

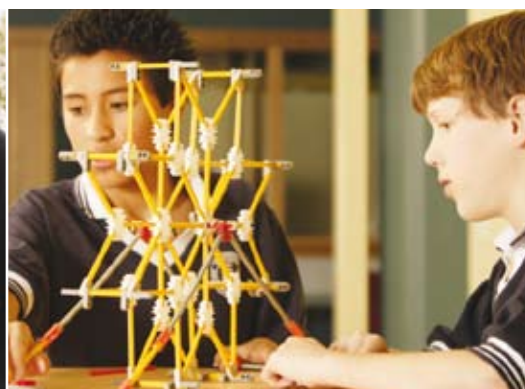


Teaching Secondary Mathematics

Module 3

Narrowing the achievement gap:
Focus on fractions

3



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Introduction to Module 3: Narrowing the achievement gap: Focus on fractions

Use of this module

This module allows for flexibility in modes of engagement with professional learning. This module booklet needs to be used in conjunction with the PowerPoint slides accompanying this resource.

Workshop approach

The materials of this module can be used by a presenter in a workshop for a school or a cluster of schools. A presenter, appointed from within or outside a school or cluster, is responsible for preparing presentations, facilitating discussions and outlining processes for collaborative planning.

Where a group is working collaboratively through these modules, a designated area is required for participants to share ideas, stories and samples in a climate of mutual respect. Regular after school meetings in a particular venue, such as the library, create a productive sense of community.

Individual use

The materials of this module are also suitable for private study and reflection. Individual users become both 'presenter' and 'participant'. While they are not able to engage in group discussions or whole-school planning, individual users can readily adapt the suggested group discussions and whole-school planning activities to private reflection, writing and classroom planning.

It is suggested that individuals identify a colleague or a buddy with whom to share their thoughts and to support the development of their understandings through ongoing dialogue. Individuals may complete all the modules or choose a combination depending on their interests or needs.

Web connections

The 'Teaching for Secondary Mathematics' resource is located at <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/default.htm>.

Before commencing to plan any elements of the program, schools are strongly advised to visit the Mathematics Domain page to review the most up-to-date advice, resources and information relevant to each module of the program. Many elements of this resource are available online in a downloadable format. There are links to assist schools to locate relevant information.

- [Mathematics Domain](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm>)

See the website for further details about this additional information or contact the student learning help desk on studentlearning@edumail.vic.gov.au

Content of the module

This module comprises Module 3: Narrowing the achievement gap: Focus on fractions booklet and the accompanying slide presentations which can be downloaded from <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/module3.htm>

The following are included in this document:

- the **User's Guide** that assists the user through the professional learning program
- hard copies of the **slide presentations** and **resource sheets**
- selected **resources**.

Organisation of the module

Computer access is required for all modules. If a group is completing the modules, a data projector and tables that enable people to sit together and work collaboratively are also necessary. The presenter should encourage participants to raise questions throughout the ensuing presentation. This presentation should take approximately two and a half hours, depending on the depth of discussion and types of activities that facilitators incorporate.

Required resources

This module requires the resources listed below.

Handouts

- Developmental Overview of Proportional Reasoning and Multiplicative Thinking
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdopropreason.pdf>)
- Mathematics Developmental Continuum – Mapping the 'Indicators of Progress'
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.pdf>)
- Comparing Fractions Diagnostic Test
See Resource 2 attached or download diagnostic test worksheet (<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/DiagFrTest.doc>)
- Number lines document
See Resource 4 – Fraction Number Strips attached

Optional

- Teachers may consider giving the 'Comparing Fractions Diagnostic Test' to their students prior to this session.
- See Resource 2 attached or download Comparing Fractions Diagnostic Test worksheet (<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/DiagFrTest.doc>)
- Reading:
Siemon, D 2002. 'Partitioning – the missing link in building fraction knowledge and confidence' (<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/ppppartitioning.pdf>)
- Fractions and Algebra DVD
DEECD 2007. 'Effective Mathematics Teaching DVDs', Department of Education and Early Childhood Development: Melbourne.

Icons

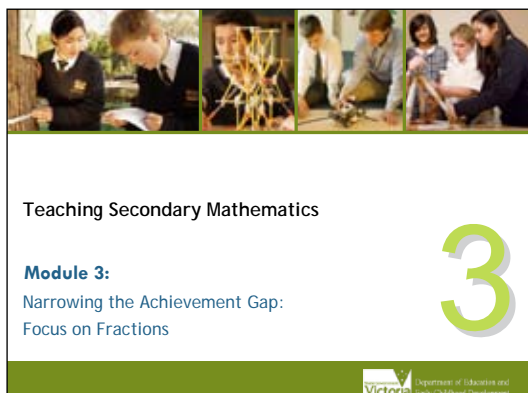
The following icons have been used in this workshop program:

Distribute handout: 

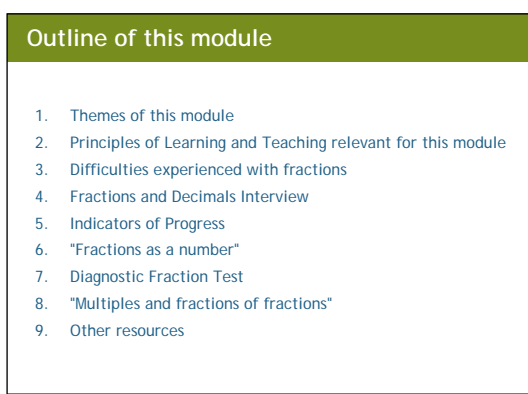
Group discussion: 

Group activity: 

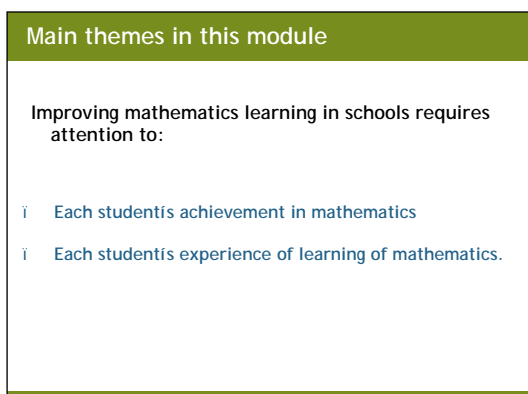
User's Guide to Module 3: Narrowing the achievement gap: Focus on fractions



Slide 1: Title slide



Slide 2: Outline of this module



Slide 3: Main themes in this module

Success in certain topics in the Number dimension (including fractions) is essential for continued progress in Mathematics. This module gives examples of diagnostic assessment and teaching which research has shown to be effective in improving student learning and in narrowing the achievement gap between students. It aims to do this by using teaching strategies which increase the mathematical understanding of the lowest achievers.

This module will model a variety of strategies that will meet teachers' needs and interests and assist them to respond to the needs of their students.

Slide 1 is the title slide

Outline

Slide 2 provides an outline of module 3: Fractions.

1. Themes of the module
2. Principles of Learning and Teaching relevant for this module
3. Difficulties experienced with fractions
4. Fractions and decimals on-line interview
5. Mathematics Developmental Continuum P–10
6. 'Fractions as a Number' as an indicator of progress
7. Comparing fractions diagnostic test
8. 'Multiples and fractions of fractions' as an indicator of progress
9. Other resources

Themes of the module

Slide 3 outlines two major themes of the module. Teachers will be exploring and be shown strategies which will be used to assess students conceptual understanding of Mathematics, and how to address any difficulties students may have. This will assist teachers to improve student *achievement*, and also provide a more positive *experience* in learning mathematics.

