Teaching Secondary Mathematics

Module 8

Working mathematically:
Focus on a range of challenging problems

8

State Government Victoria
Department of Education and
Early Childhood Development

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Contents

Introduction to Module 8: Working mathematically: Focus on a range of challenging problems

The Teaching Secondary Mathematics Resource provides support and guidelines for effective practice for classroom teachers and school leaders of mathematics, especially from Years 7–10.

This resource is for:

- all secondary mathematics classroom teachers to deepen their understanding of mathematics. This will inform their planning for mathematics and highlight opportunities for assessment of mathematics in other domains of the Victorian Essential Learning Standards (VELS)
- mathematics leaders in schools to plan opportunities for professional learning for the teachers of mathematics, in professional learning teams and/or for individual teachers
- differentiating the professional learning needs of mathematics teachers in schools.

Use of this module

This module allows for flexibility in modes of engagement with professional learning. The module booklet needs to be used in conjunction with the PowerPoint slides accompanying this resource.

Workshop approach

The materials of this module can be used by a presenter in a workshop for a school or a cluster of schools. A presenter, appointed from within or outside a school or cluster, is responsible for preparing presentations, facilitating discussions and outlining processes for collaborative planning.

Where a group is working collaboratively through these modules, a designated area is required for participants to share ideas, stories and samples in a climate of mutual respect. Regular after school meetings in a particular venue, such as the library, create a productive sense of community.

Individual use

The materials of this module are also suitable for private study and reflection. Individual users become both ‘presenter’ and ‘participant’. While they are not able to engage in group discussions or whole-school planning, individual users can readily adapt the suggested group discussions and whole-school planning activities to private reflection, writing and classroom planning.

It is suggested that individuals identify a colleague or a buddy with whom to share their thoughts and to support the development of their understandings through ongoing dialogue. Individuals may complete all the modules or choose a combination, depending on their interests or needs.

Web connections

The ‘Teaching for Secondary Mathematics’ resource is located at http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/default.htm.

Before commencing to plan any elements of the program, schools are strongly advised to visit the Mathematics Domain page to review the most up-to-date advice, resources and information relevant to each module of the program. Many elements of this resource are available online in a downloadable format. There are links to assist schools to locate relevant information.

- Mathematics Domain (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm)
  See the website for further details about this additional information or contact the student learning help desk on studentlearning@edumail.vic.gov.au

Content of the module

The module comprises Module 8 booklet and the accompanying slide presentations which can be downloaded from http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/module8.htm

The following are included in this document:

- the **User’s Guide** that assists the user through the professional learning program
- hard copies of the **slide presentations** and **resource sheets**
- selected **resources.**

Organisation of the module

Computer access is required for all modules. If a group is completing the modules, a data projector and tables that enable people to sit together and work collaboratively are also necessary. The presenter should encourage participants to raise questions throughout the ensuing presentation. This presentation should take approximately one hour, depending on the depth of discussion and types of activities that facilitators incorporate.

Required resources:

Indicators of progress

- Indicator of progress: Counter Examples (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/default.htm)
- Indicator of progress: Real World Investigations 4.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/W40006P.htm#4)

Mapping the Indicators of Progress (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.pdf)

Developmental Overview of Working Mathematically (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdoworkingmathly.pdf)

Principles of Learning and Teaching P–12 (http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm) Refer also to Resource 1 (attached).

Icons

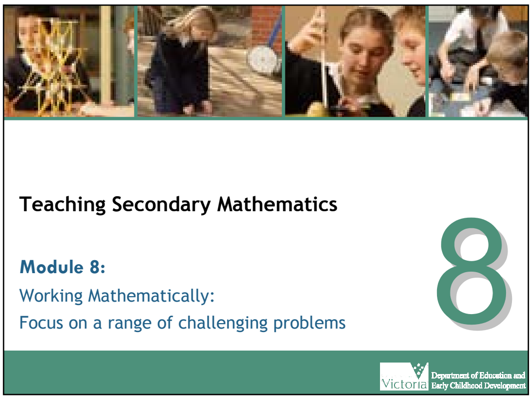
The following icons have been used in this workshop program:

Distribute handout:

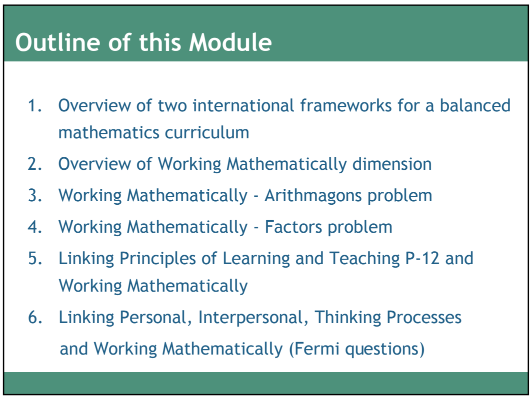
Group discussion:

