Teaching Secondary Mathematics

Module 2

Overview of the Mathematics Developmental Continuum P–10

2

State Government Victoria
Department of Education and Early Childhood Development

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Introduction to Module 2: Overview of the Mathematics Developmental Continuum P–10

The Teaching Secondary Mathematics Resource provides support and guidelines for effective practice for classroom teachers and school leaders of mathematics, especially from Years 7–10.

This resource is for:

- all secondary mathematics classroom teachers to deepen their understanding of mathematics. This will inform their planning for mathematics and highlight opportunities for assessment of mathematics in other domains of the Victorian Essential Learning Standards (VELS)
- mathematics leaders in schools to plan opportunities for professional learning for the teachers of mathematics, in professional learning teams and/or for individual teachers
- differentiating the professional learning needs of mathematics teachers in schools.

Use of this module

This module allows for flexibility in modes of engagement with professional learning. The module booklet needs to be used in conjunction with the PowerPoint slides accompanying this resource.

Workshop approach

The materials of this module can be used by a presenter in a workshop for a school or a cluster of schools. A presenter, appointed from within or outside a school or cluster, is responsible for preparing presentations, facilitating discussions and outlining processes for collaborative planning.

Where a group is working collaboratively through these modules, a designated area is required for participants to share ideas, stories and samples in a climate of mutual respect. Regular after school meetings in a particular venue, such as the library, create a productive sense of community.

Individual use

The materials of this module are also suitable for private study and reflection. Individual users become both 'presenter' and 'participant'. While they are not able to engage in group discussions or whole-school planning, individual users can readily adapt the suggested group discussions and whole-school planning activities to private reflection, writing and classroom planning.

It is suggested that individuals identify a colleague or a buddy with whom to share their thoughts and to support the development of their understandings through ongoing dialogue. Individuals may complete all the modules or choose a combination depending on their interests or needs.

Web connections

The 'Teaching for Secondary Mathematics' resource is located at http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/default.htm.

Before commencing to plan any elements of the program, schools are strongly advised to visit the Mathematics Domain page to review the most up-to-date advice, resources and information relevant to each module of the program. Many elements of this resource are available online in a downloadable format. There are links to assist schools to locate relevant information.

- Mathematics Domain (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/default.htm)
  See the website for further details about this additional information or contact the student learning help desk on studentlearning@edumail.vic.gov.au

Content of the module

This module comprises Module 2: Overview of the Mathematics Developmental Continuum P–10 booklet and the accompanying slide presentations which can be downloaded from http://www.education.vic.gov.au/studentlearning/teachingresources/maths/teachsec/module2.htm

The following are included in this document:

- the **User's Guide** that assists the user through the professional learning program
- hard copies of the **slide presentations** and **resource sheets**
- selected **resources.**

Organisation of the module

Computer access is required for all modules. If a group is completing the modules, a data projector and tables that enable people to sit together and work collaboratively are also necessary. The presenter should encourage participants to raise questions throughout the ensuing presentation. This presentation should take approximately one hour, depending on the depth of discussion and types of activities that facilitators incorporate.

Required resources

This module requires the resources listed below.

Indicators of Progress:

- Indicator of progress: Adding and taking off a percentage: 5.25 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N52503P.htm)

Mapping the indicators of progress:

- Mathematics Developmental Continuum: Mapping the 'Indicators of progress' (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/indicatorsgrid.pdf)

Developmental Overview of Proportional Reasoning and Multiplicative thinking

- Overview of Proportional Reasoning and Multiplicative Thinking (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcdopropreason.pdf)

A copy of the flowchart showing the structure of the Mathematics Developmental Continuum:

- Mathematics Developmental Continuum P–10 flowchart (http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/mathscontinuum/mcflowchart.pdf)

Scavenger Hunt

- Resource 1 (attached)

