Teaching Secondary Mathematics

Module 7

Learning through investigation:
Focus on chance and variability

7

State Government Victoria

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Introduction to Module 7: Learning through investigation: Focus on chance and variability

The Teaching Secondary Mathematics Resource provides support and guidelines for effective practice for classroom teachers and school leaders of mathematics, especially from Years 7–10.

This resource is directed to teachers of mathematics who would like to deepen their understanding of effective mathematical pedagogies. There is a need to improve mathematics learning for all students and to narrow the achievement gap within groups of students.

The Teaching Secondary Mathematics Resource provides support and guidelines for effective practice for classroom teachers and school leaders of mathematics, especially from Years 7–10.

This resource is for:

- all secondary mathematics classroom teachers to deepen their understanding of mathematics. This will inform their planning for mathematics and highlight opportunities for assessment of mathematics in other domains of the Victorian Essential Learning Standards (VELS)
- mathematics leaders in schools to plan opportunities for professional learning for the teachers of mathematics, in professional learning teams and/or for individual teachers
- differentiating the professional learning needs of mathematics teachers in schools.

This module allows for flexibility in modes of engagement with professional learning. The module booklet needs to be used in conjunction with the PowerPoint slides accompanying this resource.

Workshop approach

The materials of this module can be used by a presenter in a workshop for a school or a cluster of schools. A presenter, appointed from within or outside a school or cluster, is responsible for preparing presentations, facilitating discussions and outlining processes for collaborative planning.

Where a group is working collaboratively through these modules, a designated area is required for participants to share ideas, stories and samples in a climate of mutual respect. Regular after school meetings in a particular venue, such as the library, create a productive sense of community.

Individual use

The materials of this module are also suitable for private study and reflection. Individual users become both 'presenter' and 'participant'. While they are not able to engage in group discussions or whole-school planning, individual users can readily adapt the suggested group discussions and whole-school planning activities to private reflection, writing and classroom planning.

It is suggested that individuals identify a colleague or a buddy with whom to share their thoughts and to support the development of their understandings through ongoing dialogue. Individuals may complete all the modules or choose a combination, depending on their interests or needs.

Web connections

The 'Teaching for Secondary Mathematics' resource is located at http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/default.htm.

Before commencing to plan any elements of the program, schools are strongly advised to visit the Mathematics Domain page to review the most up-to-date advice, resources and information relevant to each module of the program. Many elements of this resource are available online in a downloadable format. There are links to assist schools to locate relevant information.

- Mathematics Domain (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm)

See the website for further details about this additional information or contact the student learning help desk on studentlearning@edumail.vic.gov.au

Content of the module

The module comprises Module 8 booklet and the accompanying slide presentations which can be downloaded from http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/module7.htm

The following are included in this document:

- the **User's Guide** that assists the user through the professional learning program
- hard copies of the **slide presentations** and **resource sheets**
- selected **resources.**

Organisation of the module

Computer access is required for all modules. If a group is completing the modules, a data projector and tables that enable people to sit together and work collaboratively are also necessary. The presenter should encourage participants to raise questions throughout the ensuing presentation. This presentation should take approximately one hour, depending on the depth of discussion and types of activities that facilitators incorporate.

Required resources

Indicators of Progress

- Indicator of Progress: Short-run variation and long-run stability: 5.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007P.htm)
- Mathematics Development Continuum – Mapping the Indicators of Progress (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.pdf)

Developmental Overviews

- Chance (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdochance.pdf)
- Data (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdodata.pdf)

DigiLearn objects

- Dice Duels (https://www.eduweb.vic.gov.au/dlrcontent/4c33363731/index.html)
- Coin Tossing (https://www.eduweb.vic.gov.au/dlrcontent/4c33353135/ec_002_utah_014/index.html)
- Spinners (https://www.eduweb.vic.gov.au/dlrcontent/4c33353436/ec_002_utah_045/index.html)

Spreadsheet simulations

- Short run variability (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mod7cointoss.xls)
- Long run variability (https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/M50007sv.xls)

Icons

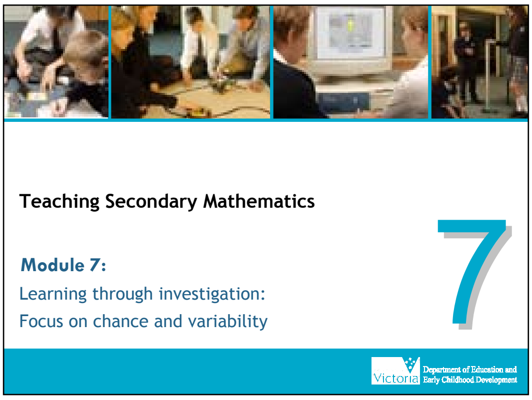
The following icons have been used in this workshop program:

Distribute handout:

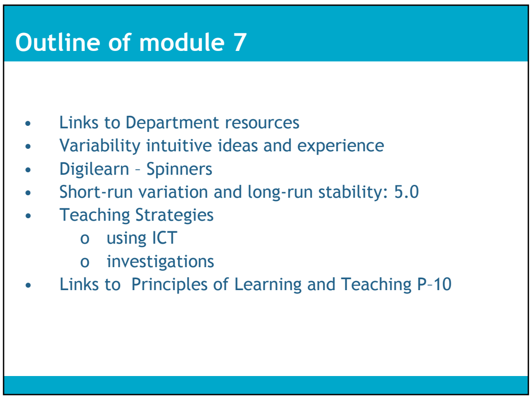
Group discussion:

Group activity:

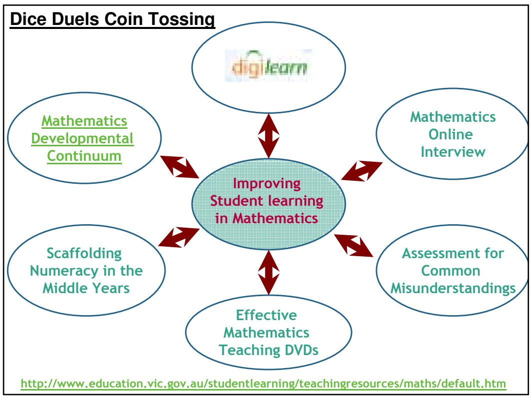
User’s Guide to Module 7: Learning through investigation: Focus on chance and variability



Slide 1: Title slide



Side 2: Outline of module 7



Slide 3; Dice Duels Coin Tossing

Slide 1 is the title slide.