Group activity:

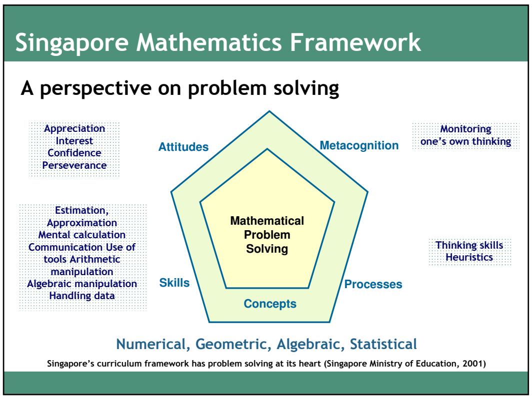
User's Guide to Module 8: Working mathematically: Focus on a range of challenging problems



Slide 1: Title slide



Slide 2: Outline of this module



Slide 3: Singapore Mathematics Framework

Introduction

Slide 1 is the title slide

"Working mathematically focuses on developing students' sense of mathematical inquiry: problem posing and problem solving, modelling and investigation. It involves students in the application of principled reasoning in mathematics, in natural and symbolic language, through the mathematical processes of conjecture, formulation, solution and communication, and also engages them in the aesthetic aspects of mathematics."(http://vels.vcaa.vic.edu.au/downloads/vels_standards/velsrevisedmathematics.pdf)

Having a good appreciation of working mathematically is an important part of achievement in mathematics. Moreover, the processes of investigation, inquiry and explanation involved build each student's capacity as a learner. These are all important outcomes of schooling. This module explores these processes in the context of some intriguing mathematical problems.

To improve mathematics learning in schools we need to improve student achievement. Mathematics (and other subjects) should offer students experiences which increase their capacity to learn. In turn, students with increased capacity to learn will do better in mathematics.

Outline of this module

Slide 2 gives an outline of this module. The module includes:

- Overview of two international frameworks for a balanced mathematics curriculum
- Overview of 'Working mathematically' dimension
- Working mathematically – Arithmagons problem
- Working mathematically – Factors problem
- Linking Principles of Learning and Teaching P–12 and 'Working mathematically'
- Linking Interpersonal, Personal, and Thinking Processes domains and 'Working mathematically'

International frameworks

Singapore Mathematics Framework

Slide 3 provides a graphic of the Singapore Mathematics Framework[1].

This diagram shows the Singapore mathematics framework, which emphasises conceptual understanding, attitudes, metacognition (thinking about thinking – step back a level and monitoring your own thinking) and problem solving processes as well as the conventional 'skills'.

The thinking skills provide strategies for tackling problems. They incorporate heuristics – an educational method in which learning takes place through discoveries that result from investigations made by students.

In Australia, practising skills is traditionally the focus in mathematics classes.

[1] Ministry of Education, Singapore. (2001). Lower Secondary Mathematics Syllabus. Singapore: Ministry of Education.

USA: Balanced Mathematics Curriculum

Slide 4 is an animated slide which shows a US model[2] of a balanced curriculum.

Explain that this model provides for five strands of mathematical proficiency. It demonstrates what it means to be a mathematical thinker, and that mathematics is much more than just procedures.

This model of the mathematics curriculum also highlights a need for balance.

Use slide 4: USA model

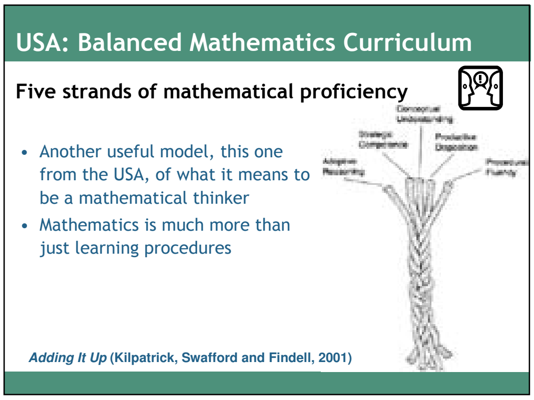
Invite participants in small groups to discuss:

- Which of the five strands are fostered in their classrooms? How is this done?
- How are these linked the 'Working Mathematically' strand?

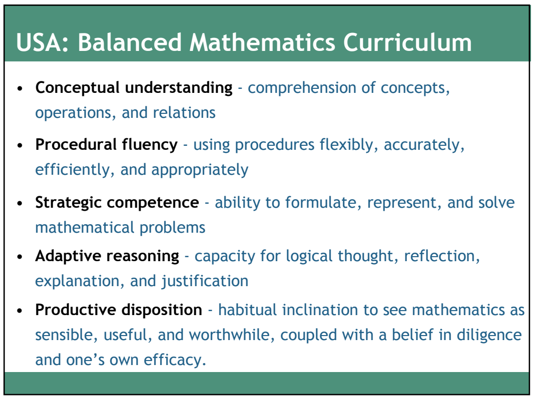
Record the suggestions made by the participants.