Icons

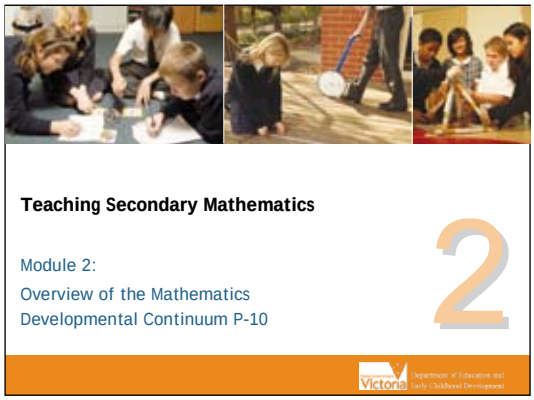
The following icons have been used in this workshop program:

Distribute handout:

Group discussion:

Group activity:

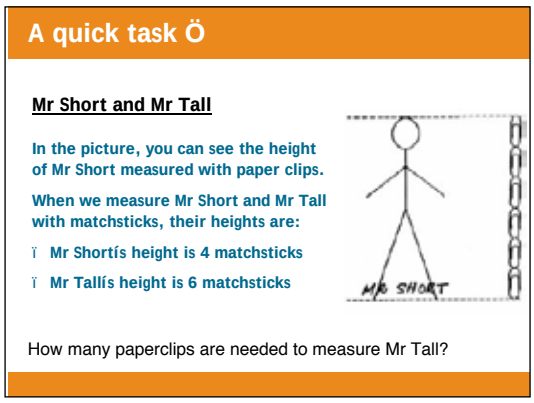
User's Guide to Module 2: Overview of the Mathematics Developmental Continuum P–10



Slide 1: Title slide

Slide 1 is the title slide.

The Mathematics Developmental Continuum P–10 is a resource to support teachers to scaffold student progress through the Victorian Essential Learning Standards. It provides powerful teaching strategies that can be selected from to provide focussed teaching.



Slide 2: A quick task

A quick task – Mr Short and Mr Tall

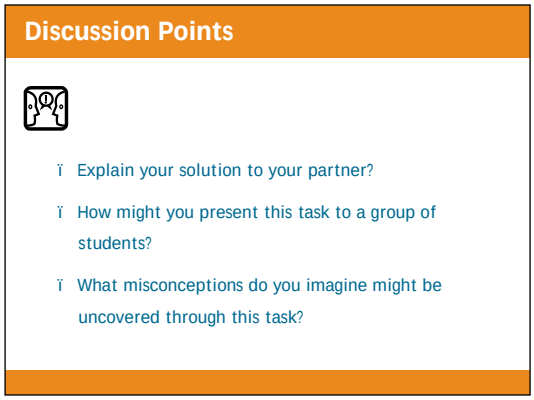
Use slides 2 and 3: Mr Short and Mr Tall

Slide 2 provides a problem. Invite participants to solve the following problem:

'How many paperclips are needed to measure Mr Tall?'

Slide 3 provides the following prompts to stimulate discussion.

- Explain your solution to a partner.
- How might you present this task to a group of students?
- What misconceptions do you imagine might be uncovered through this task?



Slide 3: Discussion Points

Purpose of the continuum

Slide 4 describes the purpose of the continuum.

The Mathematics Developmental Continuum P–10 is a resource that provides evidence-based indicators of progress to determine where a student has reached in their learning. It is linked to powerful teaching strategies that can be used to create powerful learning experiences.

The Continuum is aligned to the progression points and the standards for the Mathematics Domain of the Victorian Essential Learning Standards. The Continuum is also based on current research into student learning.

To use the Continuum effectively, it is best to read the contents with a specific student or group of students in mind. Teachers will use an on-balance judgement to determine the most appropriate level of student understanding and use this as the starting point for the resource.

The challenge for all teachers is to accurately identify where a student is located on the learning Continuum and to design learning experiences which enable all students to make progress.

The Continuum is not:

- a syllabus
- a lesson framework
- a set of activities to be used with the whole class.