Module 7 is specifically about teaching mathematics through investigations within the context of ‘Chance and variability’. The main mathematical objective is for students to develop a conceptual understanding of short-run variation and long-run stability.

There are two themes running concurrently in this module:

- how to give students the opportunity to learn through investigation (mathematical theme: Learning through investigation)
- how to provide them with the opportunity to gain a better conceptual understanding of chance and variability (pedagogical theme: Chance and variability).

Participants should be reminded that their students should be provided with the opportunity to link their own life experiences with their learning of mathematics. Connecting mathematics learning in schools with the student’s own experiences will support each student’s growth as a community member as well as raise their achievement in mathematics.

The activities are located in the Mathematics Developmental Continuum P–10, Measurement, chance and data 5.0, indicator of progress: Short-run variation and long-run stability: 5.0 found on:

- Mathematics Developmental Continuum P–10 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007P.htm)

Outline of this module

Slide 2 gives an outline of this module. The module includes:

- Links to Department resources
- Variability intuitive ideas and experience
- Digilearn – Spinners
- Continuum – Short-run variation and long-run stability: 5.0
- Teaching Strategies
  - using ICT
  - investigations
- Links to Principles of Learning and Teaching P–10

Links to Departmental resources

Slide 3 illustrates how participants could improve student learning in variation through accessing activities from:

- Mathematics Developmental Continuum P–10 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm)
- Digilearn animations – DigiLearn provides effective demonstration animations. In particular these activities allow the user to vary and bias the probabilities. (http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm).

Variability: intuitive ideas and experience

Slide 4 invites participants to discuss three questions. The aim of these questions is to link teachers' current understanding of variability with students' previous experience outside the mathematics classroom.

Use slide 4: Variability – intuitive ideas and experience

The questions are:

- Where do students encounter variability?
- What experiences and ideas do they bring to the classroom?
- What words do we use when we discuss variability in mathematics?

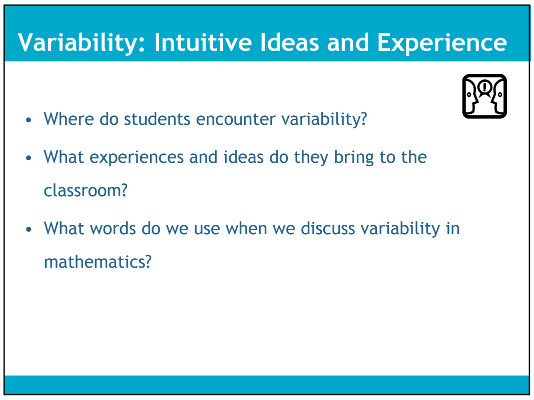
The following information may support discussion with participants.

Variability as a term causes some confusion. It has only recently been used within the context of mathematics. Teachers are encouraged to incorporate students' prior experiences in discussing chance and probability in class. Teachers could illustrate variability with the use of photographs. Students experience variability in a variety of contexts including probability. This typically occurs when they play games. Examples include board games, such as 'Monopoly'.

Participants may confuse variablitlity with variation. The term variation is used in relation to direct and indirect variation, i.e. $y=kx$.

Teachers may have used the following words when discussing variability in the mathematics classroom:

- likely
- chance
- predict
- outcome
- probability
- frequency.



Slide 4: Variability: intuitive ideas and experience

Short term experience

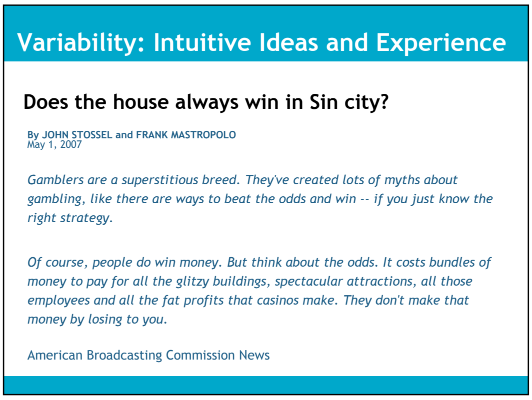
Slide 5 illustrates an example to show how easily students could draw conclusions from short term experience.

Surprise snow in Ballarat

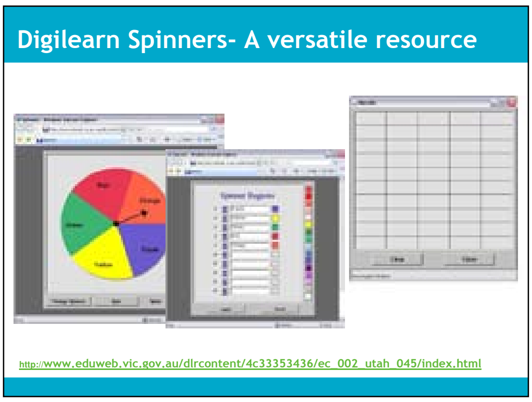
Short term experience – snowed twice in two days – reinforces people's perception that Ballarat is a cold place. But long term experience is significant – this event has only occurred 5–6 times in 30 years.

Slide 5: Variability: intuitive ideas and experience

Slide 6: Variability: intuitive ideas and experience — Does the house always win in Sin City?



Slide 7: Variability: intuitive ideas and experience — Does the house always win in Sin City?



Slide 8: Digilearn spinner

Does the house always win in Sin City?

Slides 6 and 7 illustrate an online article concerning gambling, which comments on the way gamblers create myths based on their misconceptions about probability.

Use slides 5, 6, 7: Real world experiences of variation

Invite participants to discuss whether there are any other applications of variability found in the real-world context.