Slide 5 explains these strands:

- conceptual understanding – comprehension of concepts, operations and relations
- procedural fluency – using procedures flexibly, accurately, efficiently and appropriately
- strategic competence – ability to formulate, represent and solve mathematical problems
- adaptive reasoning – capacity for logical thought, reflection, explanation and justification
- productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

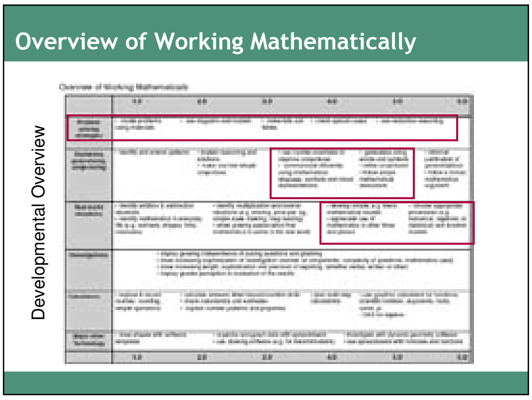


Slide 4: USA: Balanced Mathematics Curriculum

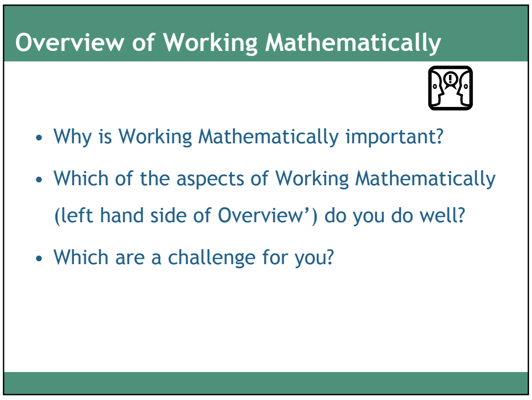


Slide 5: USA: Balanced Mathematics Curriculum

[2] Kilpatrick, J., Swafford, J. and Findell, B. (2001) Adding It Up: Helping Children Learn Mathematics. Washington, DC: National Academy Press.



Slide 6: Overview Working Mathematically — Developmental overview



Slide 7: Overview Working Mathematically

Aspects of 'Working mathematically' – Developmental overview

Slide 6 shows a copy of the 'Developmental Overview for Working Mathematically'. It also highlights the aspects featured in workshop module 7: Learning through investigation: Focus on chance and variability.

Slide 7 poses three questions about 'Working mathematically'.

Inform participants that all mathematics teaching should involve one or more aspects of the 'Working mathematically' dimension. It is important for teachers to explicitly use the aspects of:

- problem solving
- explaining, generalising and conjecturing
- real world situations
- investigations
- calculators
- technology

with their students. Developing new mathematics is important as it makes mathematics exciting, enjoyable and gives students confidence. Even though students might not invent new mathematics they are able to use old mathematical skills in new ways.

Distribute a copy of 'Overview of Working Mathematically' (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdoworkingmathly.pdf)

Use slides 6, 7: Exploring the Developmental Overview

In small groups discuss:

- Why is working mathematically important?

The following responses may assist you in your discussion:

- It is important that students are able to:
  - use known mathematics to solve problems, including unfamiliar problems
  - develop new mathematics to solve problems, from within mathematics and from the real world
  - learn to reason mathematically
  - learn strategies for tackling problems.

Ask participants to refer to the different aspects of 'Working mathematically' found on the left hand side of the 'Overview of Working Mathematically'

In small groups discuss:

- Which of the aspects of 'Working mathematically' (left hand side of 'Overview') do you do well?
- Which are challenges for you?

Working mathematically – Arithmagons problem

Counter-examples: 4.0

Slides 8 to 12 provide participants with the opportunity to delve into activities contained in the Mathematics Developmental Continuum: 'Counter-examples 4.0' in the 'Working mathematically' dimension of VELS.

The Arithmagon and Factors activities below illustrate several points about making conjectures. Conjectures often arise from systematically trying examples and the recording of results in labelled tables. This process can assist in highlighting patterns. Conjectures can be tested by checking predictions against observations/calculations, etc. If any counter-examples to a conjecture are found, then the conjecture is disproved, but the original can often be modified to make a better conjecture

Use slides 8–10: Arithmagons

Provide the following instructions to the participants.

