MATHEMATICS TEACHING toolkit

ISSUES IN THE TEACHING OF MATHEMATICS

Teaching Mathematics from a Cultural Perspective

Chris Matthews

Chris has a PhD in applied mathematics and is currently a senior curriculum officer for the Australian Curriculum, Assessment and Reporting Authority (ACARA), Associate Dean (Indigenous Leadership) Science Faculty, University Technology of Sydney and the Chair of the Aboriginal and Torres Strait Islander Mathematics Alliance (ATSIMA).

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INTRODUCTION

TEACHING CULTURE = DEEP LEARNING

Before you read this article, I would like to give the reader an understanding of who I am and my personal and professional journey into mathematics education. First, I'm from the Quandamooka people of Minjerribah (Stradbroke Island) and our Country spans the Islands and mainland of Moreton Bay near Brisbane in Queensland. I was born and raised away from my Community in Toowoomba, and there, my educational experience was marred with racism from both teachers and students. To deal with these situations, I was lucky that I had a keen interest in computing and mathematics and would disappear into this world. This world gave intellectual challenges and an appreciation for how mathematics could recreate patterns that existed in our world, that existed on Country.

Eventually this would lead me to complete a PhD in applied mathematics, but this was not a healthy educational pathway. I disconnected myself from the world, from people, from my sense of self and from my culture. Over the years to come, I had to deal with this disconnection and making a space for myself, as an Aboriginal person, in both my professional and personal life. As a research mathematician, I was continually confronted with circumstances where I was pushed aside and not treated as a valued colleague. I was also confronted with applications of mathematics (e.g. dumping of nuclear waste) that did not match my personal values and knew that these applications adversely affected Indigenous communities around the world. I started to understand that mathematics could be used as a tool of the colonizer and, for this reason, knew it was important for our people to understand this knowledge and how it affects our lives.

I also knew that we are an intellectual people who understand systems and patterns (e.g. how the world is interconnected), which are important aspects of our deep connection to Country. Our people understood the world mathematically in our own way and this knowledge has not been recognised. Our children are continually disadvantaged in the education system and current educational statistics do not reflect their intelligence and ability.

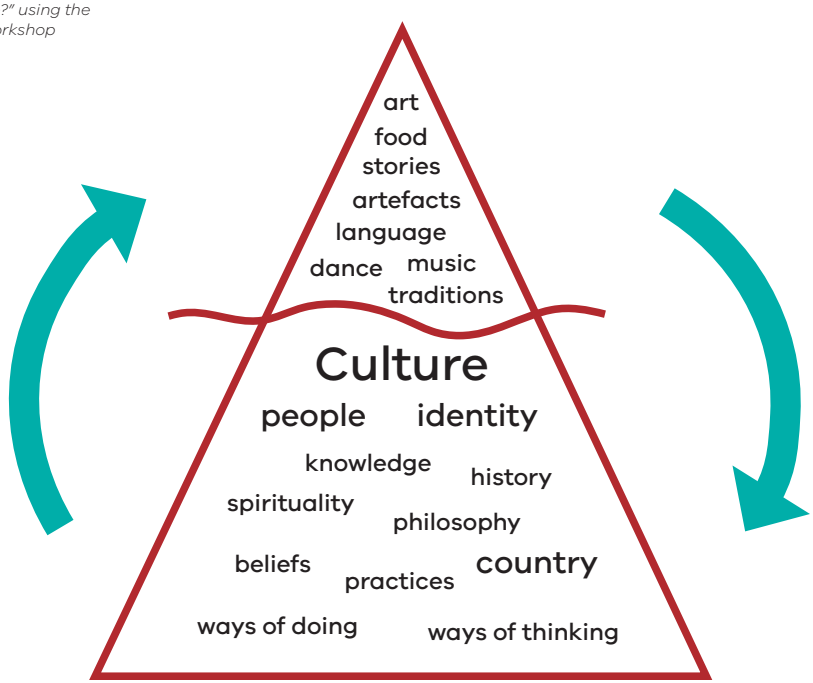
So how do we work together to change this? After many years as an applied mathematician, I decided to change fields to work in mathematics education for Aboriginal and Torres Strait Islander learners.