About the Continuum

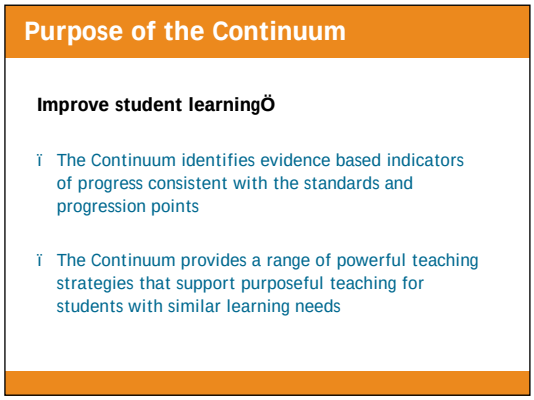
Slide 5 provides the following information about the Continuum.

The Continuum is a resource that supports improvement in teaching mathematics. It supports the Mathematics Standards and Progression Points and should be read in conjunction with these and the learning focus statements for each level. It was first published in mid-2006 and was created by a team from The University of Melbourne (K. Stacey, H. Chick, C. Pearn, L. Ball, V.Steinle), with I. Lowe (MAV) and P. Sullivan (Monash).

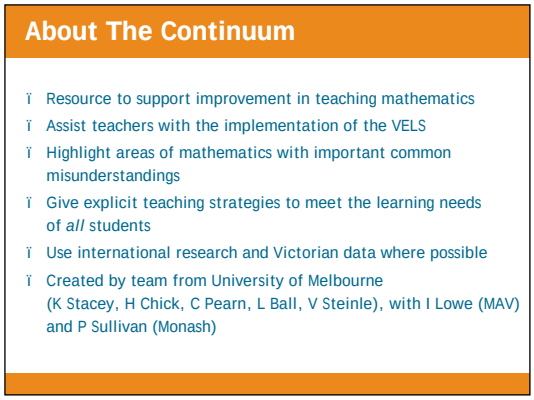
Assessment for learning

Slide 6 provides an overview of the process of assessment for learning. Assessment for learning is a cycle which aims to match instruction to a learner's needs. It begins with finding out what each student knows. This information is used in a formative way, to identify what the student needs to learn next.

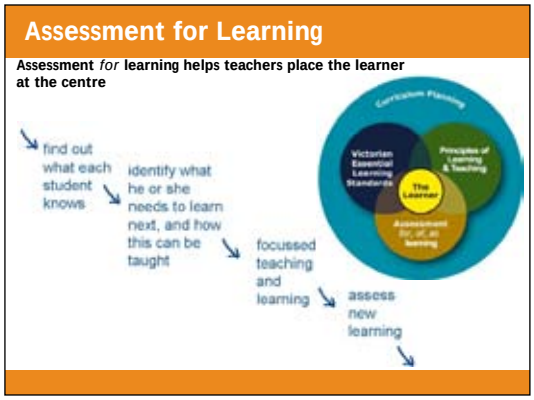
The Mathematics Developmental Continuum will provide focussed activities which give teachers insight and evidence of students' prior knowledge which will inform their teaching.



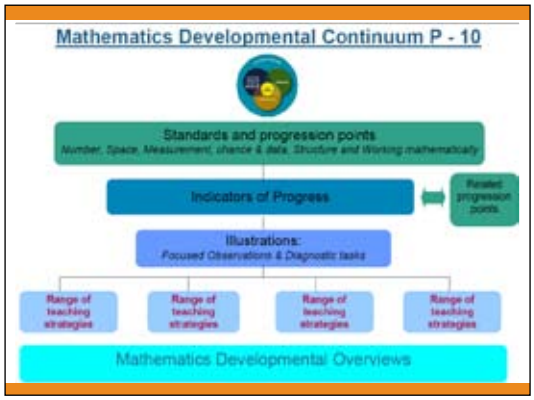
Slide 4: Purpose of the continuum]



Slide 5: About the Continuum



Slide 6: Assessment for learning



Slide 7: Mathematics Developmental Continuum P–10

Mathematics Developmental Continuum P–10

Provide participants with a copy of the structure of the Mathematics Developmental Continuum.

Slide 7 gives a visual diagram of the structure of the Continuum.

The Continuum is organised according to VELS levels and is accessed through the standards and progression points at these levels. The Mathematics Developmental Continuum groups ideas according to dimension (Number; Space; Measurement, chance and data; Structure; and Working mathematically).