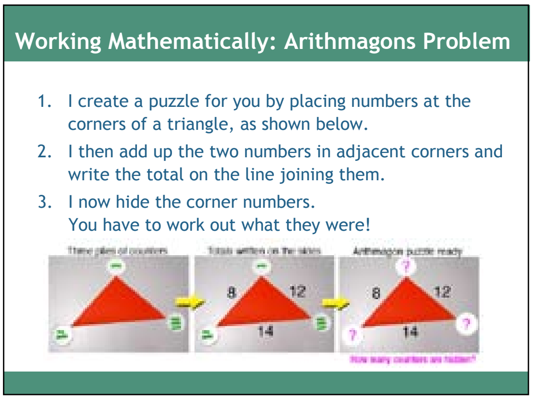
- I will create a puzzle for you by placing numbers at the corners of a triangle, as shown below.
- I will then add up the two numbers in adjacent corners and write the total on the line joining them.
- I now hide the corner numbers. You have to work out what they were!

Arithmagons example

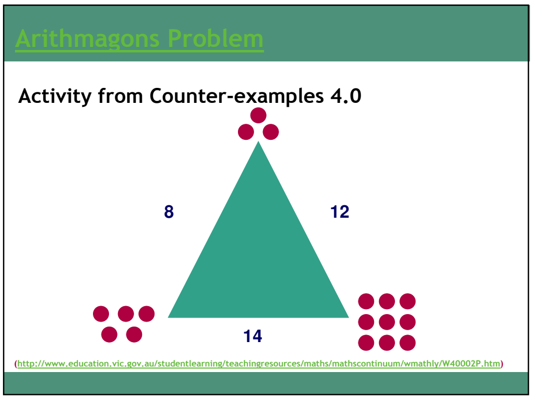
Slide 9 provides a typical arithmagons example. The slide is hyperlinked to an electronic version of the continuum:

- Indicator of progress: Counter-examples (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/W40002P.htm)

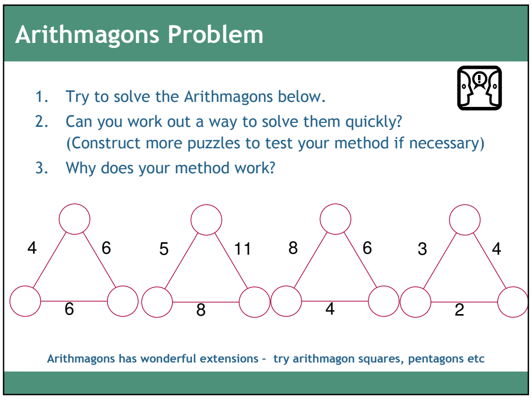
This slide has an animation to show the development of the arithmagons problem. (Note: the printed version of this slide will not show all details.)



Slide 8: Working mathematically – Arithmagons problem



Slide 9: Arithmagons Problem



Slide 10: Arithmagons Problem — Solving the arithmagons example

Solving the arithmagons example

In small groups solve the problems posed on slide 10:

- Can you work out a way to solve them quickly? (Construct more puzzles to test your method if necessary.)
- Why does your method work?

Arithmagons provide wonderful extensions – encourage participants to try arithmagon squares, pentagons, etc

Some information to consider:

- Make the vertices a, b, c and edges x, y, z (where x and y are adjacent to a, and y and z are adjacent to c …)
- To solve the arithmagons quickly, participants could add the two adjacent sides, subtract the other and divide by 2.

Other methods include simultaneous equations. For example:

- Using three simultaneous equations regardless of number, label the sides $a+c$, $b+c$, $a+b$ (a, b, c labelled already)
  - Total $=2a+2b+2c$
  - Half total$= a+b+c$
  - Then can use $a+b$ value to find c. etc

Participants will come up with a wide range of methods to solve this type of problem. Give them an opportunity to explain their method to the group.

If participants solve the problem quickly, see if they can solve arithmagon squares, or pentagons.

Challenge participants to solve four and five sided arithmagons and see if their reasoning could be applied to all kinds of arithmagons.

Principles of Learning and Teaching P–12 (PoLT)

Use slide 11: Linking with PoLT Principle 1

Slide 11 links this form of student learning to PoLT Principle 1 – The learning environment is supportive and productive.

Distribute the summary of 'Principles of Learning and Teaching P–12'.

Note: a summary of the Principles of Learning and Teaching P–12 is found in the resources section of this document.

- Principles of Learning and Teaching P–12 (http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm)

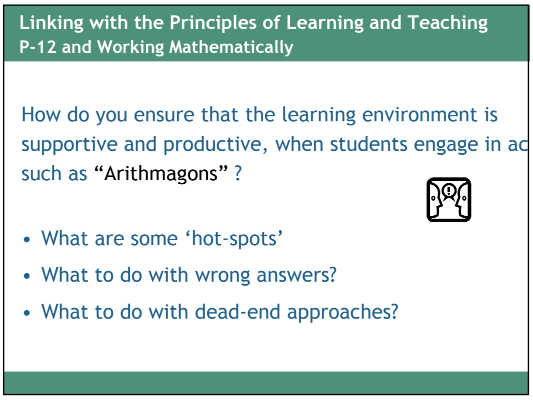
In small groups, invite participants to discuss:

- How could teachers ensure that the learning environment is supportive and productive, when students engage in activities such as arithmagons?