I was keen to explore what it meant to teach mathematics that valued our cultures. I wanted to explore the connections between our cultures and mathematics to further inform mathematics education for all Aboriginal and Torres Strait Islander learners.

In this monograph, I will argue that a richer mathematics education comes from teaching mathematics from a cultural perspective. From this way of teaching, a different type of mathematics education can be created; the potential for students to develop a deeper conceptual understanding of mathematics as well as develop a connection with mathematics. To do this, we must understand the connection between mathematics and culture. Drawing on my experience from delivering professional learning workshops for in-service and pre-service teachers, the following section briefly explores two fundamental questions, "What is Culture?" and "What is Mathematics?"

Figure 1: Typical example of "What is Culture?" using the Iceberg Model from professional learning workshop

Iceberg Model



Activity 1:

Take the time to reflect and write down what culture means to you and compare this to the written text in Figure 1.

WHAT IS CULTURE?

Figure 1 is a typical example from a professional learning workshop exploring the question "What is Culture?" using the metaphor of an iceberg, represented by the triangle, and the water line represented by the wavy line.

The iceberg metaphor has two main parts: the visible part situated above the water line and the invisible part situated below the water line. The visible part represents the tangible aspects of culture that we can experience through our senses like art, music and language. The invisible part represents the intangible aspects of culture like history, values and philosophy.

There is a dynamic relationship between these two aspects of culture represented by the blue arrows as in Figure 1. In this dynamic, the intangible aspects of culture inform the tangible aspects of culture. In other words, our history, beliefs, philosophy and so on inform how we express ourselves and interact with the world around us. This dynamic is represented by the blue arrow on the right. As we interact with the world through our tangible culture, we also have an opportunity to learn from a multitude of experiences and learn from other people's cultural expressions. These experiences provide us with an opportunity to reflect and evaluate our beliefs, our history and other aspects of our intangible culture and the cycle continues.

This dynamic happens on all different scales from the individual scale to the national scale. A good example of this is the debate around celebrating Australian Day on the 26th January.

This debate is an opportunity to listen to a broad range of opinions and to decide for yourself, at the individual scale, whether you personally choose to celebrate on the 26th January or not. Also, some local councils have decided to shift the day while others have not. While at the national scale the leadership has decided to uphold tradition and continue to celebrate Australia Day on 26th January. It is also important to note that Aboriginal and Torres Strait Islander peoples have been undertaking this process for 80,000+ years and over that time have developed complex societal structures, complex knowledge systems and a plethora of languages across Australia. In essence, culture encompasses all aspects of our lives. It informs our identity, our beliefs, the way we view the world, and how we behave and interact with the world.

WHAT IS MATHEMATICS?

Mathematics is often portrayed as an abstract knowledge that is free from bias i.e. free from influence of people and culture. In other words, mathematics is objective, and through this objectivity, mathematics is often linked to "finding the truth". Within this viewpoint, the beauty of mathematics comes from its precision, its unwavering logic and its power to solve "real world" problems. This view has also resulted in a mathematics education that is focused on students' learning mathematical processes and procedures to solve a set of problems from textbooks and worksheets with a proportion of these problems claiming to have "real world" application.

This type of education has several important implications. First, it teaches students to be slave to mathematical processes where students search for the right procedure to apply to a particular problem. In other words, the mathematics is telling the student how to act. It also leads to the notion that mathematics is always either right or wrong, which will inevitably result in a student experience where they feel either successful at mathematics or that they are failing at mathematics.

This dynamic also acts as a filter that only allows certain individuals, who are perceived to have special innate abilities in mathematics, to progress in mathematics education. In essence, mathematics education has created a process of disconnection which is often reflected in statements from students like "What are we learning this for?" and "I will never use this again".

