



Teaching Secondary Mathematics

Module 6

Using a range of strategies and resources:
Focus on percentages

6



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Introduction to Module 6: Using a range of strategies and resources: Focus on percentages

This module allows for flexibility in modes of engagement with professional learning. The module booklet needs to be used in conjunction with the PowerPoint slides accompanying this resource.

The Teaching Secondary Mathematics Resource provides support and guidelines for effective practice for classroom teachers and school leaders of mathematics, especially from Years 7–10.

This resource is for:

- all secondary mathematics classroom teachers to deepen their understanding of mathematics. This will inform their planning for mathematics and highlight opportunities for assessment of mathematics in other domains of the Victorian Essential Learning Standards (VELS)
- mathematics leaders in schools to plan opportunities for professional learning for the teachers of mathematics, in professional learning teams and/or for individual teachers
- differentiating the professional learning needs of mathematics teachers in schools.

Use of this module

Workshop approach

The materials of this module can be used by a presenter in a workshop for a school or a cluster of schools. A presenter, appointed from within or outside a school or cluster, is responsible for preparing presentations, facilitating discussions and outlining processes for collaborative planning.

Where a group is working collaboratively through these modules, a designated area is required for participants to share ideas, stories and samples in a climate of mutual respect. Regular after school meetings in a particular venue, such as the library, create a productive sense of community.

Individual use

The materials of this module are also suitable for private study and reflection. Individual users become both 'presenter' and 'participant'. While they are not able to engage in group discussions or whole-school planning, individual users can readily adapt the suggested group discussions and whole-school planning activities to private reflection, writing and classroom planning.

It is suggested that individuals identify a colleague or a buddy with whom to share their thoughts and to support the development of their understandings through ongoing dialogue. Individuals may complete all the modules or choose a combination depending on their interests or needs.

Web connections

The 'Teaching for Secondary Mathematics' resource is located at <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/default.htm>.

Before commencing to plan any elements of the program, schools are strongly advised to visit the Mathematics Domain page to review the most up-to-date advice, resources and information relevant to each module of the program. Many elements of this resource are available online in a downloadable format. There are links to assist schools to locate relevant information.

- [Mathematics Domain](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm>)

See the website for further details about this additional information or contact the student learning help desk on studentlearning@edumail.vic.gov.au

Content of the module

This module comprises *Module 6: Using a range of strategies and resources: Focus on percentages* and the accompanying slide presentation which can be downloaded from <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/module6.htm>

The following are included in this document:

- the **User's Guide** that assists the user through the professional learning program
- hard copies of the **slide presentations** and **resource sheets**
- selected **resources**.

Organisation of the module

Computer access is required for all modules. If a group is completing the modules, a data projector and tables that enable people to sit together and work collaboratively are also necessary. The presenter should encourage participants to raise questions throughout the ensuing presentation. This presentation should take approximately two hours, depending on the depth of discussion and types of activities that facilitators incorporate.

Required resources

This module requires the resources listed below.

Indicators of progress:

- Indicator of progress: Adding and taking off a percentage: 5.25
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N52503P.htm>)
- Indicator of progress: Solving percentage problems: 5.5
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55003P.htm>)
- Indicator of progress: Easy and hard ratio and proportion questions: 5.5
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55004P.htm>)

Other resources:

- Mathematics Development Continuum – Mapping the Indicators of Progress
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.pdf>)
- Developmental Overview of Proportional Reasoning and Multiplicative Thinking
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdopropreason.pdf>)

Icons

The following icons have been used in this workshop program:

Distribute handout: 

Group discussion: 

Group activity: 

User's Guide to Module 6: Using a range of strategies and resources: Focus on percentages



Teaching Secondary Mathematics

Module 6:

Using a range of strategies and resources

Example: Percentages

6

Victoria Department of Education and Early Childhood Development

Slide 1: Title slide

*Instruction is powerful only when it is sufficiently precise and focused to **build** directly on what students already know and to **take** them to the next level.*

*While a teacher does and must do many things, the most **critical** is designing and organising instruction so that it is **focused**.*

—Breakthrough! Fullan, Hill & Crévola (2006)

Slide 2: Introduction to this resource

Outline of Module 6: Percentages

1. Mathematics Developmental Continuum
 - ñ Solving percentage problems- 5.5
 - ñ Adding and Taking of a percentage- 5.25
 - ñ Easy and hard ratio and proportion questions- 5.5
2. Digilearn
 - ñ Bar charts
 - ñ Designing a neighbourhood
3. Scaffolding Numeracy in the Middle Years (SNMY)
 - ñ Learning and Assessment Framework for Multiplicative Thinking
4. Assessment for Common Misunderstandings
 - ñ Percentage Tool (Task)

Slide 3: Outline of Module 6

This module provides teachers with the opportunity to learn about multiplicative thinking and apply this in the teaching of percentages. The topic 'percentage' is one aspect of multiplicative thinking, which is a major component of the mathematics curriculum in the middle years. The module will encourage participants to explore departmental resources which help them further develop their students' understanding of percentages and multiplicative thinking.

Slide 1 is the title slide

Slide 2 introduces this resource.

The main message of this module is conveyed by Fullan, Hill and Crévola (2006)¹:

Instruction is powerful only when it is sufficiently precise and focused to build directly on what students already know and to take them to the next level.

While a teacher does and must do many things, the most critical is designing and organising instruction so that it is focused.