Digilearn spinner

Slide 8 is an animated slide demonstrating a diagram of the Digilearn spinner resource. The spinner can be modified, allowing students to create a spinner to suit their particular needs. The resource records the results of the spin.

- Spinner (https://www.eduweb.vic.gov.au/dlrcontent/4c33353436/ec_002_utah_045/index.html)

Knowing your students – diagnostic task

The activity on slides 9, 10 and 11 will provide an opportunity for participants to discuss a diagnostic task. The question below is posed on slide 9. This task was taken from:

- Mathematics Developmental Continuum – Short-run variation and long-run stability: 5.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007P.htm)

Background information

An important step in developing student conceptual knowledge about variation is that they know how random processes exhibit short-run variation and they also exhibit long-run stability.

They understand that if they roll a fair die 10 times then the proportion of sixes is hard to predict, but if they roll it 10000 times, the relative frequency is highly likely to be very close to 1/6, the theoretical probability.

In this way, the relative frequency gives the experimental (empirical) probability, which approximates the theoretical probability.

Use slides 9–11: Diagnostic task

Invite participants to answer each question, shown one at a time on slide 10.

- A fair coin is tossed. Which outcome is more likely, or are they equally likely?

Invite participants to discuss the four possibilities on slide 10.

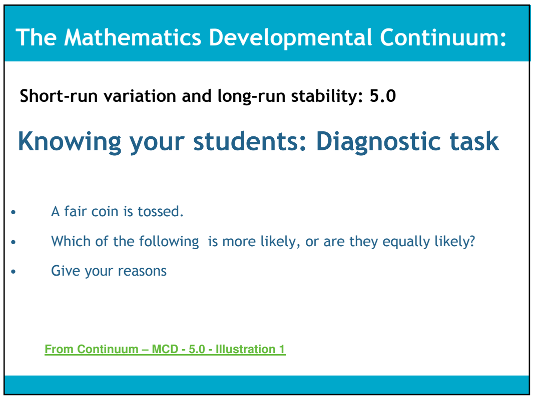
Ask the participants to respond and give reasons for their answers.

Slide 10 is animated, which will allow the participants the opportunity to answer the highlighted question.

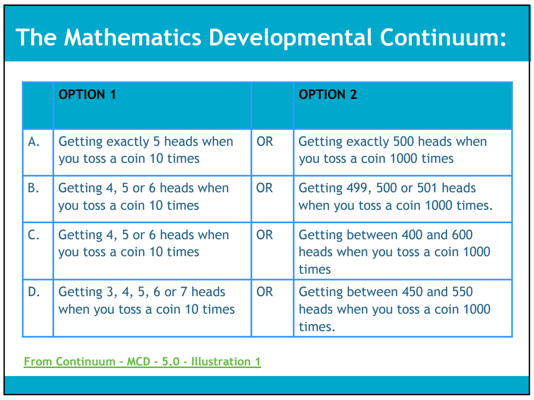
The correct responses are highlighted in green on slide 11.

Invite participants to respond to the next two questions:

- Why are the wrong answers appealing?
- What misconceptions do students bring into the classroom?

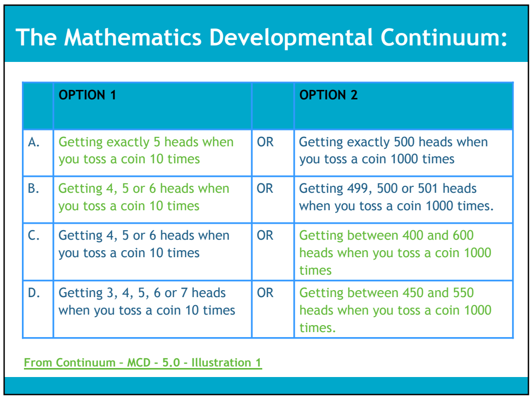


Slide 9: The Mathematics Developmental Continuum



	OPTION 1		OPTION 2
A.	Getting exactly 5 heads when you toss a coin 10 times	OR	Getting exactly 500 heads when you toss a coin 1000 times
B.	Getting 4, 5 or 6 heads when you toss a coin 10 times	OR	Getting 499, 500 or 501 heads when you toss a coin 1000 times.
C.	Getting 4, 5 or 6 heads when you toss a coin 10 times	OR	Getting between 400 and 600 heads when you toss a coin 1000 times
D.	Getting 3, 4, 5, 6 or 7 heads when you toss a coin 10 times	OR	Getting between 450 and 550 heads when you toss a coin 1000 times.

Slide 10: The Mathematics Developmental Continuum



	OPTION 1		OPTION 2
A.	Getting exactly 5 heads when you toss a coin 10 times	OR	Getting exactly 500 heads when you toss a coin 1000 times
B.	Getting 4, 5 or 6 heads when you toss a coin 10 times	OR	Getting 499, 500 or 501 heads when you toss a coin 1000 times.
C.	Getting 4, 5 or 6 heads when you toss a coin 10 times	OR	Getting between 400 and 600 heads when you toss a coin 1000 times
D.	Getting 3, 4, 5, 6 or 7 heads when you toss a coin 10 times	OR	Getting between 450 and 550 heads when you toss a coin 1000 times.

Slide 11: The Mathematics Developmental Continuum

Background information

The following information may provide participants with some insights about how students think about the concepts in underpinning variation.

One of the difficult things for students is that they need to keep track of what is being considered:

1. First there is an individual coin toss (which can come up either heads or tails).
2. A set of 10 (or 1000) coin tosses then make up a trial (which has an outcome of a certain number of heads and tails).
3. Finally, they need to imagine what kinds of outcomes are going to happen more often than others (e.g., are they more likely to get 6 heads or 2 heads?). This means they have to imagine running lots of trials of 10 (or 1000) coin tosses.

Students need to be able to picture all of this in order to make comparisons between the 10 coin toss and the 1000 coin toss situation. In addition they need to have an understanding of the long-term predictability of coin-tossing (we expect the coin to come up heads about half of the time) as well as the short-term variability of coin-tossing (we cannot predict what it will actually do next).