Teaching mathematics from a cultural perspective allows students to connect with the meaning and purpose of mathematics.

Activity 2:

Reflect on the above two paragraphs, write down what mathematics means to you and your experience with mathematics. Explore the connections between your mathematics story and the concepts in the Iceberg Model from Activity 1.

THE GOOMPI MODEL:

Connection Between Mathematics and Culture

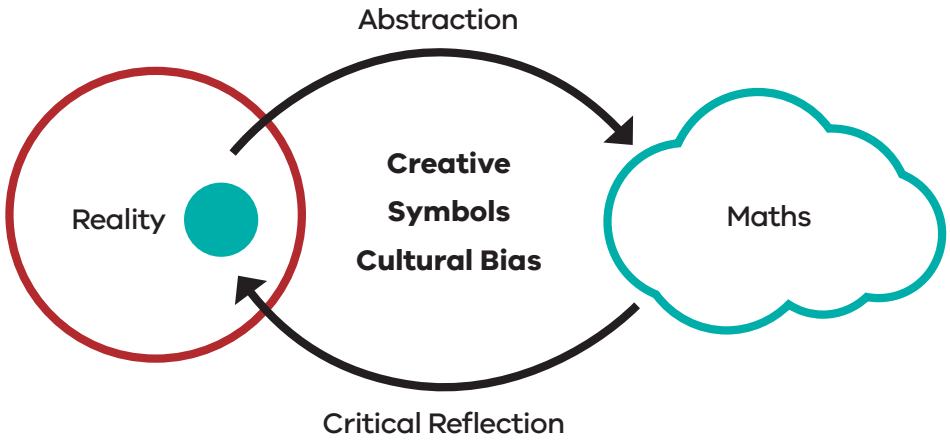


Figure 2: The Goompi Model: teaching mathematics from a cultural perspective

TEACHING MATHEMATICS FROM A CULTURAL PERSPECTIVE

My first mathematics education project was at Dunwich State School on Minjerribah (Stradbroke Island), Queensland. For this project, I wanted to explore how to improve the teaching and learning of mathematics for Indigenous students that support their cultural identity. To do this, I had to develop an understanding of mathematics and its connection to culture.

As illustrated in Figure 2, mathematics is about reflecting on the world around us and identifying a part of our reality (blue circle) we want to understand. We then naturally go through an abstraction process (top arrow) where we create multiple representations (i.e. language, symbols, drawings, stories and so on) of concepts and processes we are interested in. These abstract representations become part of a body of knowledge that we call mathematics (the cloud) and allows us to explore the world in abstract form to gain further insights. Given that mathematics has been created by people, we must then critically reflect (bottom arrow) on the mathematical constructs to ensure that we have represented concepts and processes accurately.

Figure 2 forms a cycle that can be interpreted as an evolutionary cycle that has driven the development of mathematics over time. There are three key concepts from Figure 2 that highlight the connection between mathematics and culture: creative, symbols and cultural bias.

When I first reflected on this process in Figure 2, it was evident that the process fits many different cultural expressions. My first thought was art, which then implied that mathematics could be taught through creativity and a creativity that goes beyond creative thinking. I was keenly interested in whether we could teach mathematics through ***creativity*** that allows self-expression i.e. the students can express their ideas using their language and their way they see the world while learning key mathematical concepts. In essence, students express their culture as part of the teaching and learning of mathematics.

Symbols are fundamental to the process in Figure 2 since symbols carry meaning from our reality into the abstract world of mathematics. In mathematics education, I would argue that we do not teach enough about the meaning of symbols, the connection between multiple symbols and how they connect to mathematical concepts and concepts from reality. Another inherent aspect of Figure 2 is cultural bias. ***Cultural bias*** can often be perceived as a negative, but I would argue that it is a natural consequence of any abstraction whether it be art, music or mathematics. In addition, mathematics has been created, translated and interpreted by many cultures around the world. If we recognise the cultural diversity of mathematics, it provides an opportunity to see mathematics from different perspectives and consequently see different ways to teach mathematics and create innovative pedagogy.