Outline

Slide 3 provides an outline of this module. The module includes:

- Mathematics Developmental Continuum P–10
- Digilearn
- Scaffolding Numeracy in the Middle Years (SNMY)
- Assessment for common misunderstandings

Range of Resources

Slide 4 provides an overview of the range of resources used to inform participants when completing this module. These resources include:

- [Mathematics Developmental Continuum P–10](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm>).

Within the continuum there are two indicators of progress which focus on the study of percentages. These are

- Level 5.25: Adding and taking off a percentage
- Level 5.5: Solving percentage problems

It may be useful for teachers to spend 5 minutes to overview these indicators of progress at this stage

Distribute copies of:

- [Level 5.25: Adding and taking off a percentage](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N52503P.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N52503P.htm>)
- [Level 5.5: Solving percentage problems](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55003P.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55003P.htm>)
- [Scaffolding Numeracy in the Middle Years: Learning Plans for Multiplicative Thinking](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmyplans.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmyplans.htm>)
- [Digilearn](http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm>)
- [Common misunderstandings – Introduction](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm>)

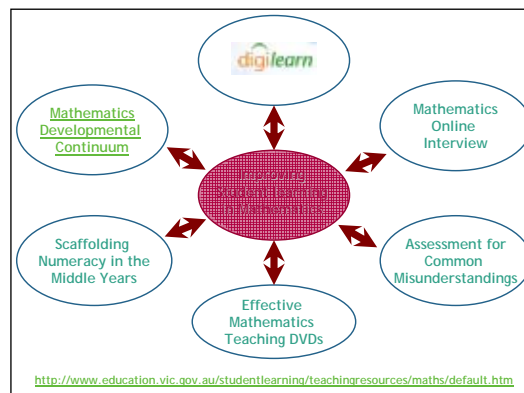
Solving percentage problems – 5.5

Slide 5 gives three different kinds of percentage problems which can be found in 'Solving percentage problems 5.5' from the Mathematics Developmental Continuum P–10.

Invite participants to complete the problems posed on the slide.

1. There are 60 books on the shelf and 20% are cookbooks. How many cookbooks?
2. There are 60 books on the shelf and 12 are cookbooks. What percentage are cookbooks?
3. There are some books on the shelf and 12 are cookbooks. If 20% are cookbooks, how many books are on the shelf?

Use slide 5: Solving percentage problems



Slide 4: Range of Resources

Solving Percentage Problems

Write a (correct) solution that you might expect from Year 7 or Year 8 students.

1. There are 60 books on the shelf and 20% are cookbooks. How many cookbooks?
2. There are 60 books on the shelf and 12 are cookbooks. What percentage are cookbooks?
3. There are some books on the shelf and 12 are cookbooks. If 20% are cookbooks, how many books are on the shelf?

These examples are illustrated in the Solving percentage problems - 5.5 Mathematics Developmental Continuum P-10

Slide 5: Solving percentage problems – 5.5

Solving Percentage Problems

Success for students depends on their understanding of the three basic types of percentage problems:

- ñ **MISSING PART:** Finding what quantity is a given percentage of another (Refer to Question 1)
- ñ **MISSING PERCENT:** Finding what percentage one quantity is of another (Refer to Question 2)
- ñ **MISSING WHOLE:** Finding the whole quantity (ie the 100%) Given what percent a certain quantity is. (Refer to Question 3)

Slide 6: Solving percentage problems – 5.5

Ask the participants to:

- Consider some sample solutions for each of the problems.
- If all three solutions use the same method then ask for alternative solutions.

Possible solutions:

It is interesting to discuss and note the different solutions to the problems. Some possible approaches:

Problem 1:

- a) 20% is $\frac{1}{5}$ and $\frac{1}{5}$ of 60 is 12.
- b) 10% of 60 books will be 6 books, so 20% is double this. 12 books.

Problem 2:

- a) Calculation: $12/60 \times 100\%$ then simplify.
- b) 12 books.... $\frac{1}{5}$ of 60 books.... $\frac{1}{5}$ is $20/100$ or 20%

Problem 3:

- a) 20% of some number is 12 cookbooks. $\frac{1}{5}$ of a number is 12, so the number of books is 12×5 or 60.
- b) Could use a diagram. Show a box representing 12 books and also labelled as 20%. Need 5 of these to get the total number of books, so 5×12 or 60. Similar to the method in (a), but using a diagram.

What does success look like?

Slide 6 describes what success for students would look like. Success depends on students' understanding of the three basic types of percentage problems:

- **missing part:** finding what quantity is a given percentage of another
- **missing percent:** finding what percentage one quantity is of another
- **missing whole:** finding the whole quantity (ie the 100%) given the percentage of a certain quantity.

The indicator of progress 'Solving percentage problems' distinguishes between these three different types of percentage problems.

Students usually find the third type of problem more difficult than the other two. Some students find further difficulty when working with percentages over 100%.

Many students will be able to complete problems with easy numbers but not with larger numbers, even when they have a calculator.

Common student difficulties in working with percentages

Slide 7 lists some of the common student difficulties. These are:

- Not knowing whether to multiply or divide.
- Trying to apply a rule that is partially remembered.
- Not appreciating that percent means 'out of 100'.
- As in the last example, finding 20% of 12, instead of 12 being 20%.
- Not estimating the approximate size of the answer and checking against it.

 Use slide 5: Solving percentage problems

Invite participants to contribute to this list.

Illustrations

Slide 8 illustrates the three types of percentage problems. Students often find 'missing whole' problems (such as problem 3) harder than problems of the other two types. This is even more so when the numbers and percentages involved are not whole numbers.