Participants may need to consider:

- What could be some difficulties?
- How do students and teachers tackle wrong answers?
- What can students and teachers do with dead-end approaches?

Participants should encourage students to develop a positive attitude towards their errors. Errors are an integral part of learning process, not disasters, and students need to persist and experiment with their answers.



Slide 11 Linking with the Principles of Learning and Teaching P–12 and Working Mathematically

Reflective discussion

Slide 12 provides a reflective discussion which provides an opportunity for participants to discuss solutions and problem-solving strategies involved in solving arithmagons.

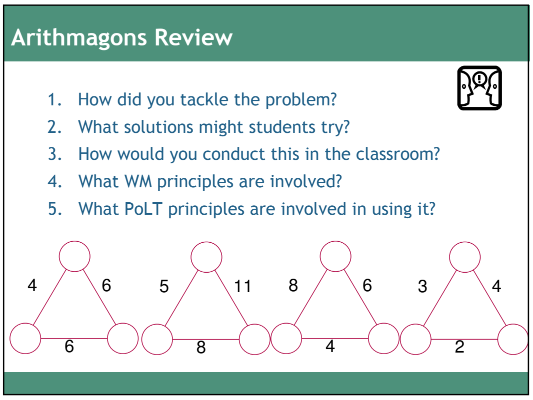
Use slide 12: Arithmagons discussion

In small groups, invite participants to discuss:

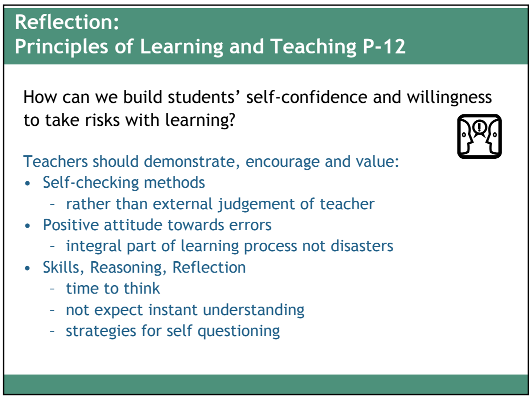
- How did you tackle the arithmagons problem?
- What solutions might students try?
- How would you conduct this in the classroom?
- What are the 'Working mathematically' principles involved?
- What PoLT principles are involved in using it?

Aspects to consider in this discussion include:

- What are the conjectures (mathematical statements that are likely to be true)?
- What are the testing procedures? Search for a reason why the method works.
- What are the limitations of the method?
- What kinds of justifications or reasoning would you expect from your students in the classroom use?



Slide 12: Arithmagons Review



Slide 13: Reflection: Principles of Learning and Teaching P–12

Building students' self-confidence and willingness to take risks with learning

Use slide 13: Taking risks

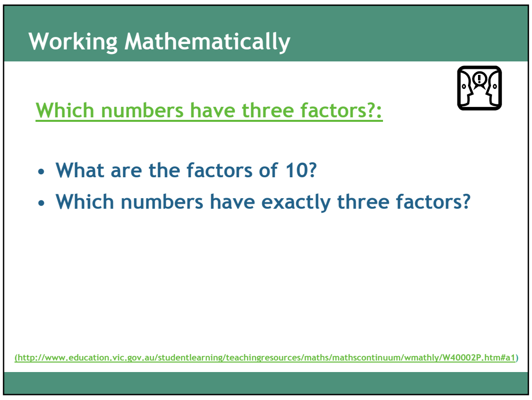
Slide 13 links this activity with one of the Principles of Learning and Teaching P–12.

Ask participants to consider the issue of 'taking risks' which is particularly relevant for 'Working mathematically'. The problem solving activities are developed to encourage self-confidence in students.

Participants may consider the following teacher behaviours (these are highlighted on the slide).

Teachers should demonstrate, encourage and value:

- self-checking methods rather than reliance on the external judgement of teacher
- positive attitude towards errors which is an integral part of learning process
- skills, reasoning and reflection. Provide students with time to think and do not expect instant understanding. Provide students with strategies for self-questioning.



Slide 14: Working mathematically – Which numbers have three factors?

Working mathematically – *Which numbers have three factors?*

Counter-examples

Use slides 14, 15: Which number has three factors

Slide 14 provides a problem which is found as a teaching strategy within the counter-examples:

- Indicator of progress : Counter-examples 4.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/W40002P.htm)

There is a place for intriguing mathematics problems that don't have any immediate real world application – they are just interesting!