As part of this project, I created the model in Figure 2. The model is called the Goompi Model because it is the traditional name of the Country Dunwich State School resides. The name keeps the connection back to Country: its place of origin. It is also about keeping all the connections alive that are represented in the model to ensure that students experience mathematics that is part of their living world and part of who they are.

EXEMPLAR:

Teaching Linear Equations using Growing Patterns

To demonstrate the used of the Goompi Model, this section will focus on the teaching and learning of linear equations. Even though linear equations are usually taught in high school, I would encourage all teachers to continue reading to get an understanding of how the Goompi Model can transform the teaching of mathematics and the potential outcomes this approach could have for students' learning. This exemplar will focus on creativity as self-expression and the potential this has to increase students' conceptual understanding of mathematical concepts.

For most students, their first introduction to linear equations is given by the following generalised equation:

$$y = mx + c$$

Where y is the dependent variable, x is the independent variable and both m and c are constants. This type of highly abstract introduction to any mathematical concept would make most students wish they could run out the door.
This problem has been recognised in mathematics education for a long time and, for linear equations, the pedagogical approach called growing patterns has been used to soften the abstraction blow and give some purpose to the study.

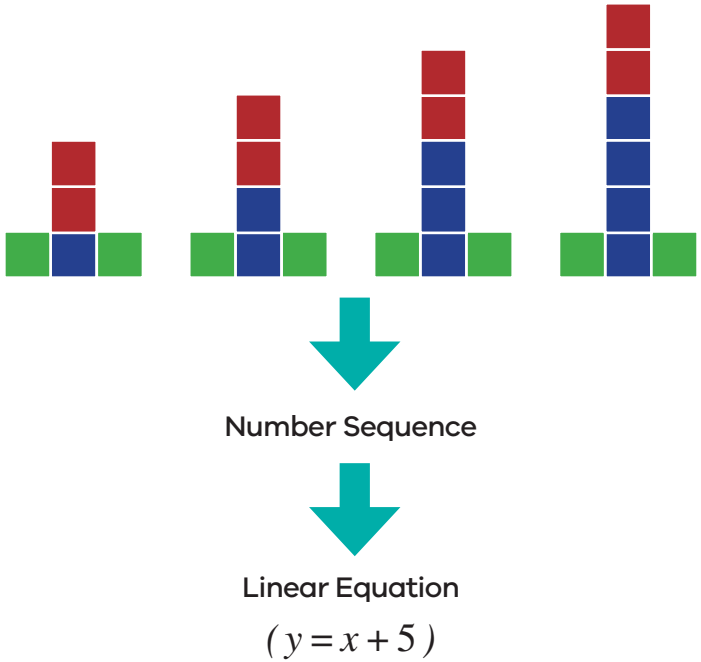


Figure 3: Typical representation of a growing pattern

TYPICAL REPRESENTATION OF A GROWING PATTERN

Figure 3 shows a typical representation of a growing pattern that is used to introduce linear equations to students. The basic process for growing patterns is for students to 1) translate the pattern into a number sequence i.e. count the number of squares at each stage; and 2) transform the number sequence into a linear equation. The first step is straight forward but, in the second step, it is not always self-evident how you can create a linear equation from the number sequence. It becomes another cognitive jump that students need to navigate.

Some of the positives of this approach is that teachers can uses these patterns as an open ended question for the students to investigate, which can lead to students' understanding the different ways to perceive the growing pattern and how they relate to a linear equation. However, by using the Goompi Model, we can create a pedagogical approach that shifts the effort from negotiating a cognitive jump to focusing on the main mathematical concepts underpinning linear equations. This allows students to create their own representation of a linear equation from their world view.