The dual number line

Slide 9 describes the dual number line teaching strategy. The dual number line is a teaching strategy which has a strong visual impact which as a thinking tool can assist student learning. It can be used to organise thinking, sorting out what is known from what is unknown. This approach can be used by teachers to assist students to solve all three types of percentage problems.

Solving Percentage Problems

- Not knowing whether to multiply or divide
- Trying to apply a rule that is partially remembered
- Not appreciating that percent means 'out of 100'.
- In the last example, finding 20% of 12, instead of 12 being 20%.
- Not estimating the approximate size of the answer and checking against it.



Slide 7: Solving percentage problems – 5.5 — Common student difficulties in working with percentages

Solving Percentage Problems

- Problem 1: Missing Part**
20% of the books on a shelf of 60 books are cookbooks. How many cookbooks are there? *What number is 20% of 60?*
- Problem 2: Missing Percent**
12 out of 60 books on a shelf are cookbooks. What percentage of the books are cookbooks? *What % is 12 of 60?*
- Problem 3: Missing Whole**
20% of the books on a shelf of books are cookbooks. If there are 12 cookbooks, what is the total number of books on the shelf? *Finding the whole given 12 is 20%*

Use all types of problems when assessing student understanding

Slide 8: Solving percentage problems – 5.5 — Illustrations

Solving Percentage Problems

Strategy to solve percentage problems:
The dual number line

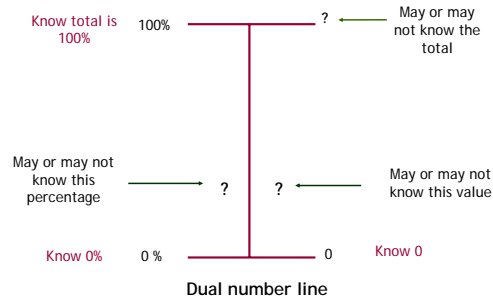
- Has strong visual impact
- Emphasises multiplication
- Is used to organise thinking, sorting out what is known and what is unknown.

This is an important general thinking tool



Slide 9: Solving percentage problems – 5.5 — The dual number line

Solving Percentage Problems



Slide 10: Solving percentage problems – 5.5 — Explaining the dual number line

Explaining the dual number line

Slide 10 is animated. It provides a visual representation of the dual number line, and explains how it is used. The dual number line is a useful organiser for solving percentage problems and other problems involving multiplicative thinking. It helps solve problems where you may not know any one of:

- the total
- the percentage
- the value.

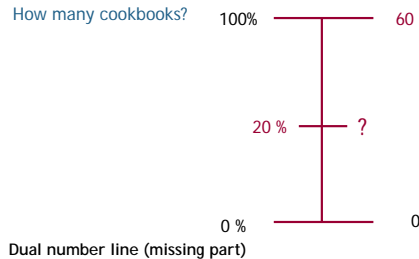
Hint- keep:

- Percents on one side.
- Raw numbers on the other.

The two zeroes are matched and located at the base of the number line. 100% is also able to be located at the top of the diagram (write it on the left hand side of the diagram).

Solving Percentage Problems

1. There are 60 books on the shelf and 20% are cookbooks.
How many cookbooks?



Slide 11: Solving percentage problems – 5.5 — Using the dual number line to solve a missing part problem

Using the dual number line to solve a missing part problem

 **Slides 11–18: Using the dual number line to solve a missing part problem.**

Slide 11 refers to the first problem:

- There are 60 books on the shelf and 20% are cookbooks. How many cookbooks?

This slide is animated, providing a step by step explanation of how to use a dual number line as a tool to solve a problem where the student is finding what quantity is a given percentage of another.

Note the following points with participants:

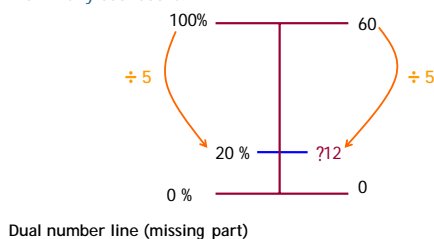
- On the diagram students can always write in 0% and 0, as well as 100%.
- We know that 60 is the total number of books (so 100%).
- That 20% are cookbooks

What is the number of cookbooks? Note the dual scale on the number line: the unknown is noted by a question mark.

Solving Percentage Problems

Approach 1

There are 60 books on the shelf and 20% are cookbooks.
How many cookbooks?



Slide 12: Solving percentage problems – 5.5 — Approach 1

Other approaches

There are various approaches that could be used to solve a missing part problem. Slides 12 to 16 provide various alternatives.

Simple division approach

Slide 12 shows one approach in solving this problem. This slide is animated. Participants will recognise that 20% is one fifth of 100%, so divide 60 by 5 to find the number of cookbooks (12).

Unitary method approach

Slides 13 and 14 develop the solution to the above problem using a **unitary method**. This slide is animated, and provides a step by step solution.

Discussion points:

- Invite participants to show the working you might expect from students using this method.
- Discuss their responses.
- Provide the solution. **Note:** It is important that students focus on the **multiplicative** relationship here.

The following information may be shared with participants whilst clicking through the animation:

- 100% divided by 100 equals 1%.
- What is done to one side of the dual number line needs to be done to the right hand side.
- 60 divided by 100 equals 0.6.
- Multiply 1% by 20 to give 20%, and on the other side.
- Multiply 0.6 by 20 give 12.

Unitary method – using known benchmarks

A variation of the unitary method is to use known benchmarks. For example, 10% is often a very convenient computational benchmark. Working through the process using this benchmark will provide you with the same answer. This slide is animated, which shows a step by step approach in solving this problem.

