies. Just because children have efficient strategies for performing two digit multiplications in the market place does not mean that they have effective strategies and to challenge, modify and extend those stratgies. Just because children have efficient strategies for dealing with division or the multiplication of decimals.

Translating an unfamiliar problem into a form that is amenable to the application of mathematical principles and procedures is a non-trivial task. Krutetskii (1976) found that successful problem solvers are more likely to be aware of the structural aspects of a problem. Encouraging students to **ASK** questions such as

What do I need to find out?
What is the problem asking?
What information is given?
Is there anything else I need to know?
What do I need to do first? ... next

by modelling such behaviour yourself, listing useful questions on an ASK-THINK-DO problem solving chart, consciously discussing the role of such questions with student, and encouraging students to write their own questions, focusses attention on the nature of the task, helps establish meaning and makes it possible to play an appropriate response. The HBJ Mathematics Series considers problem type quite overtly and encourages students to analyse problems in terms of their structural features, for example,

Sally wanted to buy a chocolate ice-cream, how much change would she have from 90 cents? (HBJ 4/26)



the difference between 90 cents and the cost of the ice-cream. But this information is not given and two options need to be considered in order to completely answer the question. When confronted with this problem for the first time, one group of fourth graders argued vociferously for twenty minutes that *"it was not mathematics because it didn't have one answer"* (Siemon, in preparation). Presenting such problems, discussing them and analysing them in terms of what questions can be asked, how many steps are involved, is there enough information, what needs to be done and so on, helps to develop an awareness of problem structure and facilitates the adoption of an appropriate plan. Some anecdotal evidence of this occurred when a grade 5 boy revisited his grade 4 classroom where he had participated in the discussion of the ice-cream problem some 16 months earlier. Seeing me in the classroom he spontaneously burst out with: *"we did one just like the ice-cream one the other day ... you had to do two answers"*.

Encouraging children to **THINK** about what they know, what they can do and what they have found useful on previous occasions, by asking questions such as

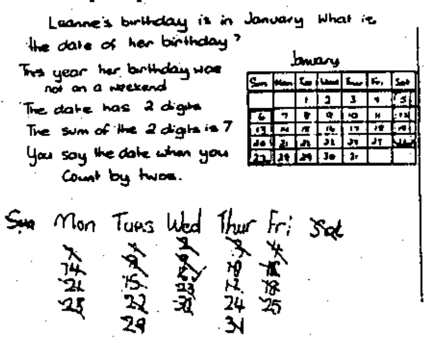
What do I need to know?
How can I find out?
What have I done before?
What can I do that might help?
What is my plan?

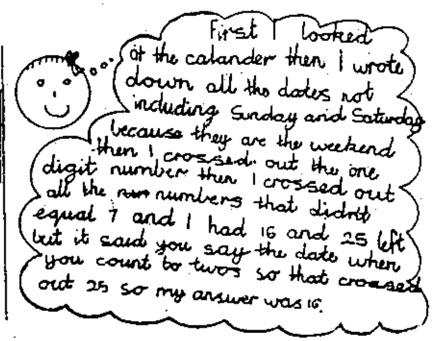
takes them onto the next stage in the problem solving process. Meaning has been negotiated and goals identified. Knowledge, skills and strategies now need to be accessed and driven purposefully towards achieving those goals. The extent to which this is done successfully depends on the solvers access to relevant strategies and techniques for monitoring his or her behaviour. There is now considerable evidence to suggest that these strategies and techniques can be taught (see review by Mayer, 1985; Siemon, in preparation). The HBJ Mathematics Series presents a range of strategies as objects of study in their own right. Problems are presented verbally usually in relation to some stimulus material such as a coloured photograph or illustration. The teacher's role in this instance is to model the problem solving process and to highlight the use of an appropriate strategy. It is the teacher's responsibility to ensure that the problem is understood by asking questions of the type already listed. Once meaning has been negotiated and goals agreed upon, the teacher prompts a consideration of relevant strategies, a particular one is agreed upon and is overtly and consciously discussed in terms of its usefulness and what it involves. For example, a photograph of the Moomba Fireworks is used to introduce the problem of planning a Fireworks Display (HBJ 6/111). The teacher negotiates the task with the students and helps formulate questions about what is needed, for example, data on different types of Fireworks (a Data Sheet is provided in the Teachers Resource Book or children could invent their own). Further discussion establishes what sort of strategies might be useful, a diagram, working backwards, recording data and so on. Strategies are then explored, discussed, practised and reviewed. The object of the activity is not to produce a prescription for an interesting display although this is a valuable outcome in itself, but to demonstrate the power and utility of particular strategies quite overtly.

Encouraging and facilitating children in their attempts to **DO** what is required in order to solve a problem completes the process. Children do not naturally check their work and need to be prompted to defend, explain and justify what they have done to others in order to evaluate the reasonableness of their response. Questions such as the following are useful.

Did it work?
Does the answer make sense?
Is there another way I could do it to make sure?
Have I thought of all possibilities?