Goompi Model Version of Growing Patterns

The following will give a brief account of the Goompi Model version of growing patterns focusing on a few key steps. I will then provide an example of a student's representation of a growing pattern to demonstrate the importance of creativity, as self-expression, in the teaching and learning of mathematics.

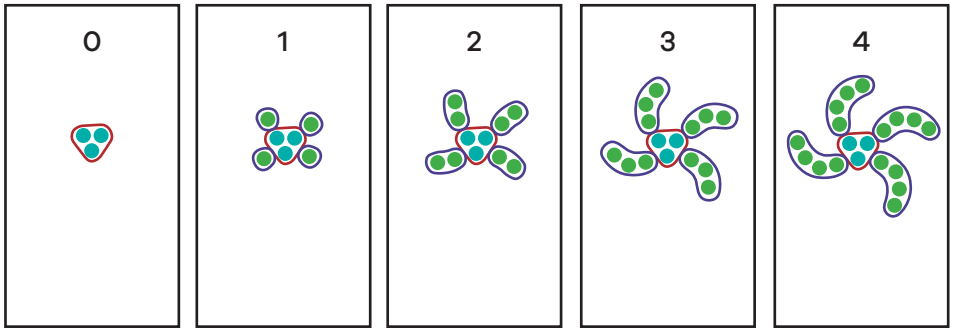


Figure 4: First stage of growing pattern pedagogy using the Goompi Model

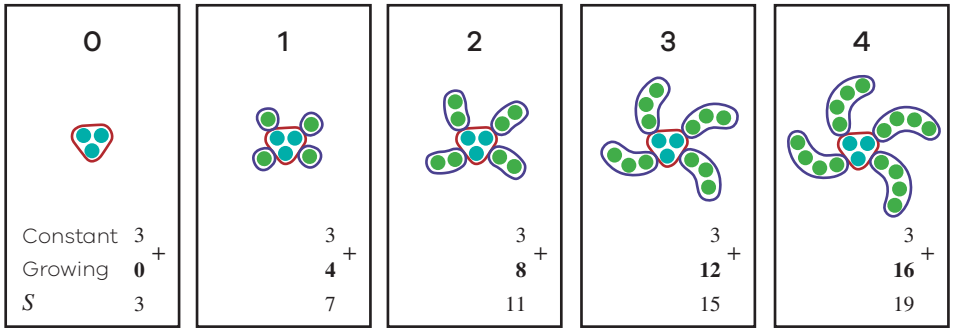


Figure 5: Second stage of growing pattern pedagogy using the Goompi Model

FIRST STAGE OF GROWING PATTERN PEDAGOGY USING THE GOOMPI MODEL

In this version of growing patterns, the first main step is to explicitly teach the connection between the structure of a pattern and the structure of the linear equation. This process starts with a series of butchers paper on the ground (5 is a good number) with each paper representing a different day. I tell a story about observing a pattern growing over several days and draw each stage of the pattern on the butchers paper. As I'm drawing the pattern, I ask the students if they recognise the pattern.

Given that I'm drawing a spiral, students start to connect this pattern with hurricanes, galaxies, flowers and so on. I then tell the student that there are two processes in my pattern, that is, there is a part that stays the same and a part that is growing. To reinforce this understanding, I ask for a volunteer to circle the part that "stays the same" and the part that is "growing" and colour code these as red and blue respectively. This is represented in Figure 4 with the pattern growing over four days.

SECOND STAGE OF GROWING PATTERN PEDAGOGY USING THE GOOMPI MODEL

The next stage is to use numbers to emphasise the structure of the pattern. I pose the question to the students, "How big will my pattern be in a year i.e. 365 days?" This is a common question in growing patterns but I emphasise here that we need to have a measure for the size of my spiral and we need to build a relationship between the size of my spiral and the number of days. I define that the size of my spiral will be the number of dots in the pattern and then get the students to tell me, for each day, how many dots are in the "growing" part and how many are in the "stays the same" part. At this point, I also introduce the mathematics language for "stays the same" which is "constant".