Ratio approach

Slide 15 provides a sample solution using a dual number line which is noted on the continuum. It shows a solution which links percent and quantity.

Notice that 60 is 0.6 or $\frac{6}{10}$ of 100, thus the solution is $\frac{6}{10}$ of 20.

There are 12 cookbooks.

See:

- **Indicator of progress: Solving percentage problems: 5.5**
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55003P.htm>)

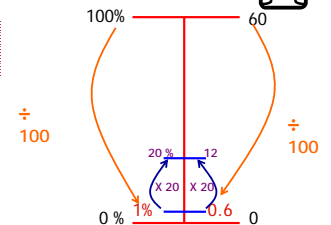
Solving Percentage Problems

Approach 2

There are 60 books on the shelf and 20% are cookbooks.
How many cookbooks?

Use the dual number line to find 1% and then find 20%.

Show the working you might expect from students using this method



Dual Number Line: Unitary method

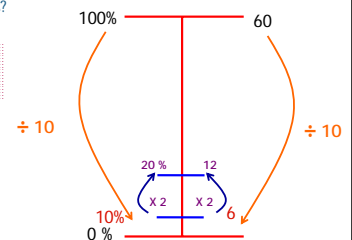
Slide 13: Solving percentage problems – 5.5 — Approach 2

Solving Percentage Problems

Approach 3

There are 60 books on the shelf and 20% are cookbooks.
How many cookbooks?

Use the dual number line to find 1% and then find 20%.



Dual Number Line: Variation 10%

Slide 14: Solving percentage problems – 5.5 — Approach 3

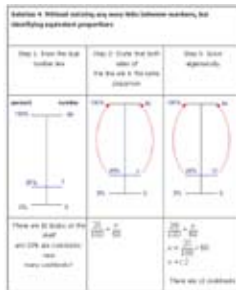
Solving Percentage Problems

Benchmark 3: Using the ratio between 100 and 60		
Step 1: Draw the dual number line	Step 2: Notice 10% from percent to quantity	Step 3: Use 10% from quantity to percent
<p>percent: 100% number: 60</p>	<p>100% 60</p> <p>10% 6</p> <p>0% 0</p> <p>÷ 10</p>	<p>100% 60</p> <p>10% 6</p> <p>0% 0</p> <p>× 2</p>
<p>There are 60 books on the shelf and 20% are cookbooks. How many cookbooks?</p> <p>$\frac{6}{10}$ of 100 = 60</p>	<p>$\frac{6}{10}$ of 20 = 12</p>	<p>There are 12 cookbooks.</p>

<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55003P.htm>

Slide 15: Solving percentage problems – 5.5

Solving Percentage Problems



<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathcontinuum/number/NS5003P.htm>

Slide 16: Solving percentage problems – 5.5

Algebraic algorithm approach

Slide 16 shows how students could use an algebraic algorithm to solve this problem.

'Missing whole' type problem

Slide 17 displays approaches to the 'missing whole' problem.

As noted earlier, 'missing whole' problems (like the problem 3 on slide 5) are often found by students to be harder than problems of the other two types. The effect is stronger if the numbers and percentages involved are not 'nice'.

Invite participants to discuss this problem:

- There are some books on the shelf and 12 are cookbooks. If 20% are cookbooks, how many books are on the shelf?

Ask participants:

- How could they use the dual number line to solve this problem?
- Explain their strategy.
- Are there other strategies?

Slide 17 provides one solution to this problem:

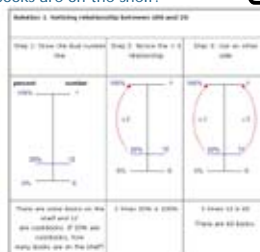
- The first approach requires the participants to notice the relationship between 100 and 20.

Dual number line (missing whole)

3. There are some books on the shelf and 12 are cookbooks. If 20% are cookbooks, how books are on the shelf?



Use a dual number line to solve this problem



Slide 17: Dual number line

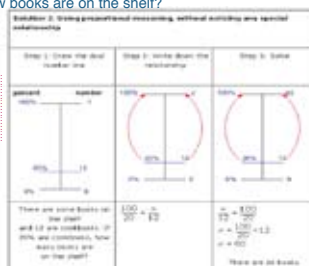
'Missing whole' type problem (contd)

Slide 18 provides another solution to this problem without noticing the relationship between 100 and 20. This approach uses proportional reasoning.

Dual number line (missing whole)

3. There are some books on the shelf and 12 are cookbooks. If 20% are cookbooks, how books are on the shelf?

Use a dual number line to solve this problem



Slide 18: Dual number line

Adding and taking off a percentage – 5.25

A dual number line approach can also be applied when working with problems where the percentages are greater than 100%. This is also an important link for students studying at VCE level, especially for ideas of compound interest.

Slide 18 refers to the following problem where the percentage is greater than 100%:

- If Fiona earns \$40 per hour in her part-time job and she gets a 5% pay rise, what is her new hourly rate?

 Use slide 19: Adding and taking off a percentage

Invite the participants to solve this problem.

The slide is animated and provides a solution to this problem.

Please inform participants that adding 5% is multiplying by 1.05. The same approach can be used for compound interest, exponential growth, decay, etc.

Additive reasoning

Slide 20 shows a solution which may have been offered by participants. It is confusing when addition/subtraction and multiplication/division are on the same dual number line. The calculation is not wrong, as such, but the dual number line is best used for proportional (hence, multiplicative) reasoning.