Mastering fractions is fundamental to success in many other areas of mathematics as it supports students in:

- general confidence for mathematics learning throughout school
- using 'multiplicative thinking', which is a critical component of mathematics in the middle years of schooling
- manipulating algebraic expressions, where algebraic fraction notation is used to note division
- working with exponentials, complex numbers, etc.

Some students may not have reached the expected achievement of VELS Level 4 before coming to secondary school and therefore this module will draw on material from VELS Level 3 from the Mathematics Developmental Continuum P–10.

The Continuum provides a range of powerful teaching strategies that support focused teaching for individuals and small groups of students with similar learning needs.

Principles of Learning and Teaching P-12 (PoLT)

PoLT principles featured in this module

Slide 4 lists the selected Principles of Learning and Teaching P-12 which are emphasised in this module. The module demonstrates the following principles and components:

1. The learning environment is supportive and productive.
The teaching strategies ensure each student experiences success through structured support, the valuing of effort, and recognition of their work.
3. Students' needs, backgrounds, perspectives and interests are reflected in the learning program.
The teaching strategies are flexible and responsive to the values, needs and interests of individual students.
5. Assessment practices are an integral part of teaching and learning.
The teacher uses evidence from assessment to inform planning and teaching.

Activities which demonstrate these Principles

Slide 5 illustrates how activities in this workshop module demonstrate principles 1, 3, and 5.

These activities are:

- Exploring the use of a linear model to help ensure students experience success with fractions (principle 1)
- Supporting teachers to respond to the needs and interests of their students (principle 3)
- Using the comparing fractions diagnostic test to identify students' misconceptions (principle 5).

Difficulties experienced with fractions

What do students find difficult about fractions?

Slide 6 explores participants' responses in what they consider students find difficult about fractions.



– Use slide 6: Difficulty with fractions

Invite participants to discuss the following question in small groups.

- What do students find difficult about fractions?
- What impact does this have on their learning?

Principles of Learning & Teaching

In this session the focus will include the following principles and components:

1. The learning environment is supportive and productive
 - ensuring each student experiences success through structured support, the valuing of effort, and recognition of their work
2. Students' needs, backgrounds, perspectives and interests are reflected in the learning program.
 - Teacher uses strategies that are flexible and responsive to the values, needs and interests of individual students
3. Assessment practices are an integral part of teaching and learning.
 - Teacher uses evidence from assessment to inform planning and teaching.

Slide 4: Principles of Learning & Teaching

Principles of Learning & Teaching

These principles are demonstrated in the module through:

- Exploring the use of a linear model to help ensure students experience success with fractions (Principle 1)
- Supporting teachers to respond to the needs and interests of their students (Principle 3)
- Using the Diagnostic Fraction Tests to identify students' misconceptions (Principle 5). Principle 5 will be a major focus of this module.

Slide 5: Principles of Learning & Teaching

Difficulties experienced with fractions

Discuss with your neighbour:



1. What do students find difficult about fractions?
2. What impact does this have?

Slide 6: Difficulties experienced with fractions

Difficulties experienced with fractions

Students often:

- i Cannot think of a fraction as one number
- i Use rules without thinking and don't check how sensible their answer is (e.g. $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$)
- i Think that $\frac{30}{80}$ is larger than $\frac{3}{8}$ (as we multiplied by 10)
- i Don't see a fraction as a result of division
- i Are unable to co-ordinate the number of parts (numerator) with the size of the parts (indicated by denominator)

Slide 7: Difficulties experienced with fractions

Difficulties experienced with fractions

Find the answers to the following:

$$\frac{4}{7} + \frac{3}{8} = \quad \frac{1}{2} \div \frac{1}{8} =$$



- i Do the rules/algorithms for the four operations (+, -, ×, ÷) emphasise the idea that a fraction is one number or two numbers?
- i Which view of fractions (as one number or two numbers) helps with estimation (number sense)?

Slide 8: Difficulties experienced with fractions

Slide 7 lists possible responses:

Possible responses include:

Students often:

- cannot think of a fraction as one number
- use rules without thinking and don't check how sensible their answer is (e.g. $\frac{1}{2} + \frac{1}{2} = \frac{2}{4}$)
- think that $\frac{30}{80}$ is larger than $\frac{3}{8}$ (as 'we multiplied by 10')
- don't see a fraction as a result of division
- are unable to co-ordinate the number of parts (numerator) with the size of the parts (indicated by denominator).

These responses are not exhaustive – in fact participants may come up with other responses not included here that are still valid and reasonable. These responses could be recorded by the facilitator.

This workshop will be focussing particularly on the concept of 'fraction as one number'. Research has shown that students often think of fractions such as $\frac{2}{5}$ as a two and a five, not as one number. This causes many difficulties for students when calculating with fractions.

Fraction as a number



Use slide 8: Thinking of fractions as one number

Slide 8 opens the discussion about how using common operations with fractions does not necessarily support students in thinking that a fraction is one number. They involve treating numerators and denominators as separate numbers.

Invite the participants find the answers to the following equations:

$$\frac{4}{7} + \frac{3}{8} =$$

$$\frac{1}{2} \div \frac{1}{8} =$$

Invite the participants to discuss in a small group:

- The solutions of these two calculations with their partner.
- Do the rules/algorithms for the four operations (+, -, ×, ÷) emphasise the idea that a fraction is one number or two numbers?

Information to support discussion:

'One number' means seeing a fraction like $\frac{3}{8}$ as the number three-eighths, which is just less than an $\frac{1}{2}$. In the examples, think about whether students try to apply an algorithm (rule) or do they use number sense to calculate or estimate an answer?

Indeed, fraction algorithms do not emphasise the idea that a fraction is one number; they involve treating numerators and denominators as separate numbers. It is also important students use number sense to estimate the answers to problems such as $\frac{4}{7} + \frac{3}{8}$. Students should be able to estimate that the answer is about 1 because $\frac{4}{7}$ is a bit more than $\frac{1}{2}$ and $\frac{3}{8}$ is a bit less than $\frac{1}{2}$.

Many students (and teachers) remember the rule that when you divide by a fraction you tip it upside down and multiply.

However in this case ($\frac{1}{2} \div \frac{1}{8} =$) it is easier to think about the task and say 'how many eighths in one-half?'

Fractions and decimals online interview

See Fractions and Decimals Online Interview on

- **Mathematics Domain - Assessment** (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/assessment.htm#1>)

Background



Use slides 9–11: Fractions and Decimals Interview

Slides 9–11 provide information about the 'Fractions and decimals online interview'.

The 'Fractions and decimals online interview' provides a means of tracking students' learning through significant big ideas in fractions, decimals, ratio and percentages.

The interview consists of appropriate hands-on and mental computation assessment tasks where students can demonstrate mathematical understanding and preferred strategies for solving increasingly complex tasks.

The 'Fractions and decimals online interview' is a powerful tool for assessing students' mathematical development. Evidence gathered through the interview allows teachers to achieve the following:

- Build a deep understanding of students' thinking in mathematics, especially rational number.
- Develop a detailed profile of student achievement.
- Inform focused teaching.

The fractions and decimals interview highlights the importance of responses and behaviours through the focus on mental computation and the explanation of strategies used. Having the interview online allows data to continually build from year to year in one assessment record.

Pizza question

Slide 11 shows an example of a task found on the 'Fractions and decimals interview'.

Invite participants to complete the question:

- Three pizzas were shared equally between five girls. How much pizza does each girl get?