Figure 5 shows the series of numbers for each day and, at this point, the students get a strong understanding that three dots are constant across all the days and the growing part is increasing by four each day.

Goompi Model Version of Growing Patterns Cont.

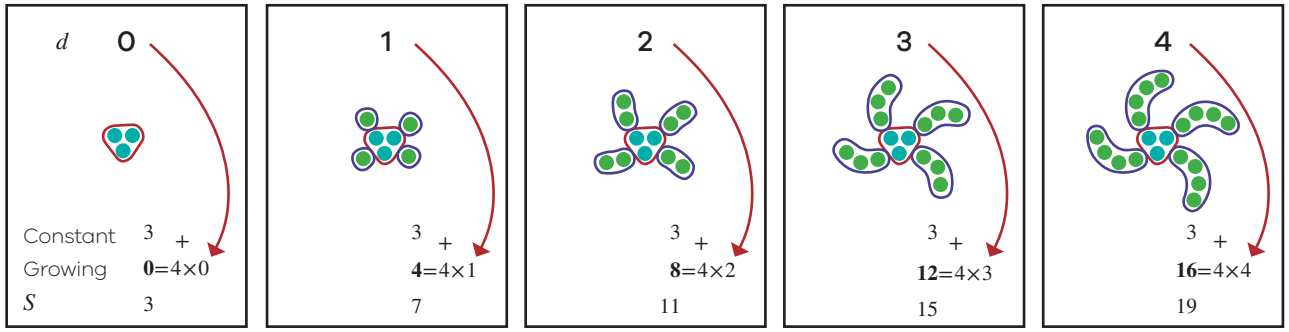


Figure 6: Third stage of growing pattern pedagogy using the Goompi Model

THIRD STAGE OF GROWING PATTERN PEDAGOGY USING THE GOOMPI MODEL

The next stage of this process is to build relationship between the size of the spiral and time (or days). Figure 5 has started the relationship building by recognising that the size of the spiral is equal to the growing and constant parts added together and I define the letter "S" to represent the size of the spiral. I discuss with the students that building a relationship directly with "days" and the size of the spiral "S" is difficult but we can explore the relationship between each separate part since they make up the spiral. We then talk through the relationship between "days" and the "constant" part drawing out the conclusion that no matter which day it is the "constant" part will always be 3.

We then explore the relationship between days and the growing part. Students already recognise that the pattern is increasing by four each day but when we write out together the groups as multiplies of four, we soon realise the number of groups of four is related to the number of days. This is represented in Figure 6.

Note that I also introduce the idea of variables to the students as a symbol that holds a concept like time or the size of the spiral. From explicitly showing days with the "growing" part with the red arrow, we can then build the linear equation as $S = 4 \times d + 3$ and reinforce that the linear equation is made up of a growing part ($4 \times d$) plus a constant part (3) that stays the same across all days.

I then give students a different linear equation, which is usually $S = 5 \times d + 2$ and ask the students to draw a pattern for the equation.

VIGNETTE:

Roma Mitchell Secondary College, Adelaide, South Australia

Amanda Pedder is a mathematics teacher from Roma Mitchell Secondary College. Amanda attended a workshop where I demonstrated the above pedagogy to teach linear equations and decided to try the idea on a class of disengaged Year 10 students. After the lesson, Amanda was astonished that the class wanted to engage in the activity (i.e. draw pictures), she was amazed at what they drew and she had never experienced this level of conceptual understanding before from any student.

To demonstrate this, Figure 7 shows a drawing from one of the students who connected their growing pattern with their foster care experience. The student's story behind his expression is that there are two caregivers who cared for and impacted the lives of five children each year. It is clear that the student understands the meaning of linear equations and how a growing pattern can be constructed for a given equation. More importantly, the student also connected, on a personal level, and wanted to share part of their life with the class and the teacher. Amanda also found that this process of creativity helped disrupt the cycle of disengagement in the class and improved her relationship with the students.