Exactly 5 heads in 10 coin-tosses vs Exactly 500 heads in 1000 coin-tosses

Part (a) will be answered incorrectly by students who over-estimate the predictability of individual events and also by students who have a shallow understanding of the short-run variability and long-run predictability.

For example, students know from their experience that the probability of tossing a head is a half and believe that in a long run experiment this exact value will arise, i.e., they expect 500 heads for 1000 coin tosses. In a sense, their understanding of long-term predictability dominates any idea of short-term variation. They also do not immediately appreciate how close proportions like $\frac{501}{1000}$ or even $\frac{520}{1000}$ are to $\frac{1}{2}$. This indicates that their conceptual understanding is quite shallow.

Parts (b) and (c) are designed to identify those students who appreciate the importance of the relative frequency, not just frequency of outcomes.

4, 5 or 6 heads in 10 coin-tosses vs 499, 500 or 501 heads in 1000 coin-tosses

Students may answer part (b) incorrectly if they know that there is long-run predictability (about a half will be heads) but they think about this in absolute terms, rather than relative frequency. It is much more likely to be within 1 of 5 (half of 10) when tossing 10 times than to be within 1 of a 500 (half of 1000) when tossing 1000 times.

4, 5 or 6 heads in 10 coin-tosses vs between 400 and 600 heads in 1000 coin-tosses

Getting the correct answer in part (c) relies on understanding the long-term predictability of coin-tossing, and that the proportion of heads gets closer to half the more coin tosses are done. This means that when 1000 tosses are done the number of heads is very likely to be close to 500, and almost certainly in the range of 400–600. Again, this can also be demonstrated with a spreadsheet. The short-term variation phenomenon means that when only tossing the coin 10 times, the number of heads may vary quite widely from 5.

Short-term variability

- An Excel spreadsheet simulation is available (Short run variability). It will need to be downloaded on to the user's computer before entering numbers. (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mod7cointoss.xls)

An important thing to realise here is that any trial of 10 coin tosses will have one result for the number of heads, e.g., you may get 4 heads or on another trial you may get 8 heads.

Let's just consider getting exactly 4 heads out of 10 coin tosses. This has a reasonable chance of occurring (use of the binomial distribution shows this to be about 0.2). However, getting exactly 5 heads out of 10 coin tosses actually has a different likelihood (about 0.25). Some students may just think that each possibility is equally likely, when in fact they aren't.

Relative probability

Now let's consider the 1000 coin tossing experiment. This time, getting exactly 499 heads out of 1000 coin tosses is not very likely (the actual probability is about 0.025). Getting exactly 500 or 501 are similarly unlikely.

Note that some students may think that there are three outcomes in each case (4, 5 and 6 for the 10 coin tossing case, and 499, 500, and 501 for the 1000 coin tossing case). Some students may therefore conclude that the two situations are equally likely.

Other students, in contrast, may think that the first case in part (b) has a probability of $\frac{3}{10}$ and the other has a probability of $\frac{3}{1000}$, thus concluding – correctly, but for the wrong reasons – that the first situation is more likely. However, students following this line of reasoning will then get question (c) incorrect (by comparing $\frac{3}{10}$ with $\frac{199}{1000}$).

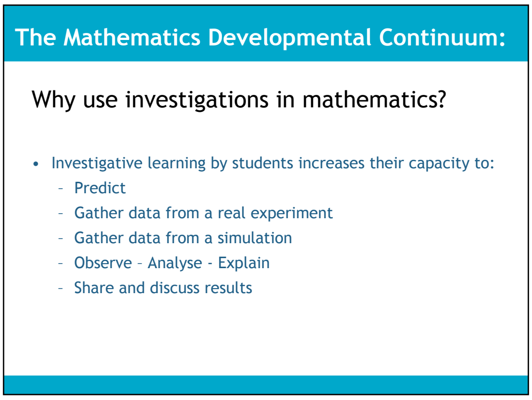
Knowing the exact probabilities helps students see that getting 4 or 5 or 6 heads when tossing a coin 10 times is much more likely than getting exactly 499, 500 or 501 heads. This can be demonstrated with a spreadsheet simulation, or even by doing some actual simulations in class (but perhaps only doing 100 coin tosses instead of 1000).

- An Excel spreadsheet simulation is available (Long run variability). It will need to be downloaded on to the user's computer before entering numbers. (https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/M50007sv.xls)
- Note there is other excellent simulation software being made available to teachers during 2008. Check the Digilearn site, or the Mathematics Domain site.

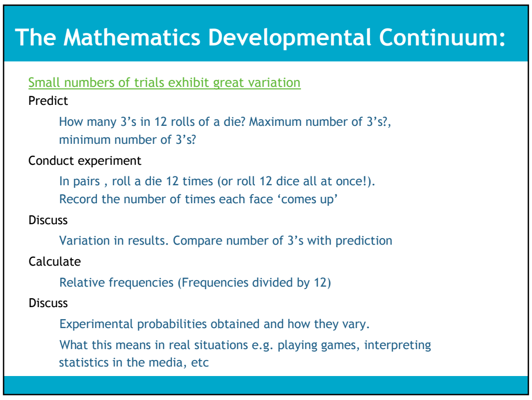
3, 4, 5, 6 or 7 heads in 10 coin-tosses vs between 450 and 500 heads in 1000 coin-tosses

Part (d) further clarifies the long-run probability idea. Correct answers should mention that the increasing number of tosses means that that the number of heads is more likely to be about a half when there are more tosses.

Even though we seem to have widened the range of possible values for the 10 coin-toss situation, and narrowed the range for the 1000 coin-toss situation, the long-term tendency to half means that the number of heads arising when a coin is tossed 1000 times is extremely likely to be between 450 and 550. On the other hand, short-term variation when tossing the coin only 10 times means that extreme values like 0, 1, 2 or 8, 9, 10 are still quite likely to arise.



Slide 12: The Mathematics Developmental Continuum — Why use investigations in mathematics?