Some general principles:

- Deep thinking occurs when convincing follows conjecturing. First you need to convince yourself and then you need to think of arguments to convince others that your conjecture is true.
- If you have partial solutions, look for counter-examples. Teach students to doubt and check.

Challenge participants with the following problems.

- What are the factors of 10?

  1, 2, 5, 10
- Which numbers have exactly three factors?

  The numbers with exactly 3 factors are those which are the square of a prime. The three factors are $1, p, p^2$, where p is prime

Slide 15 provides an example which will extend your discussion of factors by using the expression $n^2 + n + 41$.

Invite participants to consider the equation $n^2 + n + 41$ and ask them to evaluate it when:

$n = 0$

$n = 1$

$n = 2$

$n = 3$

$n = 4$.

Invite participants to respond:

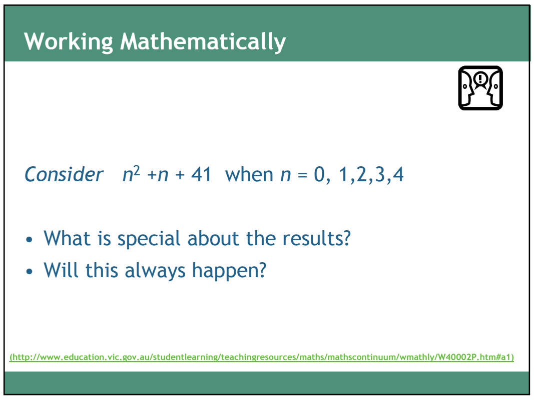
- What is special about the results?

  They generate prime numbers for all values of n between 0 and 39 inclusive.

- Will this always happen?

  It doesn't produce a prime for $n=41$: $41^2 + 41 + 41 = 1763 = 41x43$ which gives us a counter-example. A composite number is generated with $n = 40$ as well.

  The point of this example is to remind participants that just because many examples DO work, doesn't mean it will ALWAYS work. Students often think that because there are lots of specific working examples this means that there is a universally true result.



Slide 15: Working mathematically

Linking Principles of Learning and Teaching P–12 and 'Working mathematically'

Principles of Learning and Teaching P–12 (PoLT)

Use slide 16: Linking with PoLT

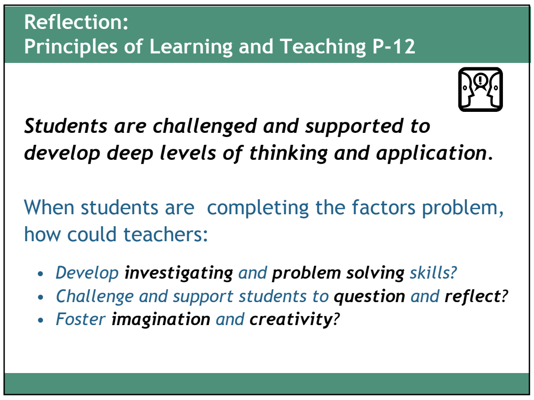
Refer to 'Principles of Learning and Teaching P–12' summary.

In small groups, **invite participants** to discuss Principle 4: 'Students are challenged and supported to develop deep levels of thinking and application' through asking them the following questions.

When students are completing the factors problems:

- How would teachers use strategies that challenge and support students to question and reflect?
- How would teachers use strategies to develop investigating and problem solving skills?
- How would teachers use strategies to foster imagination and creativity?

To promote discussion, facilitators may provide one question to each pair of participants, and then ask each pair to report back to the group through a 'jigsaw' type activity.



Slide 16: Reflection: Principles of Learning and Teaching P–12

Participants may also wish to consider the following information:

Teachers may support problem solving through:

- clarifying the purpose and context of investigations and problems
- setting learning challenges that require students to analyse, evaluate and create and that allow for student risk-taking, decision-making and time-management
- providing support and scaffolding for investigative or problem solving tasks through checklists, pro-formas, planning frameworks, teacher-student conferences, self- and peer assessment processes.

Examples where a teacher uses strategies that challenge and support students to question and reflect could include:

- setting interesting and challenging problems
- encouraging deeper thinking not only rote rule-based responses
- encouraging communication of mathematical ideas
- encouraging reasons and illustrations
- exploiting situations of cognitive conflict (i.e. allowing students to experience discomfort in order to stimulate learning)
- exploring the limitations and anomalies of technology
- providing opportunities for student or teacher-led class or small group discussion.

Teachers should be encouraged to see the creative side of mathematics and think about how to foster it in the classroom.

Furthermore, teachers could work with students to:

- explore patterns:
  - numeric, geometric
  - Pascal's triangle, etc
- encourage geometric constructions
- explore graphical creations such as coordinates, lines and transformations and vectors.