General comments about student responses to this question:

- Although 30.3% of the trial students responded correctly. The students either drew a picture or mentally divided the pizzas.
- 11.8% of students were unable to make a start on this task.
- Students need to be provided with a greater exposure to division problems and explicit discussion connecting division with their fractional answers. For example, 3 divided by 5 = $\frac{3}{5}$ may help lead students to the generalisation that 3 divided by b = $\frac{3}{b}$

Fractions and Decimals Online Interview

- i The Middle Years Numeracy Research Project (1999/2000) found that significant numbers of students in years 5 to 9 experience difficulty with fractions, decimals & multiplicative thinking concepts.
- i One of the clear implications of the research is that early diagnosis and intervention are critical.
- i In January 2007, the Mathematics Online Interview was launched on the DEECD website. It has been proven to be an effective tool in enabling teachers and schools to collect detailed information on students' understandings of mathematics.
- i The Fractions and Decimals Online Interview will replicate its success for middle years students.

Slide 9: Fractions and decimals online interview

Fractions and Decimals Online Interview

- i One-on-one interview away from the regular classroom
- i 20 questions and sub-questions
- i Should take approximately 30 minutes to conduct
- i Focus is on mental computation and developing flexible strategies
- i Awarding a task requires that the students use an appropriate strategy \odot not only giving the correct answer
- i Questions ranging from Level 3 ñ 6 (VELS)



Slide 10: Fractions and decimals online interview

Fractions and Decimals Online Interview

Example question from the Interview

6. Pizza

Show the student the pale green picture of the 5 girls and the 3 pizzas.

Three pizzas were shared equally between five girls



3) How much pizza does each girl get?

Students are invited to use pen and paper if they require it.

Slide 11: Fractions and decimals online interview

The Mathematics Developmental Continuum P-10

Indicators of Progress

- i Early fraction ideas with models (level 2.5)
- i Multiples and fractions of fractions (level 3.25)
- i Fraction as a number (level 3.5)
- i Fractions for algebra and arithmetic (level 4.5)

Slide 12: The Mathematics Developmental Continuum P-10


The Mathematics Developmental Continuum P-10

Overview - Proportional Reasoning and Multiplicative Thinking

www.education.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/overproposas070105.pdf

Department of Education

	1.0	2.0	3.0	4.0	5.0	6.0
Foundations	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines
Multiplication	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines
Division	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines	Counting Number names Number words Number symbols Number lines
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www.education.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/overproposas070105.pdf

Slide 13: The Mathematics Developmental Continuum P-10

Indicators of Progress

Slide 12 lists the four indicators of progress on the Mathematics Developmental Continuum P-10 which relate to fractions:

- This module will focus on the indicator of progress 'Fraction as a number' as it is a common misconception for students and if corrected could impact on narrowing the achievement gap.

Slide 13 shows the developmental overview for 'proportional reasoning and multiplicative thinking'.

-  **GD Use Slide 13: Overview – Proportional reasoning and multiplicative thinking**

Ask the participants to respond to the following question:

- What would the implications of this overview be on the teaching and learning of fractions?

Some of the responses may include:

- The overview illustrates the long-term developmental growth of students' understanding of fractions.
- The concept of fractions is introduced early in primary school.
- By the end of primary school, students should be able to place a fraction on a number line.
- Students entering secondary school may display a range of conceptual understandings; some student knowledge may be at VELs level 3.

- Some of the responses may include:

- 8 |
- Module 3**
- Narrowing the achievement gap: Focus on fractions

Visual map of the Mathematics Developmental Continuum P–10

Slide 14 provides a visual map of the Mathematics Developmental Continuum P–10.

Note the use of terminology:

- The **indicator of progress** provides the critical understandings required by students in their ability to recognise that the fraction is one number.
- The **illustrations** uncover the students' mathematical thinking and misconceptions.
- **Teaching strategies** are used to follow up and support conceptual understanding, building on the students' existing ideas.

 Provide participants with a copy of:

- **Fraction as a number: 3.5** (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N35007P.htm>)

Give participants time to read and familiarise themselves with this resource.

Exploring 'Fractions as a number: 3.5'

Overview

Slide 15 provides an overview of 'Fraction as a number: 3.5'.

An important step in mathematical development is that students come to see a fraction $\frac{a}{b}$ as one number, even though it is written using two whole numbers: a and b .

A linear model for fractions helps students to see a fraction as a number, with a place on the number line. To find out more about models for fractions, go to the 'more about' link found on this indicator of progress.

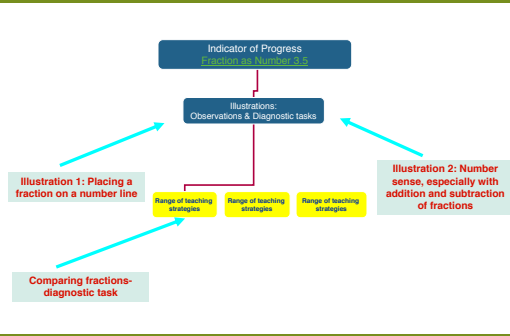
See

- **Models for fractions: 3.5**
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N35007G.htm>) Illustration – Placing a fraction on a number line

Slide 16 illustrates how students' understanding of fractions at this level may be revealed by their difficulty with placing a fraction on a number line. This is illustrated in the following ways:

- Students do not place fractions on a number line and are unable to use the number line to model operations with fractions. They have not extended their number system from their original appreciation of the set of whole numbers.
- Since students do not know that $\frac{2}{3}$ is one number; they have trouble marking this point on a number line as a point between 0 and 1. Instead of marking the point $\frac{2}{3}$, they may mark $\frac{2}{3}$ of the given line (using the fraction as an operator), or one or both of the whole numbers 2 and 3.

The Mathematics Developmental Continuum P-10



Slide 14: The Mathematics Developmental Continuum P–10

Fraction as a Number 3.5

Overview

- An important step in mathematical development is that students come to see a fraction $\frac{a}{b}$ as **one** number, even though it is written using two whole numbers, a and b .
- Using a linear model for fractions helps students to see a fraction as a number, with a place on the number line.
- To find out more about models for fractions, go to the "more about" link from the [The Continuum nModels for fractions: 3.5](#)

Slide 15: Fraction as a number 3.5

Fraction as a Number 3.5

Students understanding of fractions at this level may be illustrated through:

- Not placing fractions on a number line and being unable to use the number line to model operations with fractions. They have not extended their number system from their original appreciation of the set of whole numbers
- Not knowing that $\frac{2}{3}$ is one number, will students have trouble marking this point on a number line as a point between 0 and 1. Instead of marking the point $\frac{2}{3}$, they may mark $\frac{2}{3}$ of the given line (using the fraction as an operator), or one or both of the whole numbers 2 and 3

Slide 16: Fraction as a number 3.5

Fraction as a Number 3.5

Students understanding of fractions at this level may be illustrated through:

- i Not knowing that $\frac{2}{3}$ is one number and $\frac{7}{8}$ is another number. These students will have trouble comparing their size.
- ii Unable to estimate that the sum of $\frac{2}{3}$ and $\frac{7}{8}$ will be less than 2 because each number is less than 1.

Slide 17: Fraction as a number 3.5

Fraction as a Number 3.5

Models for fractions

Model	Fraction size represented by
3 Dimensional (volume model)	Relative volume
2 Dimensional (area model)	Relative area
1 Dimensional (linear model)	Relative length
Discrete (subset/set model)	Relative numbers
Number line	Position of a point on a line

Slide 18: Fraction as a number 3.5

Fraction as a Number 3.5

The Linear Model

Using a linear model is very important for several reasons:

- i It helps in building the concept of fraction as one number.
- ii A linear model, where the size of a fraction is modeled by the length of a line, is an important pre-requisite for student conceptual understanding of the number line.
- iii Students will need to use both horizontal and vertical number lines when they progress in mathematics to learning about Cartesian graphs.