The Continuum includes:

- Standards and progression points

  Teachers can identify where the student's current knowledge, skills and behaviours sit within the standards and progression points (assessment FOR learning) – then locate an Indicator of Progress in the Mathematics Developmental Continuum at this level. Teachers could also refer to related progression points to develop a holistic understanding of the prior learning and the future learning associated with the concept they are teaching.

- Indicators of Progress

  Indicators of progress are points on the learning continuum that highlight critical understandings required by students to progress through the standards.

- Illustrations

  Illustrations are focused observations and diagnostic tasks that are illustrative of student's prior knowledge, skills and behaviours. Teachers can use observation or a diagnostic task to delve more deeply into student thinking. This may identify any misconceptions present.

- Teaching strategies

  Teaching strategies are specific tasks designed for purposeful teaching to overcome misconceptions or develop deeper understanding of the mathematical concept.

- Developmental overviews

  Developmental overviews map out on one page the 'big ideas' in mathematics across the six levels.

- Related progression points

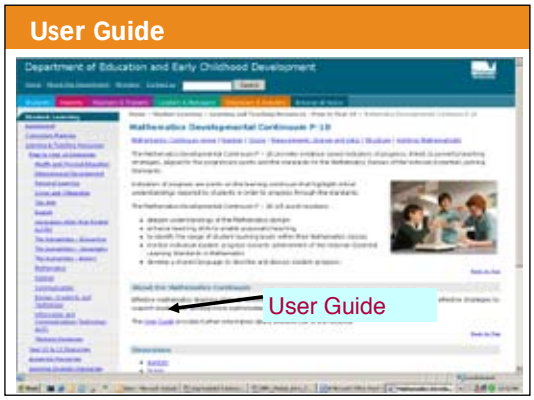
  Related progression points are a selection of progression points showing the developmental progression for one concept.

User Guide

Slide 8 shows how to access the User Guide from the Mathematics Development Continuum P–10 home page. The User Guide of the Mathematics Continuum can be accessed by clicking on the link on the Mathematics Domain page:

Mathematics Developmental Continuum P-10
(http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/default.htm)

Teachers should refer to the user guide which can provide quite detailed information about the Continuum.



Slide 8: User Guide

Indicator of progress

Slide 9 provides some more detailed information about the 'indicators of progress'.

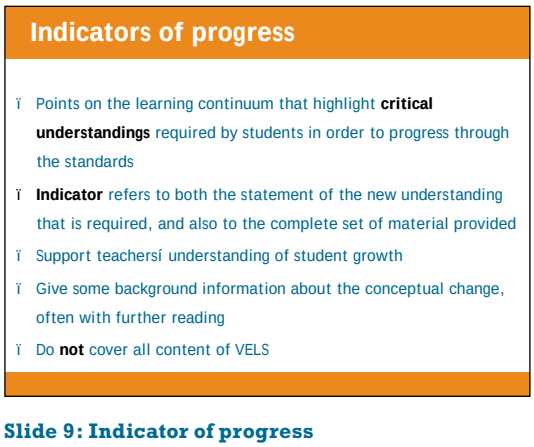
The Continuum is structured around indicators of progress. These indicators are points on the learning continuum that highlight 'critical understandings' required by students in order to progress in their mathematical learning. The indicators of progress often highlight common misconceptions of students.

The indicators of progress will support teachers to deepen their understanding of student growth in mathematics through research-based descriptors of achievement. It is important to note that they do not capture all aspects of learning within a dimension. They support purposeful teaching by informing teachers of students' mathematical thinking and the types of learning and teaching experiences necessary for further progress to occur. In this context teachers will use the indicators of progress as part of their ongoing assessment and monitoring.

Teachers do not report against the indicators of progress; they are not used in the formal reporting process. However, the indicators of progress and the illustrations that exemplify the prior knowledge, skills and behaviours of the students could assist teachers to describe student achievement when reporting on a student's mathematical knowledge and skills.