Slide 13: The Mathematics Developmental Continuum — Small number of trials exhibit great variation

Why use investigations in mathematics?

Slide 12 provides the rationale for using investigations to enhance student learning.

Students' capacity as learners is enhanced when they investigate questions.

Students will already be familiar with ideas of unpredictability of random processes and possibly also with ideas of long-run predictability, but these are subtle ideas which need development and discussion over some time.

The activities in the next set of slides are sequential, and employ the principles that teaching about probability and random processes should be based on practical experiences and that technology can be used to extend classroom experiences from the hands-on base.

The Continuum – Short-run variation and long-run stability: 5.0

Small number of trials exhibit great variation

Slide 13 describes an awareness-raising activity found on the continuum, providing hands-on experience, and moving students towards the conceptual change of considering rolling a die 12 times as one experiment (and not 12 experiments). This activity is found on

- Mathematics Development Continuum – Short-run variation and long-run stability: 5.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007P.htm) Direct participants to the Excel spreadsheet:
- Excel spreadsheet simulation. It will need to be downloaded on to the user's computer before entering numbers. (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mod7cointoss.xls)

Use slide 13: Small number of trials exhibit great variation

Invite participants to complete the following task in groups and discuss results

Predict	• How many 3s in 12 rolls of a die? Maximum number of 3s? Minimum number of 3s?
Conduct experiment	• In pairs, roll a die 12 times (or roll 12 dice once!). Record the number of times each face 'comes up'.
Discuss	• Variation in results. Compare the number of 3s with the predicted number.
Calculate	• Relative frequencies (frequencies divided by 12).
Discuss	• Experimental probabilities obtained and how they vary. • What this means in real situations e.g. playing games, interpreting statistics in the media, etc. • Alternative ways to record results, the idea of 12 tosses in this case being viewed as one experiment or one event rather than a series of 12 experiments or events.

Exploration of 60 trials

Use slides 14–16: Exploration of 60 trials

Slide 14 describes the activity. Inform participants that they are assuming the role of a student.

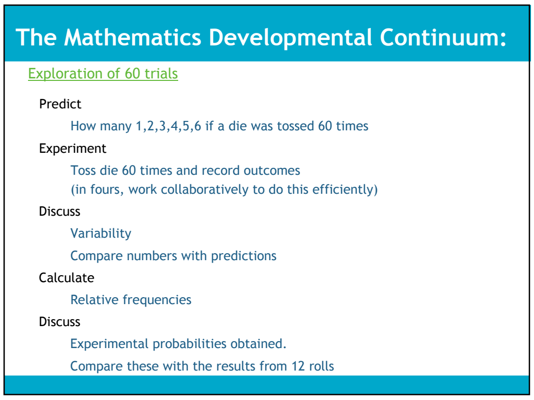
This activity extends the previous activity by using a simulation of rolling a die sixty times, which enables students to explore the variation in the relative frequencies (experimental probabilities) with a larger number of trials. Participants need to interpret both tabulated and graphical data to determine the variation in results for this situation.

Direct participants to the Excel spreadsheet:

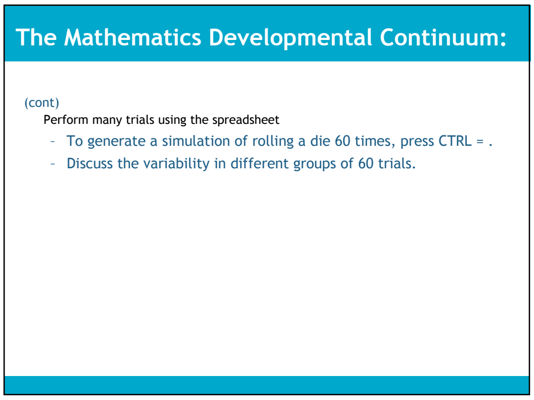
- Excel spreadsheet simulation. It will need to be downloaded on to the user's computer before entering numbers. (https://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/M50007sv.xls)

Invite participants to complete the following task in groups and discuss results

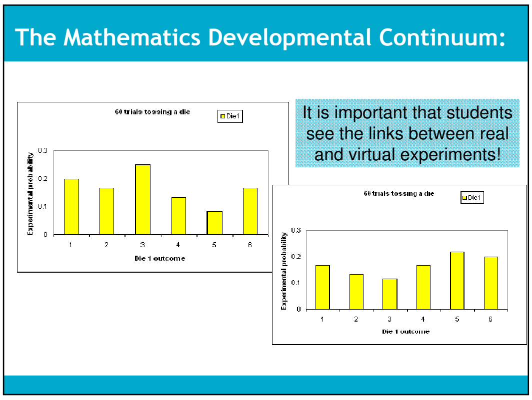
Predict	• How many ones, twos, threes, fours, fives and sixes they would they get if a die was tossed 60 times. (should be 10).
Conduct experiment	• Ask pairs of participants to roll a die sixty times and record the number of times each outcome appears. • Write the frequency of each outcome and convert to its relative frequency (i.e. divide by 60).
Discuss	• The variation in the results. For example, did anyone get exactly ten of each outcome? Did anyone not have any threes, etc? What was the highest frequency for an outcome? • Linking relative frequency and experimental probability: participants need to consider each set of 60 trials as one experiment, not as 60 experiments.



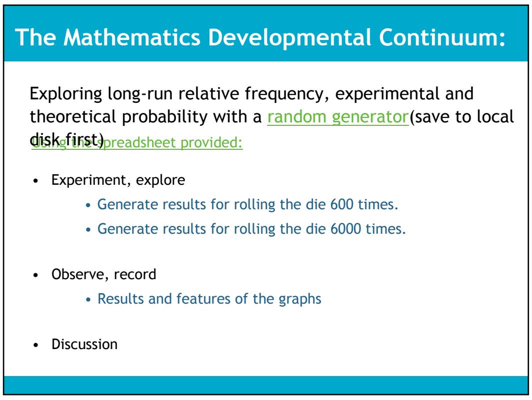
Slide 14: The Mathematics Developmental Continuum — Exploration of 60 trials



Slide 15: The Mathematics Developmental Continuum — Exploration of 60 trials



Slide 16: The Mathematics Developmental Continuum



Slide 17: The Mathematics Developmental Continuum

Simulation

Slides 16 and 17 demonstrate how this activity can be completed through a simulation.

