The Golden Ratio

The interesting thing is that it keeps popping up in strange places - places that we may not ordinarily have thought to look for it. In fact civilizations as far back and as far apart as the Ancient Egyptians, the Mayans, as well as the Greeks discovered the Golden Ratio and incorporated it into their own art, architecture, and designs. They discovered that the Golden Ratio seems to be Nature's perfect number. For some reason, it just seems to appeal to our natural instincts. For centuries, designers of art and architecture have recognized the significance of the Golden Ratio in their work.

Is what we call an **irrational number**:

- It has an infinite number of decimal places
- It never repeats itself!

Generally, we round the Golden Ratio to:

What is the Greek letter for the Golden Ratio?

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The Golden Ratio in Art

Now let's go back and try to discover the Golden Ratio in art. We will concentrate on the works of Leonardo Da Vinci, as he was not only a great artist but also a genius when it came to mathematics and invention. Your task is to find at least one of the following Da Vinci paintings on the Internet. Make sure that you find the entire painting and not just part of it.

List of paintings to look for:

* The Annunciation
* Madonna with Child and Saints
* The Mona Lisa
* St. Jerome

Directions for finding evidence of the Golden Ratio in each painting:

*The Annunciation* - Using the left side of the painting as a side, create a square on the left of the painting by inserting a vertical line. Notice that you have created a square and a rectangle. The rectangle turns out to be a Golden Rectangle, of course. Also, draw in a horizontal line that is 61.8% of the way down the painting (.618 - the inverse of the Golden Ratio). Draw another line that is 61.8% of the way up the painting. Try again with vertical lines that are 61.8% of the way across both from left to right and from right to left. You should now have four lines drawn across the painting. Notice that these lines intersect important parts of the painting, such as the angel, the woman, etc. Coincidence? I think not!
Madonna with Child and Saints - Draw in the four lines that are 61.8% of the way from each edge of the painting. These lines should mark off important parts of the painting, such as the angels and the baby Jesus in the center.

The Mona Lisa - Measure the length and the width of the painting itself. The ratio is, of course, Golden. Draw a rectangle around Mona's face (from the top of the forehead to the base of the chin, and from left cheek to right cheek) and notice that this, too, is a Golden rectangle.


Conclusions - Leonardo Da Vinci's talent as an artist may well have been outweighed by his talents as a mathematician. He incorporated geometry into many of his paintings, with the Golden Ratio being just one of his many mathematical tools. Why do you think he used it so much? Experts agree that he probably thought that Golden measurements made his paintings more attractive. Maybe he was just a little too obsessed with perfection. However, he was not the only one to use Golden properties in his work.
The Golden Ratio in Architecture

For this investigation, we are going to use images of one particular structure: The Parthenon in Greece.

You will need a ruler to complete this activity.

Let's start by looking at the following image of the Parthenon:

1. Let's start by drawing a rectangle around the Parthenon, from the left most pillar to the right and from the base of the pillars to the highest point.

2. Measure the length and the width of this rectangle. Now find the ratio of the length to the width. Is the number fairly close to the Golden Ratio?

3. Now look above the pillars. You should notice some rectangles on the face of the Parthenon. Find the ratio of the length to the width of one of these rectangles. Notice anything?

There are many other places where the Golden Ratio appears in the Parthenon, all of which we cannot see because we only have a frontal view of the structure. The building is built on a rectangular plot of land which happens to be ... you guessed it - a Golden Rectangle!
We will continue this process until each square has an arc inside of it, with all of them connected as a continuous line. The line should look like a spiral when we are done. Here is an example of what your spiral should look like (move your cursor over the images to see them change):

Now what was the point of that? The point is that this "golden spiral" occurs frequently in nature. If you look closely enough, you might find a golden spiral in the head of a daisy, in a pinecone, in sunflowers, or in a nautilus shell that you might find on a beach. Here are some examples:
The Perfect Face