Most indicators are about conceptual change but there are some that are not. An example of an indicator of progress not about conceptual change is 'attaining fluency in multiplication tables'.

Further indicators of progress will be added to the Continuum over time, continuing to strengthen this online resource.



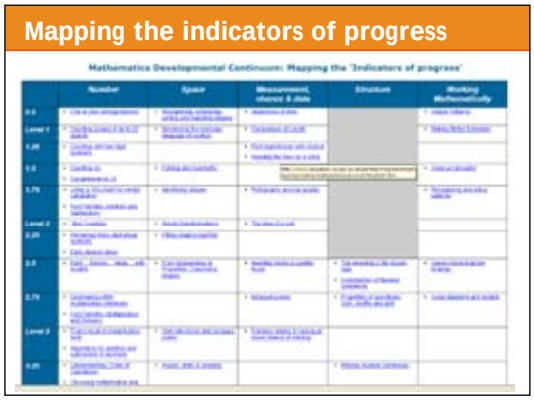
Slide 9: Indicator of progress

Mapping the indicators of progress

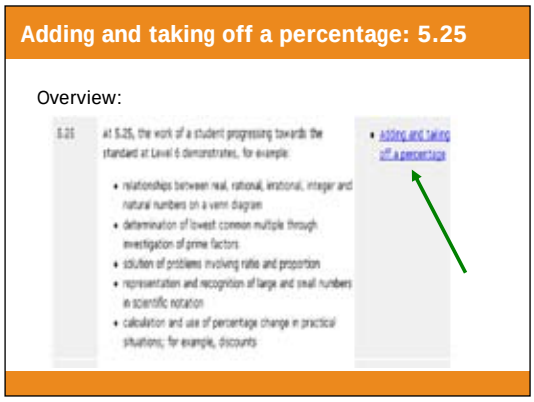
Slide 10 shows a table which maps all the indicators of progress against dimensions and VELS progression points. This is an alternative index page which links to all the indicators of progress within the Continuum across the dimensions and levels.

Provide participants with a copy of 'Mapping the indicators of progress' to illustrate the range of indicators of progress available to teachers.

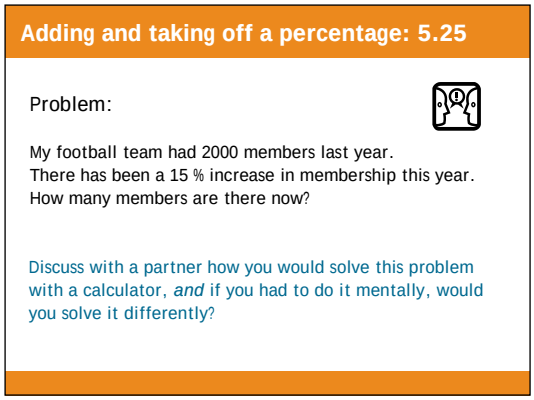
Allow participants a few minutes to scan this resource. If internet access is available, give participants the opportunity to access these documents through the Continuum using the hyperlinked headings.



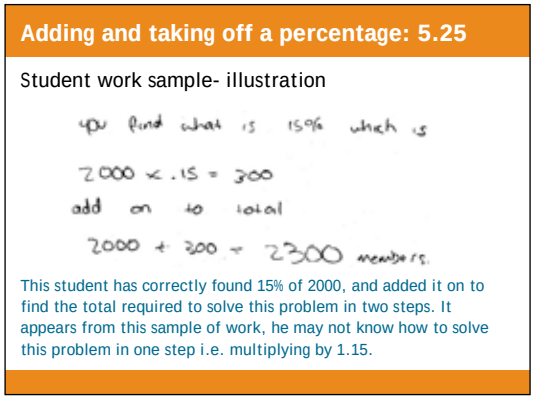
Slide 10: Mapping the indicators of progress



Slide 11: Adding and taking off a percentage: 5.25 — Overview



Slide 12: Adding and taking off a percentage: 5.25 — Problem



Slide 13: Adding and taking off a percentage: 5.25 — Student work sample illustrations

An indicator of progress- Adding and taking off a percentage: 5.25

Slide 11 provides an overview of this indicator of progress

Success at this level depends on students being able to add or subtract a percentage in one step by multiplication. A very common application of percentages is to increase or decrease a given amount by a percentage. Many students(and teachers) will calculate the mark-up or discount separately, and then adding or subtracting from the price.