Slide 19: Fraction as a number 3.5

Illustration – Number sense

Slide 17 points out difficulties student may experience if they don't have a sense of a fraction as a number. This is illustrated in the following ways:

- Students who do not know that $\frac{2}{3}$ is one number and $\frac{7}{8}$ is another number will have trouble comparing their size.
- Students are unable to estimate that the sum of $\frac{2}{3}$ and $\frac{7}{8}$ will be less than 2 because each number is less than 1.

Models to teach about fractions

Slide 18 presents a table illustrating a range of models which are used to teach about fractions. The table is taken from the Mathematics Continuum, Number dimension, level 3.5 'more about'.



Use slide 18: Range of models to teach fractions

Invite participants to draw or give an example of each of these models.

The Linear Model

Slides 19 and 20 highlight and explain the use of a linear model to teach fractions. A linear model represents numbers by length of a line.

A **fraction strip** is an example of a linear model (refer to slide 38).

A **number line** is another example of a linear model, where the size of the fraction is measured by the length a point takes from the origin.

Using a linear model is very important for several reasons:

- It helps in building the concept of fraction as one number.
- It is the best model for highlighting the concept of 'number density', that is, between any two numbers there are many other numbers. If we are using only whole numbers there is no number between 3 and 4, but once we allow fractions and decimals then there are an infinite number of possibilities.
- Many types of numbers can be represented with a linear model or shown on the number line: whole numbers, fractions and decimals. This means they are all examples of real numbers.
- Students will need to use both horizontal and vertical number lines when they progress in mathematics to learning about Cartesian graphs.

Teaching strategies

Slide 21 lists three activities which can be found on the Continuum. These activities can be used as teaching strategies which will build students' conceptual understanding:

- Activity 1: 'Comparing fractions diagnostic test' gives teachers feedback about student's conceptual understanding about fractions.
- Activity 2: 'Using area models better' provides guidance on ensuring that fraction concepts are not lost when concrete models are used. A short diagnostic task is included to see if students understand that area models show the fraction as a part-whole relationship.
- Activity 3: 'Number between' is a game that highlights the position of fractions on a number line, emphasises relative size, develops number sense and shows the property of number density for fractions.

The 'Comparing fractions' diagnostic test



Use slides 22–36 Comparing Fractions Diagnostic Test



Provide the test to participants (See resource 1).

The 'Comparing fractions diagnostic test' activity will assist teachers to identify the misconceptions about fractions held by their students. In this instance, participants will complete the test taking on the role of different students (Abby, Ben, Carol and David).

Instructions

Slide 22 gives instructions for the 'Comparing fractions diagnostic test'.

This resource can also be downloaded from:

- Comparing fractions diagnostic test
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/DiagFrTest.doc>)



Provide participants with a copy of

- Resource 3: 'Definitions of strategies used by students in comparing fractions'

This will assist participants in gaining a deeper understanding of the vocabulary associated in this activity.

Fraction as a Number 3.5

The Linear Model (cont.)

- i A linear model is the best model for highlighting the concept of *number density*, that is, between any two numbers, there are many other numbers. If we are using only whole numbers, there is no number between 3 and 4, but once we allow fractions and decimals, then there are an infinite number of possibilities.
- i Many types of numbers can be represented with a linear model or shown on the number line: whole numbers, fractions and decimals. This means they are all examples of real numbers.

Slide 20: Fraction as a number 3.5

Fraction as a Number 3.5

Teaching strategies:

Activity 1: Comparing Fractions Diagnostic Test provides a diagnostic test to determine how students are conceptualising fractions

Activity 2: Using area models better, provides guidance on ensuring that fraction concepts are not lost when concrete models are used. A short diagnostic task to see if students understand that area models show the fraction as a part-whole relationship is included.

Activity 3: Number Between, is a game that highlights the position of fractions on a number line, emphasises relative size, develops number sense and shows the property of number density for fractions.

Slide 21: Fraction as a number 3.5

Comparing Fractions Diagnostic Test

Instructions:



Complete the Fractions Diagnostic Test

i For each pair of fractions, either CIRCLE the larger fraction, OR write = between them

i We are going to think like students who are unable to correctly co-ordinate:

n The *number of parts* (numerator)

n With the *size of the parts* (denominator)

Slide 22: Comparing fractions diagnostic test

Abby:

Abby thinks that larger numerators give larger fractions

What would she choose?

$\frac{5}{8}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{3}{4}$
$\frac{4}{6}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{3}{8}$
$\frac{5}{7}$	$\frac{3}{8}$
$\frac{3}{10}$	$\frac{2}{5}$

Slide 23: Abby

Abby:

Abby's responses

$\frac{5}{8}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{3}{4}$
$\frac{4}{6}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{3}{8}$
$\frac{5}{7}$	$\frac{3}{8}$
$\frac{3}{10}$	$\frac{2}{5}$

Slide 24: Abby

Ben:

What is Ben thinking?

Ben thinks that smaller denominators give larger fractions

$\frac{5}{8}$	$\frac{1}{3}$
$\frac{2}{3}$	$\frac{3}{4}$
$\frac{4}{6}$	$\frac{2}{3}$
$\frac{4}{5}$	$\frac{3}{8}$
$\frac{5}{7}$	$\frac{3}{8}$
$\frac{3}{10}$	$\frac{2}{5}$

Slide 25: Ben

Abby's conceptions: Larger numerator

Slide 23 describes Abby's conceptions in relation to the size of fractions (she believes that the bigger fraction is the one with a larger numerator).

Invite participants to complete the task as Abby would.

The responses that Abby provides are provided on slide 24.

Invite participants to respond to the following questions:

- Why might students think like Abby?

Students apply the ordering of numbers that they have learnt from the set of natural numbers to fractions. Also when they compare numbers (e.g. 527 with 381) they are comparing just the first digits and nothing else.

- Why do you think she left two items blank?

These fractions both had the same numerator.

- What else might she do for these two items?

She may have written '=' or numbers with larger denominators.

She may have compared the size of the numbers of the denominators.

- Does how many she gets correct (between seven and nine) tell us anything?

Having the majority of responses correct does not necessarily indicate that a student has understood the comparing fractions concept.

Ben's thinking: Larger denominator

Slide 25 is an animated slide which demonstrates Ben's thinking. He thinks that smaller denominators give larger fractions.

Invite participants to respond to the following questions:

- Why do you think Ben left two items blank?

Denominators are the same, so he may not be able to choose. For example, if looking at the first pair of fractions, he knows that $\frac{1}{3}$ is more than $\frac{1}{8}$, so he thinks 'thirds are bigger than eighths', but he is not attending to the numerator (number of parts).

- What else might he do for these two items?

Compare the numerators in order to determine size of fractions, write '=', or may even correctly choose the fraction with larger numerator.

- How many does Ben get correct? Does this tell us anything?

Five correct; could be up to seven if he guessed the two blanks correctly.

Having the majority of responses correct does not necessarily indicate a grasping of the mathematical concept.

Carol's thinking: 'Gap' thinking

The question on slide 26 questions participants on what Carol is thinking. She thinks that the fraction with the smaller 'gap' is larger. This is an animated slide.

Invite participants to respond to the following questions:

- Why might other students think like Carol?