Invite participants to use a random number generator spreadsheet to simulate 60 rolls of a die:

- Download random generator spreadsheet (Excel – 821Kb). The random number generator updates an automatic graphical display. (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/M50007sv.xls).
- Complete 60 trials with the random generator. Use the 60 trials page of the spreadsheet.

Each time the spreadsheet file opens, 60 rolls of a die are automatically generated. Although the relevant frequencies are provided, ask the participants to calculate the relative frequencies (experimental probabilities) and use the summary table and graph to describe the situation for one group of 60 rolls. Two graphs for 60 rolls are shown on slide 16.

Conduct experiment	• Perform many trials using the spreadsheet. • To generate a simulation of rolling a die 60 times, hold down CTRL and press '='.
Discuss	• How the experimental probabilities are calculated. • The biggest differences in experimental probabilities for a sample of 60. • The variability in different groups of 60 trials. • The biggest differences in experimental probabilities for a sample of 600. • The size of the interval on the vertical axis. The vertical axis of the spreadsheet graph only goes up to 0.3 – was there a case where it was not big enough?
Predict	• A range that you might expect these values (experimental probabilities) to lie within. • Are these results what you expected? • What does this mean for real life?

Exploring long-run relative frequency, experimental and theoretical probability

Use slides 18–21: Long run relative frequency

This activity uses slides 18–21 in a simulation activity. The activity is based on:

- Mathematics Development Continuum – Short-run variation and long-run stability: 5.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007P.htm)

Exploring long-run relative frequency, experimental and theoretical probability extends the exploration using technology for experiments consisting of 6000 trials, conducted multiple times. Participants can directly compare the variability of experimental probabilities from short-run and long-run experiments

In this activity participants use the random generator spreadsheet to produce many results of many trials quickly to explore long term relative frequency and the experimental probability.

- Download random generator spreadsheet (Excel – 821Kb). The random number generator updates an automatic graphical display. [This is the same spreadsheet used for the previous activity] (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/M50007sv.xls).

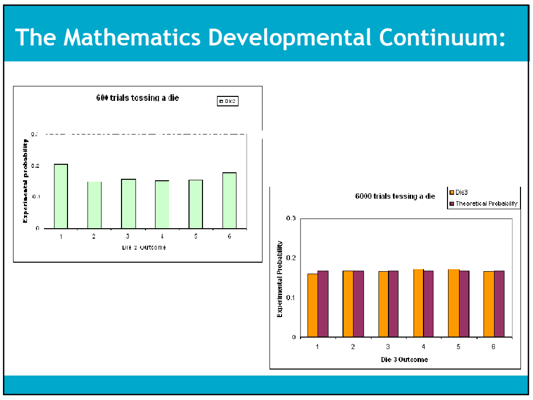
Experiment, explore:	• results from throwing dice 600 times • results from throwing dice 6000 times.
Observe, record:	• results in each case • features of the graphs.
Discuss:	• how relative frequencies compare with 12 and 60 tosses • how results compare with prediction • experimental vs theoretical probability • short run vs long run stability.

Participants will observe how the variation in results for each experiment is larger for experiments with a smaller number of trials. They will also observe that when the number of trials is very large, the value of the experimental probability (long-run relative frequency) is often very close to the theoretical probability.

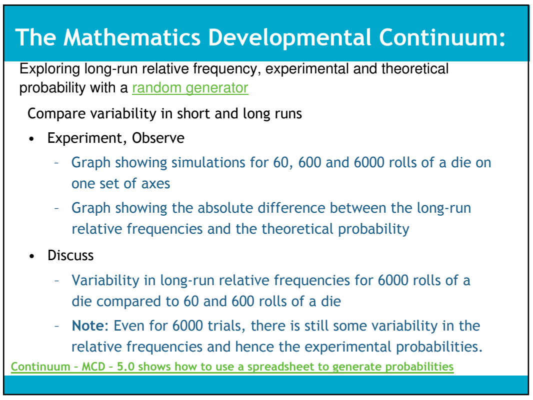
Note the importance of proportional reasoning: with the larger trials there can still be a big difference in absolute numbers between the theoretical and the experimental results, but the point is that this is a small **proportion**.

Two graphs are illustrated on slide 18 comparing the relative frequencies for experiments containing 600 and 6000 trials.

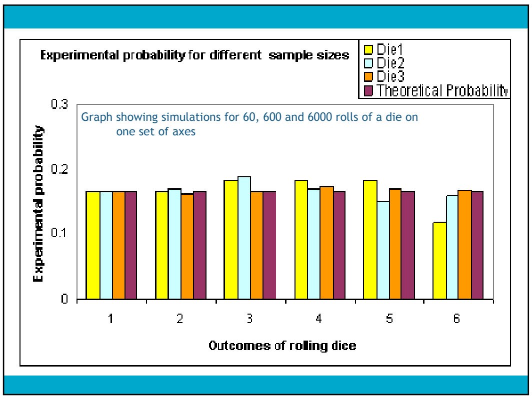
Refer also to Measurement, chance and data - How to use an Excel spreadsheet to simulate tossing a die: 5.0 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M50007a.htm)



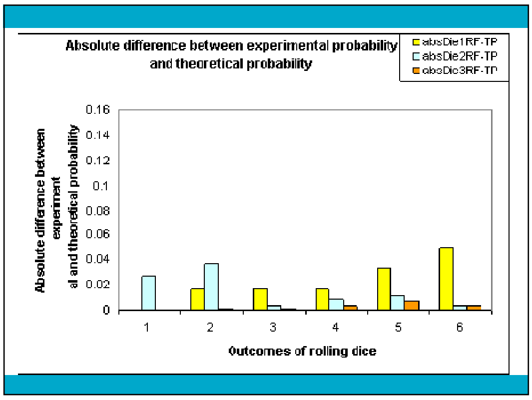
Slide 18: The Mathematics Developmental Continuum