Do these faces seem attractive to you? Many people seem to think so. Is there something specific in each of their faces that attracts us to them, or is our attraction governed by one of Nature's rules? Does this have anything to do with the Golden Ratio
Here's how to conduct the search for the Golden Ratio: measure certain aspects of each person's face. Then compare the ratios.

\[ a = \text{Top-of-head to chin} = \underline{\phantom{0}} \text{ cm} \]
\[ b = \text{Top-of-head to pupil} = \underline{\phantom{0}} \text{ cm} \]
\[ c = \text{Pupil to nosetip} = \underline{\phantom{0}} \text{ cm} \]
\[ d = \text{Pupil to lip} = \underline{\phantom{0}} \text{ cm} \]
\[ e = \text{Width of nose} = \underline{\phantom{0}} \text{ cm} \]
\[ f = \text{Outside distance between eyes} = \underline{\phantom{0}} \text{ cm} \]
\[ g = \text{Width of head} = \underline{\phantom{0}} \text{ cm} \]
\[ h = \text{Hairline to pupil} = \underline{\phantom{0}} \text{ cm} \]
\[ i = \text{Nosetip to chin} = \underline{\phantom{0}} \text{ cm} \]
\[ j = \text{Lips to chin} = \underline{\phantom{0}} \text{ cm} \]
\[ k = \text{Length of lips} = \underline{\phantom{0}} \text{ cm} \]
\[ l = \text{Nosetip to lips} = \underline{\phantom{0}} \text{ cm} \]

Now, find the following ratios:

\[ a/g = \underline{\phantom{0}} \text{ cm} \]
\[ b/d = \underline{\phantom{0}} \text{ cm} \]
\[ i/j = \underline{\phantom{0}} \text{ cm} \]
\[ i/c = \underline{\phantom{0}} \text{ cm} \]
\[ e/l = \underline{\phantom{0}} \text{ cm} \]
\[ f/h = \underline{\phantom{0}} \text{ cm} \]
\[ k/e = \underline{\phantom{0}} \text{ cm} \]

Did any of these ratios come close to being Golden? If not, then maybe this face isn't so perfect after all. Of the face above, who has the most "Golden" one?
Fibonacci’s sequence

Use your research skills to find Fibonacci’s Sequence:

1, 1,

What does this have to do with the Golden Ratio?

This sequence of numbers was first discovered by a man named Leonardo Fibonacci, and hence is known as Fibonacci’s sequence. The relationship of this sequence to the Golden Ratio lies not in the actual numbers of the sequence, but in the ratio of the consecutive numbers. Let’s look at some of these ratios:

\[
\begin{align*}
2/1 &= 8/5 = 34/21 = \\
3/2 &= 13/8 = 55/34 = \\
5/3 &= 21/13 = 89/55 = 
\end{align*}
\]

What do you notice?

What do you think will happen if we continue to look at the ratios as the numbers in the sequence get larger and larger?
The Golden Ratio in Nature

So, why do shapes that exhibit the Golden Ratio seem more appealing to the human eye? No one really knows for sure. But we do have evidence that the Golden Ratio seems to be Nature's perfect number. Take, for example, the head of a daisy:

![Daisy](image)

Somebody with a lot of time on their hands discovered that the individual florets of the daisy (and of a sunflower as well) grow in two spirals extending out from the center. The first spiral has 21 arms, while the other has 34. Do these numbers sound familiar? They should - they are Fibonacci numbers! And their ratio, of course, is the Golden Ratio. We can say the same thing about the spirals of a pinecone, where spirals from the center have 5 and 8 arms, respectively (or of 8 and 13, depending on the size)- again, two Fibonacci numbers:

![Pinecone](image)

A pineapple has three arms of 5, 8, and 13 - even more evidence that this is not a coincidence. Now is Nature playing some kind of cruel
game with us? No one knows for sure, but scientists speculate that plants that grow in spiral formation do so in Fibonacci numbers because this arrangement makes for the perfect spacing for growth. So for some reason, these numbers provide the perfect arrangement for maximum growth potential and survival of the plant.