Easy and hard ratio and proportion questions – 5.5

Non percentage problems

Slide 21 refers to the following problem:

- The recipe for strawberry jam says 3.5kg of sugar for 4kg of strawberries. I have only 3kg of strawberries. How much sugar do I need?

This is an example of a problem which can be solved using the dual number line. Note that this is a non-percentage context, and is found on the indicator of progress:

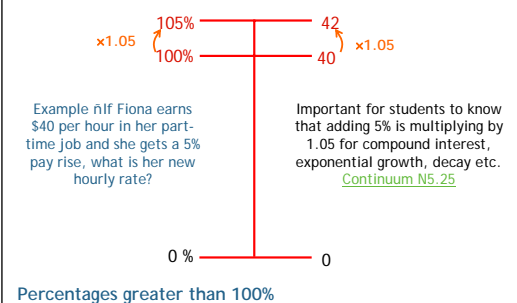
- Indicator of progress: [Easy and hard ratio and proportion questions: 5.5](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55004P.htm#a3) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55004P.htm#a3>)

 Use slide 21: Non percentage problems

Invite the participants to solve this problem and discuss possible strategies.

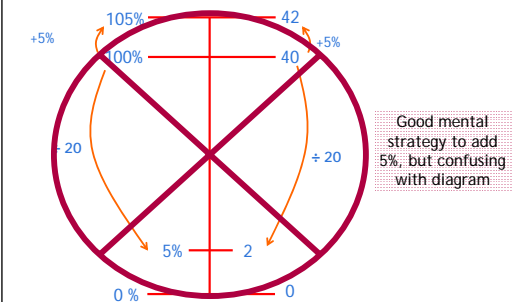
Participants may comment that their students may not always use the same method as they are using informal or formal methods. However, they can be reassured that one method may not necessarily be more effective than another.

Adding & Taking off a Percentage



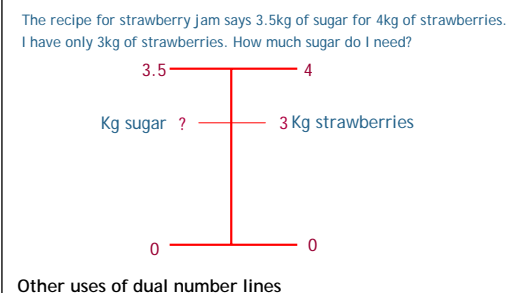
Slide 19: Adding and taking off a percentage – 5.25

Don't Divide & Add on One Diagram



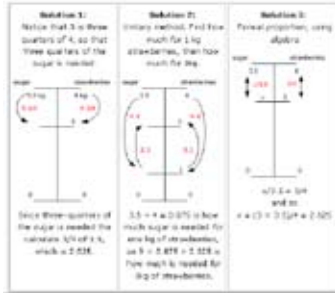
Slide 20: Don't Divide and Add on One Diagram

Easy and Hard Ratio and Proportion Questions



Slide 21: Easy and hard ratio and proportion questions – 5.5

Easy and Hard Ratio and Proportion Questions



Slide 22: Easy and Hard Ratio and Proportion Questions

Easy and Hard Ratio and Proportion Questions

Write a question, not involving percentages, that students could solve using a dual number line. For example:



1 cup is 250g. What if you need 80g? How much of a cup is this?

To make 12 muffins you use 250g of flour. How much flour will you need if you want to make 30 muffins?

A car travels 100 km in 70 minutes. How far does it travel in 80 minutes? (assuming constant speed!)

n.b. try and make the questions as authentic as you can.

Slide 23: Easy and Hard Ratio and Proportion Questions

Dual Number Line: Summary

The Dual Number Line:

- ī Useful organiser
- ī Helps formulate a problem mathematically from a worded problem
- ī Helps with estimation
- ī Use multiplication and division only on the diagram
- ī Problem solving using a mathematical diagram (but not a picture!)
- ī Focus on efficiency of various approaches
 - ñ Depends on the numbers involved, mental skills etc

Slide 24: Dual Number Line: Summary

A range of solutions

Slide 22 provides three different ways to solve this problem. Encouraging students to develop a range of strategies is consistent with the Principles of Learning and Teaching where the teacher uses a range of strategies that support the different ways of thinking and learning (PoLT 3.2). The Principles of Learning and Teaching are detailed in Resource 1.

Write your own question

Slide 23 challenges the participants to write a question, not involving percentages, which they could solve using a dual number line. This slide is animated and on clicking will provide the following responses:

Some examples:

- 1 cup is 250g. What if you need 80g? How much of a cup is this?
- To make 12 muffins you use 250g of flour. How much flour will you need if you want to make 30 muffins?
- I travel 100km in 70 minutes. How far do I travel in 80 minutes?

Further examples include:

- Calculations in Chemistry (moles, pressure, volume).
- Currency exchange rates.

Use slide 23: Write your own question

Invite participants to write problems which they could solve using a dual number line.

Share responses.

Summary of the advantages of using a dual number line to solve problems

Slide 24 provides a list of the advantages of using a dual number line.

The dual number line

- is a useful organiser
- helps formulate a problem mathematically from a worded problem
- helps with estimation
- uses multiplication and division only on the diagram
- supports problem solving by using a mathematical diagram (not a picture).

Using Digilearn objects

Barchart (L3512)

Slide 25 shows a Digilearn bar chart object. This slide is animated.

This barchart object has used the data entered by 20 students showing their favourite sport. This object can easily provide a visual representation of percentages.