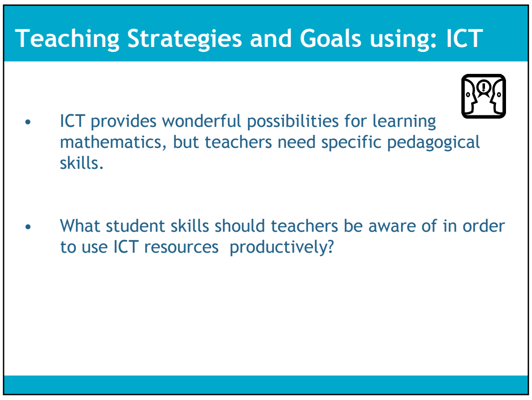
Slide 19: The Mathematics Developmental Continuum



Slide 20: Compare variability in short and long runs



Slide 21: absolute difference between the long-run relative frequencies and the theoretical probability



Slide 22: Teaching strategies and goals – Using ICT

Step 2: Compare variability in short and long runs

Participants can now consider a graph showing simulations for 60, 600 and 6000 rolls of a die on one set of axes. This is the fourth sheet, called 'CompareTrialsTheoretical' worksheet found on the spreadsheet.

Ask participants to:

Experiment, Observe	• graph showing simulations for 60, 600 and 6000 rolls of a die on one set of axes • graph showing the absolute difference between the long-run relative frequencies and the theoretical probability.
Discuss	• variability in long-run relative frequencies for 6000 rolls of a die compared to 60 and 600 rolls of a die • note that, even for 6000 trials, there is still some variability in the relative frequencies and hence the experimental probabilities.

The graph on slide 21 shows the absolute difference between the long-run relative frequencies and the theoretical probability. It is a further graphical display to show that the experimental probability is close to the theoretical probability when the number of trials is large. This further reinforces the ideas of short-run variation and long-run stability.

Teaching strategies and goals – Using ICT

Use slide 22: Using ICT

Slide 22 prompts participants to discuss the question below.

- What student skills should teachers be aware of in order to use ICT resources productively?

Suggested responses:

Technical ability to use spreadsheets (e.g. move between worksheets)

Technical ability to make changes to spreadsheets if required (e.g. vary number of trials)

Make a list of three more points which are not of a computer technical nature.

ICT provides wonderful possibilities for learning mathematics, but teachers need specific pedagogical skills.

Discuss the following issues with participants:

- Do not assume that all students will appreciate the relevance of the computer graphics, which will certainly need careful attention.
- Do not assume that students automatically understand what the computer is doing or how the computer results link to the work they carried out beforehand.
- Make sure that students engage in mathematical thinking, not just playing with the computer or just going through the data collection motions. This is a difficulty in all practical work – how to get depth in mathematical discussion, not just collecting data.
- There are classroom management issues when the class works in the computer lab. Are they doing the maths, or surfing the internet?
- Is it better to use this spreadsheet for a whole class investigation, or for students to work directly on the computers, whether alone or in pairs?

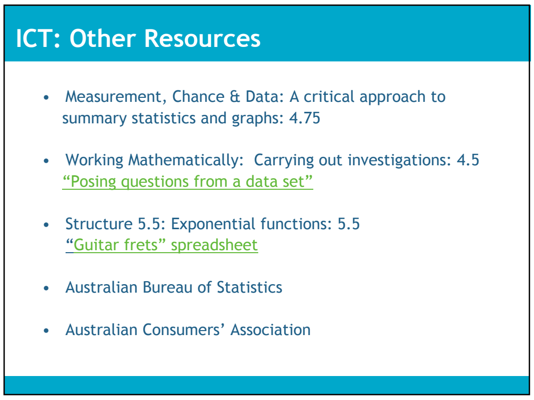
Using ICT for teaching mathematical investigations

Slide 23 provides a brief list of other resources which support the learning of mathematics through the use of ICT.

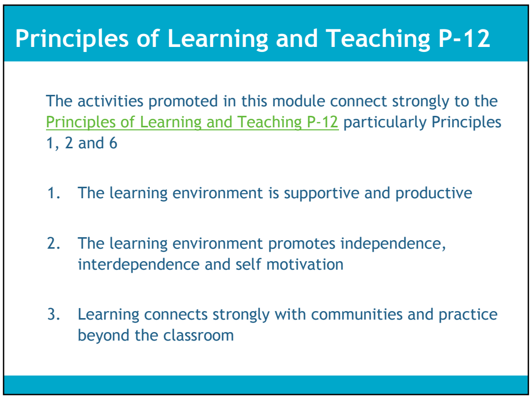
These are:

- Measurement, Chance & Data – A critical approach to summary statistics and graphs: 4.75 – activities 2 and 3 deal with Chance and Data in the news (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/mcd/M47509P.htm)
- Working Mathematically – Carrying out investigations: 4.5 – activity 6 'Posing questions from a data set' (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/wmathly/W45003P.htm)
- Structure – Exponential functions: 5.5 – activity 3: 'Exponential decay on a guitar' provides a 'Guitar frets' spreadsheet (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/structure/St55002P.htm)
- Australian Bureau of Statistics (http://www.abs.gov.au/)
- Choice (previously Australian Consumers' Association) (http://www.choice.com.au/)

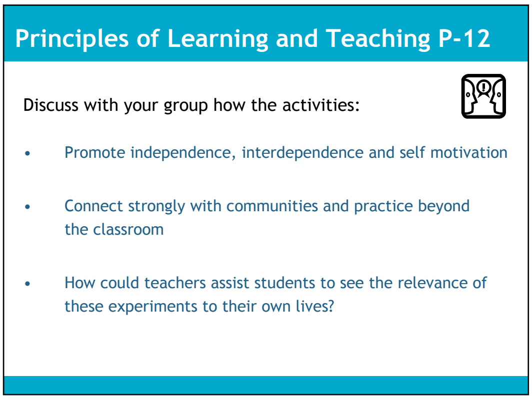
Dynamic geometry files are also extremely valuable but cannot be accessed through the Mathematics Domain page, so teachers should look elsewhere for these. Many good files are on the internet.