Figure 7: A student's expression of a linear equation connected to their foster care experience

Activity 3:

Reflect on your culture, your world view and draw a growing pattern that represents the linear equation.

$$S = 5 \times d + 2$$

CONNECTION BETWEEN ABORIGINAL CULTURE AND MATHEMATICS

The section will provide an introduction into the connection between mathematics and Aboriginal Culture.

In doing so, I must first recognise Dr Mandawuy Yunupiŋu, the first Yolŋu principal at Yirrkala School, who set a vision that all students at Yirrkala School will receive bilingual education that encompassed two-way learning. To achieve this type of education, Dr Yunupiŋu worked with Elders and a range of academics and educators to explore the connection between Yolŋu knowledge and Western knowledge. From this extensive work, Dr Manadwuy Yunupiŋu stated that the closest connection between Yolŋu knowledge and Western knowledge is mathematics and went on to say that Yolŋu mathematics is Gurruṯu. Gurruṯu is a kinship system that connects all people to all the elements of the world (e.g. animal, plants, wind, fire, water and so on). In essence, it is a system that connects people to the environmental system of a particular Place on Country, which creates complex societal structures that are inherently mathematical. For an introduction to Gurruṯu and its connection to education at Yirrkala, please refer to three articles I wrote for ACER's Teacher Magazine.

The philosophy that underpins any Aboriginal kinship system is that everything in the world is interconnected through a network of relationships. From this worldview, "pattern thinking" and "systems thinking" are essential skills and have obvious connections to mathematics and mathematics education. Note that the linear equation pedagogy outlined above is a good example of this, particularly when the pattern relates to processes and relationships on Country.

> ***"Pattern thinking is Aboriginal thinking. There is no big boss. Patterns are about belonging. Nothing is separate from anything else", David Mowaljarlai, Elder from Derby, Aboriginality a Gift: Spirituality for a Nation.***

It is also represented beautifully in the quote by David Mowaljarlai, an Elder from Derby, that was documented in the book Aboriginality a Gift: Spirituality for a Nation. David Mowaljarlai clearly states that "Pattern thinking is Aboriginal thinking". He also continues to state there is "no big boss" and "patterns are about belonging". This means that under an Aboriginal philosophy and knowledge system there can never be one person who controls everything since everyone belongs in the system and has their own agency and responsibility to strengthen and maintain the system. The last part of the quote "Nothing is separate from anything else" is a statement about how all the elements of the world are interconnected.

This is a sophisticated way to see the world and demonstrates that Aboriginal knowledge systems are about sustainability and an understanding of why Aboriginal people are the oldest living culture in the world.

I also see this as a metaphor for mathematics and mathematics education. In my opinion, mathematics and mathematics education have been a process of disconnection that ensures only a few make it through the "rigour" of mathematics education. We need a mathematics education that connects with people, embraces diversity of thinking and ensures that all people belong. To achieve this, I would encourage educators to teach mathematics from a cultural perspective so that:

- Creativity is a focus in the teaching and learning of mathematics that allows students to express their culture, their worldview while learning key mathematical concepts. Also, creativity leads to innovation.
- Students have their own voice in the mathematics classroom, which allows positive relationships to develop throughout the classroom.
- Students experience and learn to value a diversity of worldviews.
- Aboriginal Culture is valued in the classroom and students develop an understanding that mathematics is part of Aboriginal culture.

If we can achieve this, then we will truly see a difference in mathematics education for Aboriginal students.

Activity 4:

Read the second article "Indigenous perspectives in mathematics: Understanding Gurruṯu" and attempt the teacher/student activity. The activity is about drawing a family tree, using Western structures, and then connecting the tree to Gurruṯu. What patterns do you see?