An excellent resource of animations for supporting learning is:

- [National Library of Virtual Manipulatives](http://nlvm.usu.edu/en/NAV/vlibrary.html)
(<http://nlvm.usu.edu/en/NAV/vlibrary.html>)

This particular example is:

- 'Bar Chart' (L3512)
(https://www.eduweb.vic.gov.au/dlrcontent/4c33353132/ec_002_utah_011/index.html)

This is an applet which will allow students to easily represent in graphical form data they have collected.

This applet will allow students the opportunity to produce the graph (by choosing cells), label categories and then move between standard representations (no numbers shown on columns), to 'Totals' (where total number is shown) to 'Percentages'. Note that the number of columns and rows can be varied and the labels have been changed manually in this slide.

Use slides 25–28: Using Digilearn objects

Provide time for participants to explore this applet.

The applet easily allows students to vary figures, with resulting changes in the graph. The adaptive features of the applet will provide opportunities for students to explore, investigate, ask 'what if?'

Invite participants to discuss:

- What makes good classroom use of this applet?

Some possible responses may include:

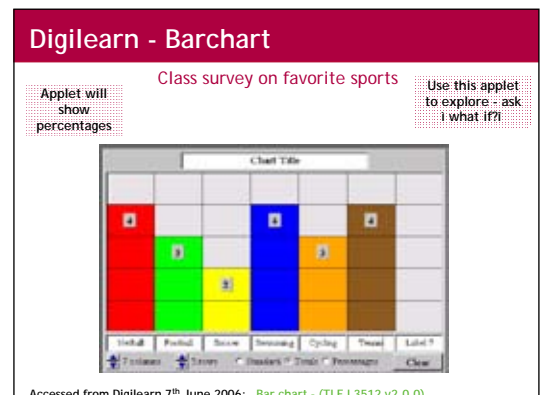
- Students can verify their calculations.
- Students can investigate what happens if two more people are added to the sample and they both choose netball as their favourite sport etc.
- The dynamic nature of the graph enables exploration.

Slide 26 shows a different approach to using this Digilearn object. In this example the teacher provides students with a completed chart and asks them to explore various methods of finding the percentage of people who chose each sport.

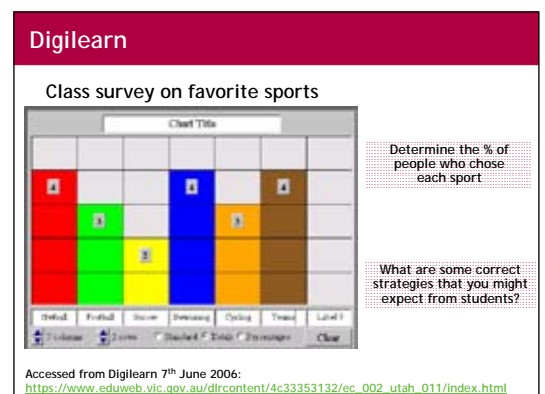
Invite participants in small groups to share their ideas in teaching with this object.

Some possible responses may include:

- Use a rule (in this case very inefficient).
- Multiply each number by 5 (noticing that $20 \times 5 = 100$).



Slide 25: Digilearn – Barchart

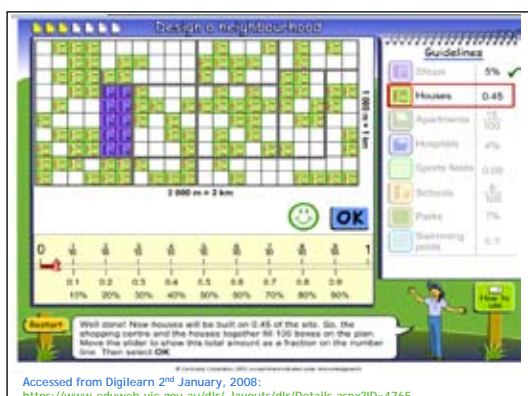


Slide 26: Digilearn



Accessed from Digilearn 2nd January, 2008:
https://www.eduweb.vic.gov.au/dlr/_layouts/dlr/Details.aspx?ID=4765

Slide 27: Digilearn – Designing a neighbourhood (L122)



Accessed from Digilearn 2nd January, 2008:
https://www.eduweb.vic.gov.au/dlr/_layouts/dlr/Details.aspx?ID=4765

Slide 28: Digilearn – Designing a neighbourhood (L122)

Digilearn – Designing a neighbourhood (L122)

Slides 27 and 28 refer to 'Designing a neighbourhood', Digilearn object L122, which could be used as a five minute lesson starter or short activity. In this activity students are able to make connections between various representations of numbers (% , fraction, decimal).

The program provides for different ways of demonstrating or showing the area. It is useful for visual learners, if they take care to observe how much area is covered. It does however provide for a fixed percentage for each type of property (that is, the numbers do not change).

- [Digilearn – Designing a neighbourhood \(L122\)](https://www.eduweb.vic.gov.au/dlr/_layouts/dlr/Details.aspx?ID=4765) (https://www.eduweb.vic.gov.au/dlr/_layouts/dlr/Details.aspx?ID=4765)

If possible, provide the opportunity for participants to explore the online Digilearn Neighbourhood object. They will be able to observe visually the conversions in the percentage/fraction/decimal for each of the shops/houses/apartments etc.

At each stage participants will need to add the new amount to the number line shown in slide 28. In this screenshot there is already 5% for shops, so the participant would need to add 0.45 to the original 5% and move the pointer on the number line to 0.5.

Sometimes participants will need to add the correct number of buildings (etc) onto the plan. They will be provided with feedback to show whether or not they have added the correct number of buildings (etc).