Often this will necessitate a recycling through the ASK-THINK-DO phrases of the problem solving process. Other techniques whcih support this sort of behaviour include written reports describing what was done and why, posters, displays, oral presentations and problem posing. The latter is particularly effective as means of assessing whether or not problem solving objectives have been achieved. A technique, which is referred to as 'Talking Heads' (Siemon, in preparation), has ben found to be a very useful device for helping students reflect on their problem solving behaviour. A blank circle and a speech or thought bubble is presented alongside a problem or task. Students are required to draw a face in the circle to indicate how they feel about the task and/or their prerformance and to record what they did, and how in the bubble. An example is presented below.





Problem solving entails much more than "strategy games and puzzles", it is not "a topic" to be considered in isolation from the rest of the school mathematics syllabus, or something which takes place only on Friday afternoons, during wet lunchtimes, in "task Centres" or during "Activity Maths". Problem solving behaviour is rarely, if ever, absorbed unconsciously from experience. It is acquired by conscious, critical reflection on past and immediate experience, the focii of which are self, task, strategy and process. Teaching and learning ABOUT problem solving can and should be integrated into all aspects of the school mathematics program (Siemon, 1985). The ASK-THINK-DO model of the problem solving process can be applied to the following problem:

If the shearers managed to shear 179 of the 368 sheep before lunch, how many sheep remained to be shorn that afternoon?

as effectively as it can be to any one of the following:

Fred Frog dived into a well that was 12 metres deep. Unfortunately, the well was empty. Each day he could only jump 4 metres up the side of the well. Each night he slid down 2 metres. How many days did it take Fred to get out of the well? (HBJ 4/119)

Can you roll a marble along the blackboard ledge as quickly as you can hop a distance of 10 metres? (HBJ 4/68)

A 5 metre length of fencing costs $8 and each fence post costs $5. How much will it cost to make a triangular pen using 9 lengths of timber? (HBJ 4/123).

How many ways can 4 postage stamps be attached so that they are joined on at least one edge? (HBJ 6/247)

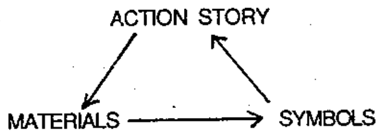
Should boys between the ages of 5 and 16 be banned from riding bikes? (HBJ 6/209).

Problems such as these provide opportunities to apply appropriate knowledge, skills and strategies, but more importantly, they provide opportunities to examine problem structure and reflect on the problem solving process. Teaching and learning ABOUT problem solving empowers students to recognise and accept responsibility for their problem solving by providing them with the knowledge and techniques to access, monitor and direct what they know and what they do.

Teaching and learning THROUGH problem solving

Problem solving may be seen as an end in itself, the natural focus of learning mathematical skills which are based on real-life needs and applications. However, this is only one of the justifications for making problem solving the centre of activity in school mathematics programs. Problem solving recontextualises mathematics. It provides a rich background against which to interpret meaning, to apply basic ideas, to explore strategies, and to evaluate personal performance. As such, it is much more likely to lead to insights into the processes of mathematical thinking than traditional approaches which treat mathematics as a set of isolated procedures to be learnt more or less by rote. In other words, new mathematical ideas can grow out of appropriately posed mathematical problems. It is in this sense that mathematics can be said to be taught and learnt THROUGH problem solving.

In early years of primary school, this has always happened. Stories and problems from the children's environment are posed to help develop the concepts associated with number and the four operations. Children's toys, familiar objects and events and other more structured materials are then used to develop a more generalised understanding of the concepts and, eventually, the associated recorded forms. A concept is said to be grasped when the match between story problems, materials and symbols is internalised:



While the initial learning is arrived at THROUGH problem solving, it is consolidated, extended and applied by teaching and learning FOR and ABOUT problem solving. For example, although the fundamental concepts associated with addition, subtraction, multiplication and division are established and linked to words and symbols through problem solving, their successful application to more generalised problems depends upon the construction of standard meanings and processes. Once these have been established and consolidated through practice and application to real problems, usually by the later primary years, the potential for further mathematical insights through problem solving is restored and new ideas, which build on these foundations, can emerge.

Unfortunately, this potential is not always realised, as evidenced by traditional approaches to the teaching and learning of algebra which do not recognise or begin with the understandings the child has arrived at after 6 or 7 years of schooling. Little wonder that children fail to grasp the significance of the new expressions in terms of the more abstract and general thinking processes that they allow.

Algebra involves the extension of general solution procedures to identified classes of problems which have essentially the same result. A sequence of experiences which leads from concrete arithmetic situations to algebriac generalisations must establish that the use of letters is a powerful means to express such results. A first use is simply as labels to identify the objects being examined and thus grows naturally out of words used to describe them in a manner analogous to the use of letters in measurement, for example, L for length or W for width. When this has been established and accepted, relationships between the objects which have been identified and labelled can also be expressed using the letters that have provied the labels, as seen in the general relationship for the area of a rectangle given by $A = L \times W$. The use of tables of values to show these relationships can then in turn suggest more concise ways of expressing the results by means of the number which identifies a particular entry (see Fig. 3).