The 'gap' is the missing portion to make a whole. $\frac{5}{8}$ has a gap of 3 ('3 more to go') while $\frac{1}{3}$ has a gap of 2 ('2 more to go') so $\frac{1}{3}$ is closer to the whole as it only has '2 to go'. An overreliance on the discrete model can lead to this thinking.

- Carol gets seven correct. Does this score tell us anything?

Having the majority of responses correct does not necessarily indicate a grasping of the mathematical concept.

David's thinking: 'Common denominators' strategy

Slide 27 prompts participants to complete the test using David's reasoning. He uses a 'common denominators' strategy.

Invite participants to respond to the following questions:

- Do students always choose the most suitable common denominator?

Students may use an algorithm that involves finding the product of both denominators. Using this method eventually produces fractions with the same denominator but this denominator can be very large.

- Give an example of unsuitable denominators that David might use, using 50 as a common denominator for $\frac{3}{10}$ and $\frac{2}{5}$.

- How could you encourage students to make suitable choices?

You could show alternative strategies like benchmarking and use of linear models on a number line (see Resource 3: 'Definitions of strategies used by students in comparing fractions' for definition of these terms).

- How could you be sure that David appreciates this strategy that using common denominators actually involves equivalent fractions?

You could explore other concepts like a linear model where you could demonstrate in a concrete way equivalence of fractions.

Use slides 28–33: Other strategies

Invite participants to share their strategies.

Other strategies: benchmarking

Slide 28 describes a range of strategies used by students. This slide highlights the strategy of comparing with known fractions such as $\frac{1}{2}$.

Diagnosing misconceptions and also teaching with a range of strategies will give the opportunity to raise the achievement of all students including those that are low achievers.

Teachers should encourage students to think **flexibly** and use more than one strategy in comparing fractions.

Carol:

Carol's responses

Carol thinks that the fraction with the smaller 'gap' is larger.

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 26: Carol

David:

David uses common denominators on each item

What would he choose?

Correct answers chosen

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 27: David

Correct answers – other strategies?

Benchmark: compare with known fractions like $\frac{1}{2}$

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 28: Correct answers – other strategies?

Correct answers ñ other strategies?

Use a known decimal or %

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 29: Correct answers – other strategies?

Other strategies: Using a known decimal

Slide 29 demonstrates using a known decimal or percentage approach.

This shows using a known percentage to match a fraction e.g. $\frac{3}{5} = 60\%$ but $\frac{3}{4} = 75\%$ so $\frac{3}{4}$ is larger.

Other strategies: More parts of the same size

An example of this strategy is shown on slide 30. For example, three-quarters is more than one-quarter, three-fifths is more than two-fifths.

Other strategies: Same number of larger parts

Slide 31 shows an example where one-third is more than one-quarter and one-quarter is more than one-fifth, so three-quarters must be more than three-fifths.

Correct answers ñ other strategies?

More parts of the same size

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 30: Correct answers – other strategies?

Correct answers ñ other strategies?

Same number of larger parts

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{8}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 31: Correct answers – other strategies?

Size of missing piece

$\frac{2}{3}$, $\frac{3}{4}$ are both missing one piece from one whole. As $\frac{3}{4}$ is missing a smaller piece (one-quarter is less than one-third), it is closer to 1.

Likewise, $\frac{7}{8}$ and $\frac{11}{12}$ are both missing one piece from one whole. As $\frac{11}{12}$ is missing a smaller piece (1 twelfth is less than 1 eighth), it is closer to 1.

More larger parts strategy

Slide 33 explains that if one-fifth is more than one-eighth, then three-fifths would be more than three-eighths, and here we have even more fifths (4 not 3). So four-fifths has more parts and they are larger parts.

If one-seventh is more than one-eighth, then three-sevenths would be more than three-eighths, and here we have even more sevenths (5 not 3). So five-sevenths has more parts and they are larger parts.

Residual thinking may also be a strategy suggested by participants.

The main message is that we need to encourage students to think flexibly and use more than one strategy.

Ranking Students

Slide 34 provides an opportunity for participants to rank Abby, Ben, Carol and David in order of their perception about fractions knowledge.

 Use slide 34: Ranking students

Invite participants to respond to the following questions:

- How did you decide?
- Did you use their scores?
- Could you use another method to rank students?

Participants may respond with many different responses. Participants need to consider the sophistication of the students' thinking rather than the test score. Consider the following:

Using scores, the ranking will be David (12), then Abby (7–9), Carol (7), and lastly Ben (5–7)

Participants may argue that Ben seems to appreciate that larger denominators give smaller fractions which is an essential feature of fractions. Ben is starting to think multiplicatively. Carol, on the other hand, is just subtracting two whole numbers and so may have no idea of partitioning a whole, which is a basic fraction idea.

Correct answers ñ other strategies?

Size of missing piece: consider the relative sizes of the missing pieces

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 32: Size of missing piece

Correct answers ñ other strategies?

More larger parts

How can we encourage students to think flexibly and use more than one strategy?

$\frac{5}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
$\frac{2}{3}$	$\frac{3}{4}$	$\frac{3}{5}$	$\frac{2}{5}$
$\frac{4}{6}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{4}$
$\frac{4}{5}$	$\frac{3}{8}$	$\frac{3}{5}$	$\frac{3}{4}$
$\frac{5}{7}$	$\frac{3}{6}$	$\frac{1}{6}$	$\frac{2}{12}$
$\frac{3}{10}$	$\frac{2}{5}$	$\frac{7}{8}$	$\frac{11}{12}$

Slide 33: More larger parts

Comparing Fractions Diagnostic Test

Ranking students



Rank Abby, Ben, Carol and David in order of their knowledge.

Discuss:

- How did you decide?
- Did you use their scores?
- Could you use another method to rank students?

Slide 34: Comparing Fractions Diagnostic Test

Comparing Fractions Diagnostic Test

Features of this diagnostic test

- i The power of a diagnostic test is to reveal how students think, by comparing with the pattern of answers on the samples provided
- i Total score on this test is a function of how many items of each i typei are included on the test, and so has little value
- i Full scores indicate the ability to complete this task by using a successful strategy (which may be applied with or without understanding)
- i Any error may be the i tip of the iceberg

Slide 35: Comparing Fractions Diagnostic Test

Comparing Fractions Diagnostic Test

Creating tasks

1. Could you imagine another task that David may not get correct?
2. Write a new item that Abby will get correct & Ben will get wrong.
3. Write a new item that Abby will get wrong & Ben will get correct

Slide 36: Comparing Fractions Diagnostic Test

Features

Features of this diagnostic test are listed on Slide 35.

The test reveals how students think by comparing their responses with the pattern of answers on the samples. The results demonstrate that a total score on this test is a function of how many items of each 'type' are on the test, and so the student score is of little value in determining understanding.

Full scores indicate students' ability to complete this task by using a successful strategy (which may be applied with or without understanding).

Any student error may be the 'tip of the iceberg' of misunderstanding.

Creating test items

Slide 36 challenges participants to create their own tasks.

Note: There is animation on this slide. Click to bring up a sample answer to question 2, click again to bring up a sample answer to question 3.



Use slide 36: Creating tasks

Invite participants to respond to the following questions:

- Could you imagine another task that David might not get correct?
- Write a new item that Abby will get correct and Ben will get wrong.
- Write a new item that Abby will get wrong and Ben will get correct.

Remind participants of David's, Abby's and Ben's thinking. David uses an algorithm approach. Abby thinks that larger numerators will give larger fractions. Ben thinks that smaller denominators give larger fractions.