Scaffolding Numeracy in the Middle Years

Background

Slide 29 provides background information about Scaffolding Numeracy in the Middle Years Project.

The Scaffolding Numeracy in the Middle Years Linkage Project 2003–2006 (SNMY) was an Australian Research Council Linkage Project awarded from July 2003 to June 2006 to:

- RMIT University
- the Victorian Department of Education and Training
- the Tasmanian Education Department.

The project produced a resource focussing on 'multiplicative thinking' as the main area of concern for mathematical understanding in the Middle Years, as found in the Middle Years Numeracy Research Project (MYNRP).

The resource determines students' multiplicative skill level through using rich assessment tasks. Based on this assessment, students are placed into one of seven zones within the SNMY learning and assessment framework. Teachers are then able to apply appropriate learning plans according each student's stage of development. This process provides a differentiated approach to teaching and learning.

Scaffolding Numeracy in the Middle Years

SNMY Research Project 2003-2006

- i Involved RMIT University + Victorian Department of Education + Tasmanian Education Department
- i Focussed on multiplicative thinking as the main area of concern for mathematical understanding in the Middle Years (as found in the MYNRP)
- i The project investigated a new assessment-guided approach to improving student numeracy outcomes.
- i It was aimed at identifying and refining a learning and assessment framework for the development of multiplicative thinking using rich assessment tasks.

Slide 29 : Scaffolding Numeracy in the Middle Years

The learning and assessment framework includes eight zones which are sequenced in order of difficulty.

The eight zones are:

- Zone 1 – Primitive Modelling
- Zone 2 – Intuitive Modelling
- Zone 3 – Sensing
- Zone 4 – Strategy Exploring
- Zone 5 – Strategy Refining
- Zone 6 – Strategy Extending
- Zone 7 – Connecting
- Zone 8 – Reflective Knowing. It is within this zone where most of the percentages concepts are taught.

See:

- [Scaffolding Numeracy in the Middle Years – Learning and Assessment Framework](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/laf.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/laf.htm>)

Multiplicative thinking

Slide 30 defines multiplicative thinking.

Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts. The multiplicative concepts which relate to percent are introduced to students in zone 5 within the SNMY learning framework, and then the concept is further extended in zone 7 through students solving and explaining percentage problems.

Development of multiplicative thinking

Slide 31 provides information of the development of multiplicative thinking. Multiplicative thinking requires students to coordinate complex reasoning including:

- groups of equal size
- the number of groups
- and the total amount.

Once students have grasped these concepts, they then need to apply them to a range of different contexts. The contexts include ratio, proportion, rational numbers, even algebra. A further application is percentages. This process is complex and may take many years to achieve.

Scaffolding Numeracy in the Middle Years

What is 'multiplicative thinking'?

Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts.

Slide 30: Scaffolding Numeracy in the Middle Years — Multiplicative thinking

Development of multiplicative thinking

From early 0

I had 3 bags of sweets with 8 sweets in each bag.
How many sweets do I have altogether?

To later multiplicative thinking skills

Julie bought a dress in an end of season sale for \$49.35.
The original price was covered by a 30% off sticker but the sign on top of the rack said 'Additional 15% off already reduced prices.' How could she work out how much she has saved?
What percentage off the original cost did she end up paying?

Slide 31: Development of multiplicative thinking

Scaffolding Numeracy in the Middle Years

How percent concepts develop through the multiplicative thinking framework?

Zone 5 - Strategy Refining

Beginning to work with decimal numbers and percent but unable to apply efficiently to solve problems

Zone 7 - Connecting

Can solve and explain solutions to problems involving simple patterns, percent and proportion. May not be able to show working and/or explain strategies for situations involving larger numbers or less familiar problems.

Slide 32: Scaffolding Numeracy in the Middle Years

Assessment for Common Misunderstandings

This resource:

- i Will identify the learning needs of students who teachers believe are at risk or likely to be at risk in relation to i Number
- ii Is comprised of level-based assessment tasks which are linked to appropriate VELS standard. However, these tasks recognise that these students are underachieving and direct teachers to choose at a level below the standard.

Slide 33: Assessment for common misunderstandings

Where can percentages be found in the SNMY?

Slide 32 indicates how and when the learning of the percentages occurs within in the multiplicative framework. Additional notes are found on:

- Scaffolding Numeracy in the Middle Years: An introduction to the Learning and Assessment Framework for Multiplicative Thinking (LAF) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmylafintro.htm>)

Assessment for common misunderstandings

Background

Slide 33 describes the Assessment for Common Misunderstandings resource, which is another resource on the departmental website which may be of use when teaching percentages.

See:

- Common Misunderstandings – Introduction (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm>)

This is an 'intervention resource' for 'at risk' students who are not achieving at the expected level within Number.

This resource assesses for student misunderstandings using set tasks. The tools (tasks) have been chosen to probe for key ideas at the level which, if not understood, could seriously undermine the student's capacity to engage meaningfully with core aspects of the Number dimension in subsequent years.

The idea behind the use of the tools is to provide teachers with a set of easy-to-use diagnostic tasks that expose critical aspects of student thinking in relation to key aspects of the Number dimension. Research has shown this area to be most responsible for the huge differential in student performance by the middle years.

The following are the most relevant to the teaching of percentages:

- Common Misunderstandings Level 5: Proportional reasoning (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/commisslv5.htm>)
- Common Misunderstandings Level 6: Generalising (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/commisslv6.htm>)

Assessment for Common Misunderstandings – Levels

Slide 34 indicates the various levels and sublevels found within the 'Assessment for common misunderstandings' resource. Students at level 5 and 6 are struggling with proportional reasoning and generalising, which will impact on the development of their conceptual understanding of percentages.