A diagnostic task

Slide 12 provides the following problem:

- My football team had 2000 members last year. There has been a 15 % increase in membership this year. How many members are there now?

Use slide 12– Football team problem

Invite participants to solve this problem. Then ask them:

- How you would solve this problem with a calculator?
- If you had to do it mentally, would you solve it differently?

A student response to this diagnostic task

Slide 13 provides a student work sample which illustrates their thinking about this task. This would be a commonly used strategy to solving this problem.

This student has correctly found 15% of 2000, and added it on to find the total required to solve this problem in two steps. It appears from this sample of work that the student may not know how to solve this problem in one step i.e. multiplying by 1.15.

Illustrations

Slide 14 provides the purposes for using the 'illustration' which can be found within each indicator of progress. The previous slide showed an example of an illustration that is found on the Continuum (under Number, level 5.25). Illustrations have been developed to exemplify the prior knowledge, skills and behaviours of the students. The illustrations often give focused observations or diagnostic tasks. The illustrations are effective in determining the level students have achieved in their mathematical thinking and uncovering any misconceptions present.

Teaching strategies

Slide 15 provides an explanation of the teaching activities and strategies found within each indicator of progress.

Teaching strategies and activities are specific tasks that are designed to support conceptual understanding, building from the students' existing ideas. While more than one activity has been given to support student progress, these activities do not represent a unit plan or a whole class lesson.

The teaching strategies and activities within the Continuum can be used to demonstrate, highlight and exemplify teaching ideas, or as a model for designing similarly purposeful activities. In planning for differentiation (i.e. placing the learner at the centre) teachers should consider the range of understanding within the cohort and where students may be placed according to the standards and progression points.

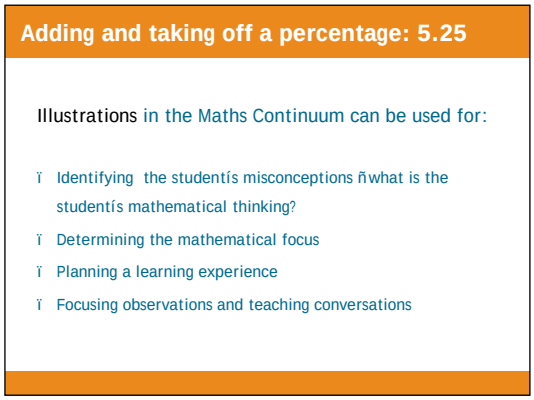
Teaching strategies may include generic ways of how problems of this nature can be effectively addressed by the teacher. The Continuum also provides ideas for very specific teaching, which can be used to address the skills in the indicator of progress. These tasks are designed for purposeful teaching.

The activities provided as teaching strategies:

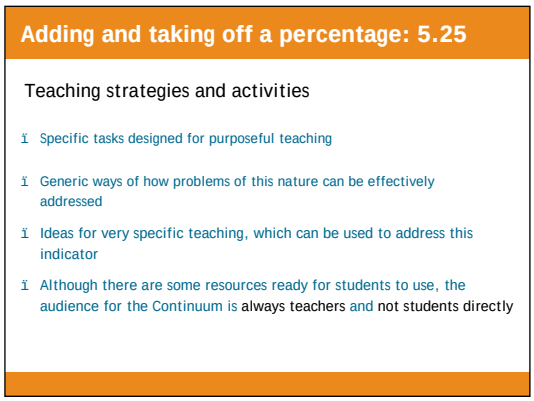
- use short lesson segments, sometimes whole lessons
- use teacher language – not children's language
- stress pedagogical content knowledge for teachers.