Suggested responses include:

- David may not provide a correct answer?
When placing a fraction on a number line.
Giving a number between $\frac{3}{10}$ and $\frac{4}{10}$.
Completing operations with fractions, including estimation.
- Abby will get the correct answer and Ben will get the wrong answer.
 $\frac{6}{7}$ and $\frac{4}{5}$.
- Abby will get the wrong answer and Ben will get the correct answer.
 $\frac{6}{11}$ and $\frac{4}{5}$

Multiples and fractions of fractions: 3.25

Fraction wall



Use slides 37–41: Using fraction strips to create a fraction wall

Making a fraction wall and marking number lines are good introductory activities with diagnostic potential. Participants will need a copy of the blank fraction wall and number lines (see Resource 4). They will need 12 paper strips the same lengths as the blank fraction wall and number walls (e.g. 20cm). An accurate model of fractions can be made electronically by inserting a table in Word (e.g. 1 column, 20 rows), then using the split cells command in each row. Cut the page into Fraction Strips.

Use Resource 4 to guide participants through the Fraction Strip activity. This resource is adapted from:

- [Multiples and fractions of fractions: 3.25](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N32508P.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N32508P.htm>).

Part A: Making the fraction strip

Slide 37 describes how to create a fraction strip. Each fraction strip has the length of one unit.

The fraction strips activity is based on the linear model of fractions, where the size of a fraction is represented by the length of a strip of paper. This activity produces a visual tool which symbolises the sizes of fractions.

This activity is adapted from

- [Multiples and fractions of fractions: 3.25](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N32508P.htm) (<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N32508P.htm>)

Direct participants to fold the fractions slips using these instructions:

- Use 12 paper strips.
- Leave one strip as the one or whole.
- Fold another strip to show halves.
- Fold another strip to show thirds.
- Continue to fold strips to twelfths.

Invite participants to discuss folding strategies. For example:

- To make one-sixth we can halve one-third or take one-third of one-half.
- Fractions with a factor of two (2, 4, 8 ...) are easier to fold.

Multiples and fractions of fractions 3.25

Make a Fraction Wall using [Fraction Strips Activity](#):

- Use 12 paper strips
- Leave one strip as the one/whole
- Fold another strip to show halves
- Fold another strip to show thirds
- Continue to twelfths.

[Multiples and fractions of fractions Indicator of Progress- 3.25](#)

Slide 37: Multiples and fractions of fractions 3.25

Multiples and fractions of fractions 3.25

Complete Fraction Wall using Fraction Strips

- Use your Fraction Strips to mark the Fraction Wall with vertical lines.
- Label each part with unit fractions.

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
---------------	---------------	---------------	---------------	---------------

- Why do some students write non-unit fractions incorrectly, for example:

		$\frac{3}{5}$		
--	--	---------------	--	--

Incorrect!

Slide 38: Multiples and fractions of fractions 3.25

Invite participants to discuss whether they can find equivalent fractions with these strips.

Ask participants why it is important for students to find equivalent fractions using fraction strips.

- A suggested response may be so that students may be able to see that fractions may look different despite being the same size.

Complete the fraction wall using the fraction strips

Slide 38 instructs participants to complete the fraction wall using the fraction strips.

Instruct them to use fraction strips to mark the fraction wall with vertical lines. Label each part with unit fractions.

Ask participants the following question:

- Why do some students write non-unit fractions incorrectly? For example:

Students confuse the $\frac{1}{5}$ portion with the location of $\frac{3}{5}$ on the number line.

$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
---------------	---------------	---------------	---------------	---------------

		$\frac{3}{5}$		
--	--	---------------	--	--

Part B: Using the fraction strips to locate various fractions on a number line

Slide 39 provides directions to locate various fractions on a number line. This second task is to use fraction strips to:

- place fractions onto a number line
- use with computations with fractions.

Participants will need another set of number lines (Resource 4).

The aim of this activity is to:

- reinforce the concept of equivalent fractions
- estimate fractions on a number line by using both scale measurements and paper strips
- use a visual model to compute with fractions.

Using the fraction strip to locate a fraction on a number line

Direct participants with the following instructions:

On a number line handout, estimate and then check (with fraction strips) the location of various fractions:

- On a line marked 0, 1 show $\frac{2}{3}$, $\frac{7}{8}$
- On a line marked 0, 2 show $\frac{2}{3}$, $1\frac{7}{8}$
- On a line marked 1, 2 show $1\frac{1}{3}$, $1\frac{3}{8}$
- On a line marked 0, 10 show $3\frac{1}{3}$, $9\frac{1}{2}$

Using operations with fractions

Slide 40 directs participants to use fraction strips to complete operations with fractions.

Invite participants to use the fraction strips to solve the following operation problems:

Addition:	$\frac{1}{5} + \frac{2}{5}$	$\frac{1}{5} + \frac{3}{10}$	$\frac{2}{3} + \frac{3}{4}$...
Subtraction:	$\frac{3}{5} - \frac{1}{5}$	$\frac{3}{5} - \frac{1}{10}$	$\frac{3}{4} - \frac{2}{3}$...
Multiplication:	$2 \times \frac{1}{3}$	$7 \times \frac{1}{3}$	$\frac{1}{4} \times 10$	$\frac{1}{3} \times \frac{1}{4}$
Division:	$2 \div \frac{1}{3}$	$10 \div \frac{1}{4}$	$\frac{1}{3} \div 2$	$\frac{1}{3} \div \frac{1}{2}$

Suggested Answers:

- For example: to add $\frac{1}{5}$ and $\frac{2}{5}$, you can think of $\frac{1}{5}$ as a point on the line, $\frac{2}{5}$ as a distance of $\frac{2}{5}$ along the line, and think of adding as a movement of a distance of $\frac{2}{5}$ to the right, to reach the point that represents $\frac{3}{5}$. Calculations on number lines are rather complex to understand because of these transitions between representations.

Multiples and fractions of fractions 3.25

Fractions on Number Lines

Use the Number Line handout to estimate and then check (with Fraction Strips) the location of various fractions:

- ñ On a line marked 0, 1 show $\frac{2}{3}$, $\frac{7}{8}$
- ñ On a line marked 0, 2 show $\frac{2}{3}$, $1\frac{7}{8}$
- ñ On a line marked 1, 2 show $1\frac{1}{3}$, $1\frac{3}{8}$
- ñ On a line marked 0, 10 show $3\frac{1}{3}$, $9\frac{1}{2}$

Slide 39: Multiples and fractions of fractions 3.25

Multiples and fractions of fractions 3.25



How could students use number lines to calculate?