Thinking processes allow for multiple entry points and develop higher-order thinking skills such as synthesis, evaluation, etc. They provide students with questions or challenges as the impetus for learning, encouraging and supporting students to construct their own responses. Thinking mathematically also supports students to develop the language and other representational tools (such as graphs, diagrams, tables, algebraic rules, reporting templates) needed to conduct investigations.

Linking Interpersonal, Personal and Thinking Processes domains and 'Working mathematically'

Fermi Questions

Use slide 17: Fermi Questions

Slide 17 introduces 'Fermi Questions'. This activity comes from the Mathematics Continuum, 'Working mathematically' section:

- Carrying out investigations – level 4.5 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/W40006P.htm#a4)

Invite the participants to try at least the first of the two Fermi questions:

- Can you fit the world's population into Victoria?
- How many soccer balls could fit in a classroom?

Suggested responses

FERMI PROBLEM 1: World Population

- It may be useful to know that the world's population is between 6 and 7 billion … but rather than tell this, see if anyone in the group knows. Knowing such a 'social number fact' is particularly important.
- Area of Victoria 228 000 square km = 228 x 10^3 km^2 = 228 x 10^9 m^2. (could be approximated as 300 km x 1000 km – Melbourne to Albury and Gippsland to Mildura).
- 7 billion = 7 x 10^9.
- If everyone is in Victoria, they would have 228 x 10^9 ÷ 7 x 10^9 sq m per person = 30 m^2 per person. Which is plenty of standing room, but not enough space to do anything much else!

FERMI PROBLEM 2: Soccer balls

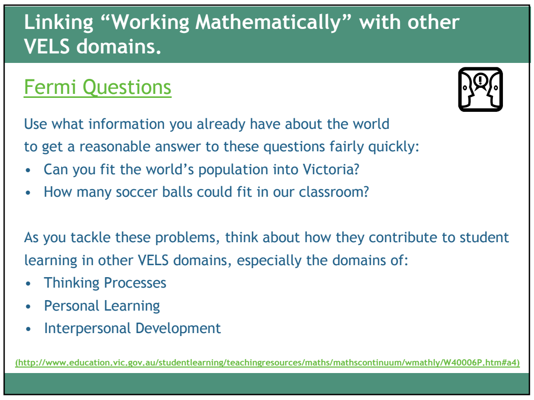
- A soccer ball is about 30cm in diameter.
- If the classroom is 9m wide and 12m long then I can fit 30 soccer balls across the width, and 40 down the length, which means there is room for 30 × 40 or 1200 soccer balls on the floor.
- Then, if the room is 3m high, there is room for 10 layers of soccer balls, or 12000 soccer balls altogether.
- In fact, there is room for even more, because they can be more closely packed in the hexagonal packing that is used for oranges at a fruit shop.

Invite the participants to respond:

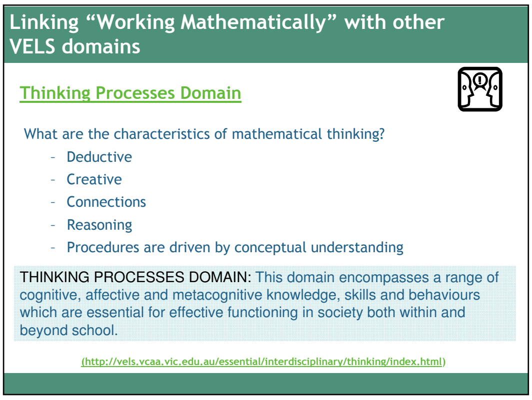
- What information were you given? What would you like to have been given?

  (In real life you aren't always told what you need to know and what you need to find out)

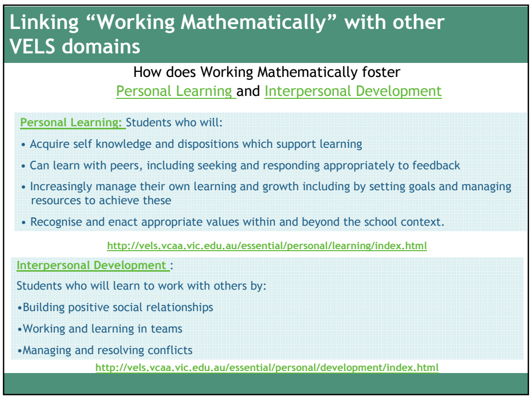
- It is useful for students to have knowledge of 'social number facts' – benchmark numbers – for example, the population of world, Australia, etc, that they can use in many contexts.



Slide 17: Linking 'Working mathematically' with other VELS domains — Fermi Questions



Slide 18: Linking 'Working mathematically' with other VELS domains — Thinking processes



Slide 19: Linking 'Working mathematically' with other VELS domains — Interpersonal and Personal Domains

Thinking processes

Use slides 18 & 19: Linking with VELS

Slide 18 describes the 'Thinking processes' domain.