Start	Result
2	10
6	26
12	50
25	
7	
32	

Figure 3, Guess My Numbers (HBJ 7/222)

In this way, the use of letters to express relationships occurs somewhat naturally and provides the basis for using the letters themselves to find and verify patterns (see Fig. 4).

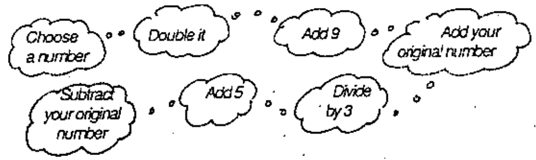
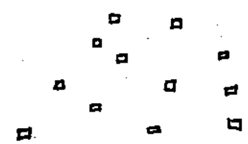


Figure 4. A Number Puzzle (HBJ 7/223)

Only when the development of a generalised arithmetic has established the need for and power of algebraic symbols can algebra be extended to a topic in its own right and meaningful procedures for manipulating the symbols be considered. So while the conceptual basis is arrived at through problem solving, its extension and consolidation is achieved through a mix of teaching and learning FOR, ABOUT and THROUGH problem solving.

Teaching and learning THROUGH problem solving can be a springboard to further learning in geometry and measurement also. For example:

The electricity supply to a new housing estate is to be put underground. The SEC wants to connect the houses using the shortest path to minimise cost and power loss. What is the shortest route connecting all houses



A shoe manufacturer can produce 10,000 pairs of fashion shoes per month. If the company produces shoes in sizes 1 to 12 and in fittings A, B, C, D and E, how many of each size and fitting should be produced to minimise waste and maximise profit?

In the first of these problems, a number of strategies may be called into play, trial and error, estimate, measure, experiment and so on. But it will become apparent quite quickly that there are very many different paths, any one of whic might be the shortest. The solver is then prompted to pose further quesitons. Can it be done for fewer houses or for houses in a more regular pattern, such as the vertices of a convex polygon? Under what conditions can a route be constructed so that every point is visited once only and no path is retraced? Questions such as this identity what is not known and motivate the construction of new learning. In this case, some aspects of elementary topology.

The second problem requires the solver to identify what information is needed, decide how it could be obtained and needs to be done with it once it is obtained. The experience and results of such an investigation are bound to reveal the need for different measures of central tendency, so prompting a consideration of mode, median and mean in more general contexts.

Thus, teaching and learning THROUGH problem solving is necessary:

- to practise the application of knowledge and strategies to unfamiliar problems, situations or issues,
- to exercise the capacity to critically reflect, monitor and direct what is known and done during a problem solving episode,
- to generate links between old and new learning, and
- to justify, motivate new learning by revealing what is known or canot be done.

such an approach actively engages learners in their own learning by providing a powerful context in which to construct and exercise meaning.

Summary

In conclusion, while it has been argued elsewhere that:

"Problem solving can be both medium and message; it can be both content and context; and we believe it is at its most effective when providing the structure for learning" (Lovitt and Clarke, 1988, p.469)

this implies that only two aspects of the model described earlier are required to promote problem solving behaviour, namely, teaching and learning ABOUT problem solving and teaching THROUGH problem solving (see Fig. 1). But teaching and learning FOR problem is just as important and just as necessary to successful problem solving. Without the appropriate knowledge, skills and processes, meaning cannot be constructed and problem solving cannot proceed. While it is undoubtedly true that the

"most elaborate and extensive tool-kit is of litte use if it remains unopened" (Lovitt and Clarke, 1988, p. 467)

it is also true that the most elaborate and extensive set of keys is of little use if the tool-box is empty.

Teaching and learning **FOR** problem solving is needed to ensure the availability of appropriate knowledge, skills and strategies, built on understanding and exercised with confidence.

Teaching and learning **ABOUT** problem solving is needed to provide the means to access, monitor and direct what is known and what can be done.

Teaching and learning **THROUGH** problem solving is needed to provide a context for further learning and to exercise the application of the knowledge, skills and processes acquired as a result of the first two approaches.

Each approach has a vital and critical role to play in the acquisition of application of mathematical thinking at all levels. We simply cannot afford to concentrate on any one or two at the expense of all three.

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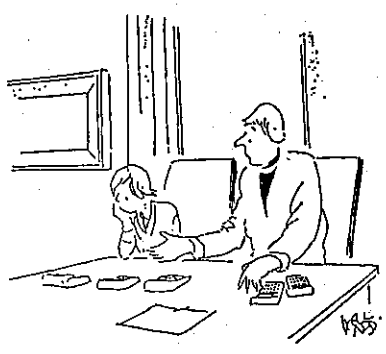

"Look — if you have five pocket calculators and I take two away, how many have you got left!"