Addition: $\frac{1}{5} + \frac{2}{5}$ $\frac{1}{5} + \frac{3}{10}$ $\frac{2}{3} + \frac{3}{4}$

Subtraction: $\frac{3}{5} - \frac{1}{5}$ $\frac{3}{5} - \frac{1}{10}$ $\frac{3}{4} - \frac{2}{3}$

Multiplication: $2 \times \frac{1}{3}$ $7 \times \frac{1}{3}$ $\frac{1}{4} \times 10$ $\frac{1}{3} \times \frac{1}{4}$

Division: $2 \div \frac{1}{3}$ $10 \div \frac{1}{4}$ $\frac{1}{3} \div 2$ $\frac{1}{3} \div \frac{1}{2}$

Slide 40: Multiples and fractions of fractions 3.25

Multiples and fractions of fractions 3.25

Summary

Linear models such as fraction walls and number lines are useful for:

- Comparing fractions
- Adding
- Subtracting
- Multiplying
- Dividing

... as they focus on fractions as one number,
(not two whole numbers)

Slide 41: Multiples and fractions of fractions 3.25

Other DEECD resources

- i [Digilearn](#)
- i [Scaffolding Numeracy in the Middle Years](#)
- i [Assessment for Common Misunderstandings](#)

Slide 42: Other DEECD resources

- Alternatively, all numbers can be thought of as translations. So think of $\frac{1}{5}$ as translating zero by $\frac{1}{5}$ to the right, and adding $\frac{2}{5}$ is thought of as translating a further distance to the right, and the result is translating zero by $\frac{3}{5}$ to the right.
- We could multiply fractions using the number lines by taking the fraction and moving it. For example $2 \times \frac{1}{3}$ actually means moving $\frac{1}{3}$ twice along the number line. In $\frac{1}{3} \times \frac{1}{4}$ it means to locate $\frac{1}{4}$ and partition it by 3 to form $\frac{1}{12}$. This illustrates that multiplying numbers does not necessarily make the number larger.
- With dividing fractions, for example 2 divide by $\frac{1}{3}$, we could phrase the problem by asking how many thirds there are in 2, using a linear model of thinking.
- Similarly in the challenging problem $\frac{1}{3}$ divided by $\frac{1}{2}$, teachers could pose the question 'how many halves are there in a third?' Then compare a linear model of a half with a model of a third. It will show that the solution will be a number less than one. By comparing the thirds with the halves, show the answer to be about two thirds.

Summary

Slide 41 provides a summary of the activities associated with this activity.

Linear models such as fraction walls and number lines are useful for:

- comparing fractions
- adding fractions
- subtracting fractions
- multiplying fractions
- dividing fractions.

Linear models focus on fractions as one number, not two whole numbers.

Other online resources

Slide 42 demonstrates other online resources which can support the teaching of fractions. These resources are hyperlinked on the slide including:

- [Digilearn](http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm)
(<http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm>)
- [Scaffolding Numeracy in the Middle Years](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmylplans.htm)
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmylplans.htm>)
- [Assessment for Common Misunderstandings](http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm)
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/common/default.htm>)

Digilearn – Cassowary fractions



Use slide 43: Cassowary fractions

Slide 43 is an illustration of Cassowary Fractions, an example of a Digilearn object.

- **Digilearn** (<http://www.education.vic.gov.au/studentlearning/teachingresources/elearning/digilearn.htm>)

Provide participants with the opportunity to 'play'. Cassowary fractions give students the opportunity to represent fractions as parts of shapes and model equivalent fractions.

Scaffolding Numeracy in the Middle Years Project

Slide 44 describes the Scaffolding Numeracy in the Middle Years Project (SNMP). This resource provides an assessment-guided approach to improving student numeracy outcomes by supporting teachers with assessment materials to assess students' multiplicative thinking. Students are located and identified against the Learning and Assessment Framework. This framework places students into one of eight zones, various materials assist teachers to encourage students to move from one zone to the next. Learning plans are provided to assist teachers in planning and with authentic tasks.

- **Learning Plans for Multiplicative Thinking** are designed to scaffold student learning from one zone of the Learning and Assessment Framework for Multiplicative Thinking (LAF) to the next.
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmylplans.htm>)
- **Authentic Tasks** are designed to encourage students to find ways of solving the task themselves, revealing different knowledge and skill sets.
(<http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/snmyauth.htm>)

The concept of fractions is introduced in Zone 2 (Intuitive Modelling) and is developed across the higher zones.

Please direct teachers to Prof. Di Siemon's discussion paper (Siemon, 2002):

- **'Partitioning – The missing link in building fraction knowledge & confidence'**.
(<http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/pppartitioning.pdf>)




Slide 43: Cassowary fractions – Digilearn

Scaffolding Numeracy in the Middle Years

An assessment-guided approach to improving student numeracy outcomes including:

- [Learning Plans for Multiplicative Thinking](#)
 - i Designed to scaffold student learning from one zone of the Learning and Assessment Framework for Multiplicative Thinking (LAF) to the next.
- [Authentic Tasks](#)
 - i Developed by teachers and students
 - i Designed to encourage students to find ways of solving the task themselves, revealing different knowledge and skill sets.



Slide 44: Scaffolding Numeracy in the Middle Years

Links to Principles of Learning and Teaching P-10

1. The learning environment is supportive and productive
 - 1.4 Ensures each student experiences success through structured support, the valuing of effort, and recognition of their work
2. Students' needs, backgrounds, perspectives and interests are reflected in the learning program
 - 3.1 Uses strategies that are flexible and responsive to the values, needs and interests of individual students
3. Assessment practices are an integral part of teaching and learning
 - 5.5 Uses evidence from assessment to inform planning and teaching.

Slide 45: Links to Principles of Learning and Teaching P-10

Module 3: Narrowing the achievement gap. Fractions

In conclusion

There are 8 more professional learning modules:

1. Overview of Learning in the Mathematics Domain
2. Overview of the Mathematics Developmental Continuum P - 10
3. Conducting practical and collaborative work (focus on contours)
4. Understanding students' mathematical thinking (focus on algebra the meaning of letters)
5. Using a range of strategies and resources (focus on percentages)
6. Learning through investigation (chance and variability)
7. Working mathematically (focus on a range of challenging problems)
8. Conclusion: Reviewing key ideas and supporting planning

Slide 46: Narrowing the achievement gap. Fractions

Reflection

Slide 45 challenges participants to reflect on how this module supports the three Principles of Learning and Teaching P-10 noted at the start of this presentation:

- The learning environment is supportive and productive.
- Students' needs, backgrounds, perspectives and interests are reflected in the learning program.
- Assessment practices are an integral part of teaching and learning.

Wrap up – Teaching Secondary Mathematics

Slide 46 'wraps up' this session through listing the other modules in this professional learning module.

These modules will describe many resources available on the DEECD website which will further provide strategies to help teachers:

- conduct formative assessment
- analyse student responses for thinking rather than a score
- learn about visual and organizing techniques
- share resources and evaluate them collaboratively.

There are eight more professional learning modules in the Teaching Secondary Mathematics resource:

1. Overview of learning in the Mathematics Domain
2. Overview of the Mathematics Developmental Continuum P-10
4. Conducting practical and collaborative work: Focus on contours
5. Understanding students' mathematical thinking: Focus on algebra and the meaning of letters
6. Using a range of strategies and resources: Focus on percentages
7. Learning through investigation: Focus on chance and variability
8. Working mathematically: Focus on a range of challenging problems
9. Conclusion: Planning for improvement in mathematics

Resource 1: Principles of Learning and Teaching P-12 and their components

- [Principles of Learning and Teaching P-12 and their components](http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm) (<http://www.education.vic.gov.au/studentlearning/teachingprinciples/principles/principlesandcomponents.htm>)

Students learn best when:

The learning environment is supportive and productive. In learning environments that reflect this principle the teacher:

- 1.1) builds positive relationships through knowing and valuing each student
- 1.2) promotes a culture of value and respect for individuals and their communities
- 1.3) uses strategies that promote students' self-confidence and willingness to take risks with their learning
- 1.4) ensures each student experiences success through structured support, the valuing of effort, and recognition of their work.

The learning environment promotes independence, interdependence and self motivation. In learning environments that reflect this principle the teacher:

- 2.1) encourages and supports students to take responsibility for their learning
- 2.2) uses strategies that build skills of productive collaboration.

Students' needs, backgrounds, perspectives and interests are reflected in the learning program. In learning environments that reflect this principle the teacher:

- 3.1) uses strategies that are flexible and responsive to the values, needs and interests of individual students
- 3.2) uses a range of strategies that support the different ways of thinking and learning
- 3.3) builds on students' prior experiences, knowledge and skills
- 3.4) capitalises on students' experience of a technology rich world.