Slide 23: ICT: Other Resources



Slide 24: Principles of Learning and Teaching P–12



Slide 25: Principles of Learning and Teaching P–12

Principles of Learning and Teaching P–12

The experiments in this module are carried out with dice, but the implications go far beyond playing games.

Refer to Resource 1: Principles of Learning of Teaching P–10, also available online.

Slide 24 reminds participants that the activities presented in this module offer a rich resource for teachers to provide for:

- a supportive and productive learning environment that promotes students' self confidence through the building of success (PoLT principle 1)
- a collaborative classroom (PoLT principle 2)
- an environment that provides opportunities for students to link their classroom experiences with their local and broader community (PoLT principle 6).

Use slides 24–25: Principles of Learning and Teaching P–12

Slide 25 invites participants to discuss how the activities link with:

- PoLT principle 2: The learning environment promotes independence, interdependence and self-motivation.
- PoLT principle 6: Learning connects strongly with communities and practice beyond the classroom.

The investigation explored in this module requires students to:

- work cooperatively in pairs by:
  - recording data
  - reporting findings
- work cooperatively as a class through:
  - sharing results
  - reporting findings
  - drawing conclusions
  - directing their own work.

In response to linking PoLT principle 6 (Learning connects strongly with communities and practice beyond the classroom), remind participants that probabilities can help people analyse important information and make decisions that affect their lives. In other words, probability is not just 'stuff' that happens at school!

Examples of probabilities affecting decisions in real life include the probabilities that

- an operation will be successful
- a child will have a genetic disease
- a particular strategy will win in a game of chance
- a certain behaviour such as smoking will lead to a bad outcome.

These probabilities are generally obtained as 'long run' probabilities.

Participants could:

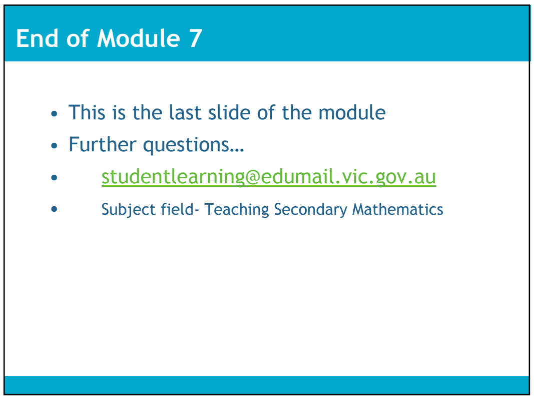
- Discuss with their students how the anecdotal experiences of everyday life often fall into the 'short run' category (all my grandparents smoke and none of them have lung disease).
- Note the modelling issues – a properly thrown die has exactly equal probabilities of getting any of the numbers on every occasion. In real life, the probabilities are not as clear as this, and will be affected by many other things, however the basic findings of the experiment will still hold.
- Note that the longer that people play a game (e.g. gamble) the more likely it is that they will end up at the average outcomes.
- Note that things like 'winning streaks' are just part of the variation observed in statistics.

Conclusion

Slide 26 is the concluding slide to the module.

There are 8 more professional learning modules:

1. Overview of learning in the Mathematics Domain
2. Overview of the Mathematics Developmental Continuum P–10
3. Narrowing the achievement gap: Focus on fractions
4. Conducting practical and collaborative work: Focus on contours
5. Understanding students' mathematical thinking: Focus on algebra and the meaning of letters
6. Using a range of strategies and resources: Focus on percentages
8. Working mathematically: Focus on a range of challenging problems
9. Conclusion: Planning for improvement in mathematics



Slide 26: End of Module 7

Resource 1: Principles of Learning and Teaching P–12

- Principles of Learning and Teaching P–12 and their components (http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm)

Students learn best when:

The learning environment is supportive and productive. In learning environments that reflect this principle the teacher:

1.1) builds positive relationships through knowing and valuing each student

1.2) promotes a culture of value and respect for individuals and their communities

1.3) uses strategies that promote students' self-confidence and willingness to take risks with their learning

1.4) ensures each student experiences success through structured support, the valuing of effort, and recognition of their work.

The learning environment promotes independence, interdependence and self motivation. In learning environments that reflect this principle the teacher:

2.1) encourages and supports students to take responsibility for their learning

2.2) uses strategies that build skills of productive collaboration.

Students' needs, backgrounds, perspectives and interests are reflected in the learning program. In learning environments that reflect this principle the teacher:

3.1) uses strategies that are flexible and responsive to the values, needs and interests of individual students

3.2) uses a range of strategies that support the different ways of thinking and learning

3.3) builds on students' prior experiences, knowledge and skills

3.4) capitalises on students' experience of a technology rich world.

Students are challenged and supported to develop deep levels of thinking and application. In learning environments that reflect this principle the teacher:

4.1) plans sequences to promote sustained learning that builds over time and emphasises connections between ideas

4.2) promotes substantive discussion of ideas

4.3) emphasises the quality of learning with high expectations of achievement

4.4) uses strategies that challenge and support students to question and reflect

4.5) uses strategies to develop investigating and problem solving skills

4.6) uses strategies to foster imagination and creativity.

Assessment practices are an integral part of teaching and learning. In learning environments that reflect this principle the teacher:

5.1) designs assessment practices that reflect the full range of learning program objectives

5.2) ensures that students receive frequent constructive feedback that supports further learning

5.3) makes assessment criteria explicit

5.4) uses assessment practices that encourage reflection and self assessment

5.5) uses evidence from assessment to inform planning and teaching.

Learning connects strongly with communities and practice beyond the classroom. In learning environments that reflect this principle the teacher:

6.1) supports students to engage with contemporary knowledge and practice

6.2) plans for students to interact with local and broader communities and community practices

6.3) uses technologies in ways that reflect professional and community practices.