- Thinking processes (http://vels.vcaa.vic.edu.au/essential/interdisciplinary/thinking/index.html)

The study of thinking enables students to acquire strategies for thinking related to inquiry, processing information, reasoning, problem-solving, evaluation and reflection. The VELS domain encompasses a range of cognitive, affective and metacognitive knowledge, skills and behaviours which are essential for effective functioning in society both within and beyond school.

Invite the participants to respond to the following question:

- What are the characteristics of mathematical thinking?

The slide is animated and provides the following characteristics including:

- deductive
- creative
- connections
- reasoning
- procedures are driven by conceptual understanding.

Interpersonal and Personal Domains

Slide 19 links 'Working mathematically' with the domains of:

- Interpersonal Development (http://vels.vcaa.vic.edu.au/essential/personal/development/index.html)
- Personal Learning (http://vels.vcaa.vic.edu.au/essential/personal/learning/index.html)

Invite participants to discuss the contribution a 'Working mathematically' oriented mathematics classroom can make to these domains. Other domains can be easily incorporated such as Civics and Citizenship (understanding society). A question may be posed such as if the whole world's population **can** fit into Victoria, what are the implications for resource distribution?

Discuss with participants the following question:

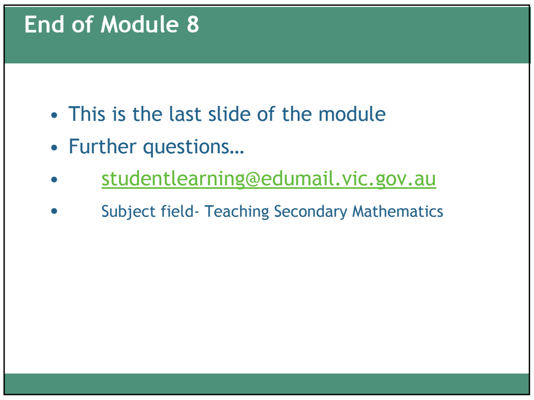
- How does 'Working mathematically' foster Personal Learning and Interpersonal Development?

Conclusion

Slide 20 is the final slide of module 8: Working mathematically: Focus on a range of challenging problems.

There are eight more professional learning modules:

1. Overview of learning in the Mathematics Domain
2. Overview of the Mathematics Developmental Continuum P–10
3. Narrowing the achievement gap: Focus on fractions
4. Conducting practical and collaborative work: Focus on contours
5. Understanding students' mathematical thinking: Focus on algebra and the meaning of letters
6. Using a range of strategies and resources: Focus on percentages
7. Learning through investigation: Focus on chance and variability
9. Conclusion: Planning for improvement in mathematics



Slide 20: End of Module 8

Resource 1: Principles of Learning and Teaching P–12 and their components

- Principles of Learning and Teaching P–12 and their components (http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm)

Students learn best when:

The learning environment is supportive and productive. In learning environments that reflect this principle the teacher:

1.1) builds positive relationships through knowing and valuing each student

1.2) promotes a culture of value and respect for individuals and their communities

1.3) uses strategies that promote students' self-confidence and willingness to take risks with their learning

1.4) ensures each student experiences success through structured support, the valuing of effort, and recognition of their work.

The learning environment promotes independence, interdependence and self motivation. In learning environments that reflect this principle the teacher:

2.1) encourages and supports students to take responsibility for their learning

2.2) uses strategies that build skills of productive collaboration.

Students' needs, backgrounds, perspectives and interests are reflected in the learning program. In learning environments that reflect this principle the teacher:

3.1) uses strategies that are flexible and responsive to the values, needs and interests of individual students

3.2) uses a range of strategies that support the different ways of thinking and learning

3.3) builds on students' prior experiences, knowledge and skills

3.4) capitalises on students' experience of a technology rich world.

Students are challenged and supported to develop deep levels of thinking and application. In learning environments that reflect this principle the teacher:

4.1) plans sequences to promote sustained learning that builds over time and emphasises connections between ideas

4.2) promotes substantive discussion of ideas

4.3) emphasises the quality of learning with high expectations of achievement

4.4) uses strategies that challenge and support students to question and reflect

4.5) uses strategies to develop investigating and problem solving skills

4.6) uses strategies to foster imagination and creativity.

Assessment practices are an integral part of teaching and learning. In learning environments that reflect this principle the teacher:

5.1) designs assessment practices that reflect the full range of learning program objectives

5.2) ensures that students receive frequent constructive feedback that supports further learning

5.3) makes assessment criteria explicit

5.4) uses assessment practices that encourage reflection and self assessment

5.5) uses evidence from assessment to inform planning and teaching.

Learning connects strongly with communities and practice beyond the classroom. In learning environments that reflect this principle the teacher:

6.1) supports students to engage with contemporary knowledge and practice

6.2) plans for students to interact with local and broader communities and community practices

6.3) uses technologies in ways that reflect professional and community practices.