Students are challenged and supported to develop deep levels of thinking and application. In learning environments that reflect this principle the teacher:

- 4.1) plans sequences to promote sustained learning that builds over time and emphasises connections between ideas
- 4.2) promotes substantive discussion of ideas
- 4.3) emphasises the quality of learning with high expectations of achievement
- 4.4) uses strategies that challenge and support students to question and reflect
- 4.5) uses strategies to develop investigating and problem solving skills
- 4.6) uses strategies to foster imagination and creativity.

Assessment practices are an integral part of teaching and learning. In learning environments that reflect this principle the teacher:

- 5.1) designs assessment practices that reflect the full range of learning program objectives
- 5.2) ensures that students receive frequent constructive feedback that supports further learning
- 5.3) makes assessment criteria explicit
- 5.4) uses assessment practices that encourage reflection and self assessment
- 5.5) uses evidence from assessment to inform planning and teaching.

Learning connects strongly with communities and practice beyond the classroom. In learning environments that reflect this principle the teacher:

- 6.1) supports students to engage with contemporary knowledge and practice
- 6.2) plans for students to interact with local and broader communities and community practices
- 6.3) uses technologies in ways that reflect professional and community practices.

Resource 2: The comparing fractions diagnostic test

Name: _____ Class: _____

For each pair of fractions, either CIRCLE the larger fraction, OR write = between them

$5 \frac{5}{8}$	$1 \frac{1}{3}$
$2 \frac{2}{3}$	$3 \frac{3}{4}$
$4 \frac{4}{6}$	$2 \frac{2}{3}$
$4 \frac{4}{5}$	$3 \frac{3}{8}$
$5 \frac{5}{7}$	$3 \frac{3}{8}$
$3 \frac{3}{10}$	$2 \frac{2}{5}$
$1 \frac{1}{4}$	$3 \frac{3}{4}$
$3 \frac{3}{5}$	$3 \frac{3}{5}$
$1 \frac{1}{3}$	$1 \frac{1}{4}$
$3 \frac{3}{5}$	$3 \frac{3}{4}$
$1 \frac{1}{6}$	$2 \frac{2}{12}$
$7 \frac{7}{8}$	$11 \frac{11}{12}$

Name: _____ Class: _____

For each pair of fractions, either CIRCLE the larger fraction, OR write = between them

$5 \frac{5}{8}$	$1 \frac{1}{3}$
$2 \frac{2}{3}$	$3 \frac{3}{4}$
$4 \frac{4}{6}$	$2 \frac{2}{3}$
$4 \frac{4}{5}$	$3 \frac{3}{8}$
$5 \frac{5}{7}$	$3 \frac{3}{8}$
$3 \frac{3}{10}$	$2 \frac{2}{5}$
$1 \frac{1}{4}$	$3 \frac{3}{4}$
$3 \frac{3}{5}$	$3 \frac{3}{5}$
$1 \frac{1}{3}$	$1 \frac{1}{4}$
$3 \frac{3}{5}$	$3 \frac{3}{4}$
$1 \frac{1}{6}$	$2 \frac{2}{12}$
$7 \frac{7}{8}$	$11 \frac{11}{12}$

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Resource 3: Definitions of strategies used by students in comparing fractions

Students use a range of strategies (which may correct or incorrect) when comparing the size of fractions. Here are definitions that will support teachers in being able to describe these strategies, and could be used to identify student misconceptions.

Benchmarking

Correct benchmarking is evidence that a student understands the relative size of fractions. It is also useful for comparing decimals. When benchmarking, a student will compare a fraction to another well known fraction, usually a half, or to a whole number such as zero or one.

For example, when comparing $\frac{5}{8}$ and $\frac{3}{7}$, $\frac{5}{8}$ is greater than a half, and $\frac{3}{7}$ is less than a half, therefore $\frac{5}{8}$ is bigger.

Residual thinking

The term residual refers to the amount which is required to build up to the whole. For example, $\frac{5}{8}$ has a residual of $\frac{1}{8}$.

This thinking is useful for comparing the size of fractions such as $\frac{5}{8}$ and $\frac{7}{8}$. $\frac{5}{8}$ has a residual of $\frac{1}{8}$ and $\frac{7}{8}$ has a residual of $\frac{1}{8}$. Therefore $\frac{7}{8}$ is a larger fraction because it has the smaller residual – the smaller amount to make the whole.

Sometimes, however, residual thinking alone is not an efficient strategy. When comparing $\frac{3}{7}$ and $\frac{5}{8}$, measuring up the residuals of $\frac{4}{7}$ and $\frac{3}{8}$ is not a helpful strategy as you are left with two residuals that are no easier to compare than the original pair. In this case, the residuals then need to be benchmarked to $\frac{1}{2}$ and 1 to prove which is larger. If students use residual thinking alone with this pair, it should be classified as an unsatisfactory explanation.

Residual thinking with equivalence

In order to use residual thinking effectively, creating an equivalent residual sometimes makes the justification clearer. For example, when comparing $\frac{3}{4}$ and $\frac{7}{9}$, a student may state that $\frac{3}{4}$ has a residual of $\frac{1}{4}$ or $\frac{2}{8}$. Therefore the residual for $\frac{7}{9}$ ($\frac{2}{9}$) is smaller than the residual for $\frac{3}{4}$ ($\frac{2}{8}$). The fraction with the smaller residual is the larger fraction.

Residual thinking with some other proof

Sometimes residual thinking alone is not the most appropriate strategy. For example, if a student uses residual thinking alone to compare $\frac{3}{4}$ and $\frac{7}{9}$, they must then convince the interviewer that they can justify which of the residuals is bigger ($\frac{1}{4}$ or $\frac{2}{9}$).

An example of residual with proof might be: 'I know one quarter of nine is more than 2 because 2 is a quarter of eight, so $\frac{2}{9}$ must be less than $\frac{1}{4}$ therefore $\frac{7}{9}$ is the bigger fraction'.

Gap thinking

This strategy is a form of whole number thinking, where the student compares the whole number difference between the numerator and denominator.

For example, $\frac{5}{8}$ and $\frac{7}{9}$ both have a difference of 'one' between the numerator and denominator. A student using 'gap thinking' might claim therefore that these fractions are the same size. When comparing $\frac{3}{4}$ and $\frac{7}{9}$, a student using gap thinking would choose $\frac{3}{4}$ as larger because it has a smaller 'gap', thereby choosing incorrectly.

There are some instances where 'gap thinking' will lead students to a correct choice, for example, comparing $\frac{3}{8}$ and $\frac{7}{8}$. This is an inappropriate strategy for comparing the size of fractions.

'Higher' or 'larger' numbers

With this strategy, fractions are deemed to be bigger if they contain larger digits. For example, when comparing $\frac{4}{7}$ and $\frac{4}{5}$ students may incorrectly claim that $\frac{4}{7}$ is larger because it has a 'larger number'. Also in comparing $\frac{2}{4}$ and $\frac{4}{8}$, a student would choose $\frac{4}{8}$ as it has 'higher numbers'.

Sometimes students will directly compare the numerators or denominators and conclude a larger digit at the top or bottom of a fraction means that it is a larger fraction.

This is an inappropriate strategy for comparing the size of fractions.

Ten tenths	
Nine ninths	
Eight eighths	
Seven sevenths	
Six sixths	
Five fifths	
Four quarters	
Three thirds	
Two halves	
One whole	