Teaching strategy activity

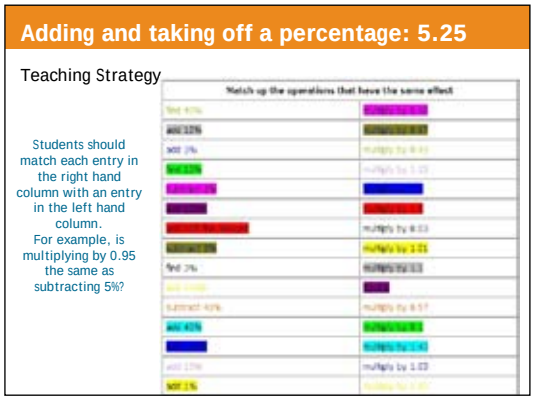
Slide 16 provides an example of an activity found in this indicator of progress. 'Adding and taking off a percentage: 5.25' demonstrates an activity where students have to match operations that have the same effect, e.g. multiplying by 0.95 is the same as subtracting by 5%.



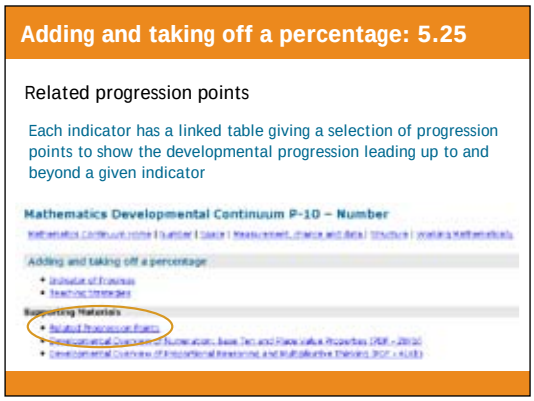
Slide 14: Adding and taking off a percentage: 5.25 — Illustrations



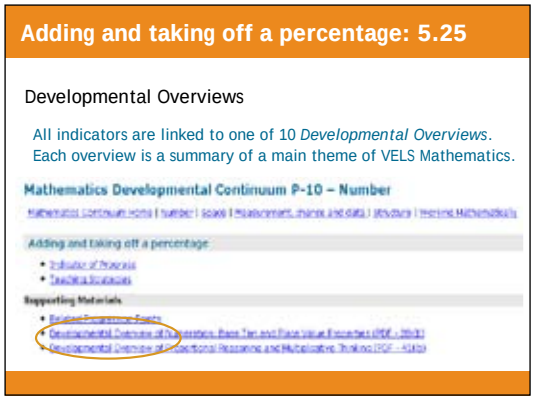
Slide 15: Adding and taking off a percentage: 5.25 — Teaching strategies



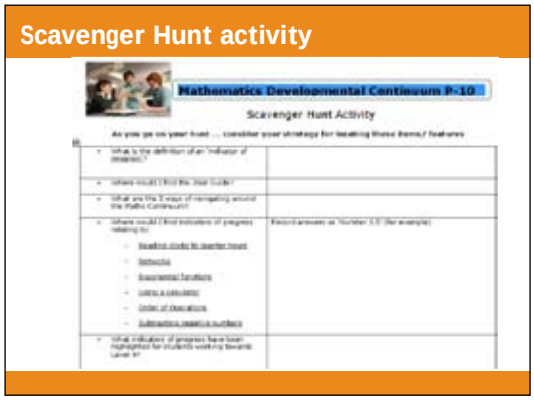
Slide 16: Adding and taking off a percentage: 5.25— Teaching strategy activity



Slide 17: Adding and taking off a percentage: 5.25— Teaching strategy activity



Slide 18: Adding and taking off a percentage: 5.25— Developmental Overviews



Slide 19: Scavenger Hunt activity

Related progression points

Slide 17 provides a visual representation of the related progression points which inform this indicator of progress. These can be accessed by the link found on the indicator of progress. Related progression points summarise key developmental learning and future learning associated with a specific indicator of progress. The points have been collated from the Standards and progression points documents.

Developmental overviews

Slide 18 highlights the 'developmental overview' link found on each indicator of progress.

Give participants a copy of the 'Overview of Proportional Reasoning and Multiplicative Thinking' resource. Give them the opportunity to read this resource and ask questions.