Assessment for Common Misunderstandings – Level 5

Slide 35 provides more detail of level 5, differentiating the sub-levels to show the scope of level 5. Levels 5.2 and 5.8 relate to the percent concept.

Example of an assessment item

Slide 36 provides an example of an assessment item which can be used to diagnose misunderstandings.

This item is designed to be used individually with students, and requires 5 to 10 minutes of class time to implement. Students must use proportional reasoning to solve percentage problems. There are four other diagnostic items on the resource sheet:

- [Common Misunderstandings – Level 5.8 Using Per Cent Tool](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/58percent.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/58percent.htm>).

Once the misconception is identified, teachers can apply an appropriate teaching strategy which will address the misconception and move students to a new level of understanding.

Assessment for Common Misunderstandings

The key ideas addressed are:

Level 1 ñ	Trusting the Count
Level 2 ñ	Place value
Level 3 ñ	Multiplicative thinking
Level 4 ñ	Partitioning
Level 5 ñ	Proportional reasoning
Level 6 ñ	Generalising

Slide 34: Assessment for Common Misunderstandings – Levels

Assessment for Common Misunderstandings

Level 5: Proportional Reasoning

- [5.1 Relational Thinking](#)
- [5.2 Sense of per cent](#)
- [5.3 Understanding scale factors](#)
- [5.4 Relative proportion](#)
- [5.5 Interpreting rational number](#)
- [5.6 Understanding ratio](#)
- [5.7 Working with rate](#)
- [5.8 Using per cent](#)

Slide 35: Assessment for Common Misunderstandings – Level 5

Assessment for Common Misunderstandings

[5.8 Using per cent](#): diagnostic question

What was the price of the skate board before the sale?



<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.html>

Slide 36: Assessment for Common Misunderstandings – Example of an assessment item

End of Module 6

- ī This is the last slide of the module
- ī Further questions
- ī studentlearning@edumail.vic.gov.au
- ī Subject field- Teaching Secondary Mathematics

Slide 37: End of Module 6

Conclusion

Slide 37 is the last slide of workshop module 6: Using a range of strategies and resources: Example Percentages.

There are many resources available on the DEECD website. The activities shown in this module will help teachers examine:

- a range of mathematical tasks that could be used to teach percentages
- the use of a graphic organiser to solve three different types of percentages
- resources to see the mathematical possibilities and also their limitations (e.g. can bar charts be used for 3 types of percentage questions)
- ways of presenting ideas visually.

There are 8 more professional learning modules:

The Teaching Secondary Mathematics workshop modules include:

1. Overview of learning in the Mathematics Domain
2. Overview of the Mathematics Developmental Continuum P–10
3. Narrowing the achievement gap: Focus on fractions
4. Conducting practical and collaborative work: Focus on contours
5. Understanding students' mathematical thinking: Focus on algebra and the meaning of letters
7. Learning through investigation: Focus on chance and variability
8. Working mathematically: Focus on a range of challenging problems
9. Conclusion: Planning for improvement in mathematics

Resource 1: Principles of Learning and Teaching P–12

- [Principles of Learning and Teaching P–12 and their components](http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm) (<http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm>)

Students learn best when:

The learning environment is supportive and productive. In learning environments that reflect this principle the teacher:

- 1.1) builds positive relationships through knowing and valuing each student
- 1.2) promotes a culture of value and respect for individuals and their communities
- 1.3) uses strategies that promote students' self-confidence and willingness to take risks with their learning
- 1.4) ensures each student experiences success through structured support, the valuing of effort, and recognition of their work.

The learning environment promotes independence, interdependence and self motivation. In learning environments that reflect this principle the teacher:

- 2.1) encourages and supports students to take responsibility for their learning
- 2.2) uses strategies that build skills of productive collaboration.

Students' needs, backgrounds, perspectives and interests are reflected in the learning program. In learning environments that reflect this principle the teacher:

- 3.1) uses strategies that are flexible and responsive to the values, needs and interests of individual students
- 3.2) uses a range of strategies that support the different ways of thinking and learning
- 3.3) builds on students' prior experiences, knowledge and skills
- 3.4) capitalises on students' experience of a technology rich world.

Students are challenged and supported to develop deep levels of thinking and application. In learning environments that reflect this principle the teacher:

- 4.1) plans sequences to promote sustained learning that builds over time and emphasises connections between ideas
- 4.2) promotes substantive discussion of ideas
- 4.3) emphasises the quality of learning with high expectations of achievement
- 4.4) uses strategies that challenge and support students to question and reflect
- 4.5) uses strategies to develop investigating and problem solving skills
- 4.6) uses strategies to foster imagination and creativity.

Assessment practices are an integral part of teaching and learning. In learning environments that reflect this principle the teacher:

- 5.1) designs assessment practices that reflect the full range of learning program objectives
- 5.2) ensures that students receive frequent constructive feedback that supports further learning
- 5.3) makes assessment criteria explicit
- 5.4) uses assessment practices that encourage reflection and self assessment
- 5.5) uses evidence from assessment to inform planning and teaching.

Learning connects strongly with communities and practice beyond the classroom. In learning environments that reflect this principle the teacher:

- 6.1) supports students to engage with contemporary knowledge and practice
- 6.2) plans for students to interact with local and broader communities and community practices
- 6.3) uses technologies in ways that reflect professional and community practices.