Teachers should refer to the developmental overviews and the related progression points when planning for differentiation to see how concept development occurs across dimensions and levels. These will be particularly useful for students achieving above or below the expected level.

Developmental overviews show the growth of important concepts and the interconnected nature of mathematics learning across dimensions. Ten developmental overviews have been created to demonstrate progression and development of the 'big ideas' in mathematics across the six VELS levels:

- Overview of Numeration: Base Ten and Place Value Properties
- Overview of Numbers and Operations
- Overview of Methods of Calculation
- Overview of Proportional Reasoning and Multiplicative Thinking
- Overview of Measurement Attributes
- Overview of Chance
- Overview of Data
- Overview of Space
- Overview of Working Mathematically
- Overview of Structure.

Scavenger hunt activity

Use slide 19: Scavenger Hunt and Resource 1: Scavenger Hunt

Slide 19 refers to the Scavenger Hunt. Invite participants to complete this activity

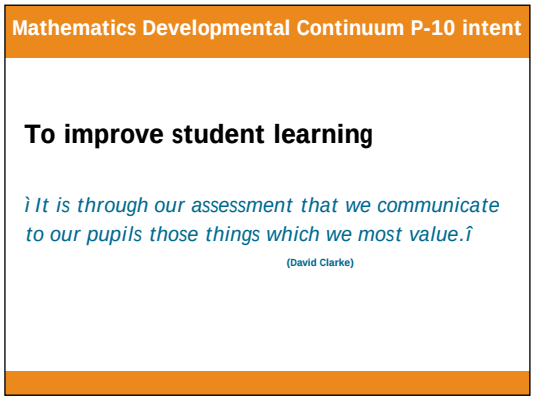
Summary

Slide 20 sums up the intent of this module. The intent of the Continuum is highlighted by a quote made by David Clarke: 'It is through our assessment that we communicate to our pupils those things which we most value.'

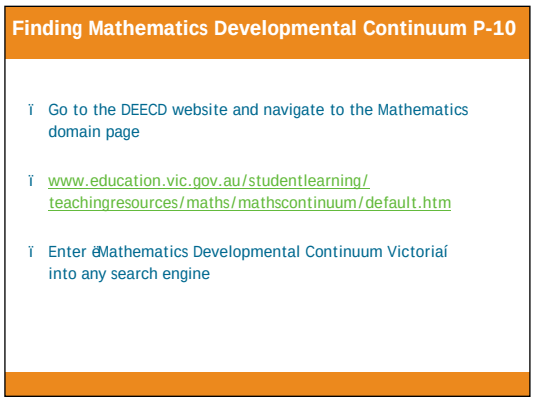
Conclusion

Slide 21 is the final slide which shows different ways to access the Continuum. There are eight further modules:

1. Overview of learning in the Mathematics Domain
3. Narrowing the achievement gap: Focus on fractions
4. Conducting practical and collaborative work: Focus on contours
5. Understanding students' mathematical thinking: Focus on algebra and the meaning of letters
6. Using a range of strategies and resources: Focus on percentages
7. Learning through investigation: Focus on chance and variability
8. Working mathematically: Focus on a range of challenging problems
9. Conclusion: Planning for improvement in mathematics



Slide 20: Mathematics Development Continuum P–10 intent



Slide 21: Finding Mathematics Development Continuum P–10

Resource 1: Scavenger hunt activity

As you go on your hunt … consider your strategy for locating these items/ features

Mathematics Developmental Continuum P–10

What is the definition of an 'indicator of progress'?	
Where would I find the User Guide?	
What are the 2 ways of navigating around the Maths Continuum?	
Where would I find indicators of progress relating to: • Reading clocks to quarter hours • Networks • Exponential functions • Using a calculator • Order of Operations • Subtracting negative numbers	Record answers as 'Number 1.5' (for example)
What indicators of progress have been highlighted for students working towards Level 4?	
List all 10 Developmental Overviews	1. 2. 3. 4. 5. 6. 7. 8. 9. 10.

List 4 indicators of progress that relate to learning about fractions	
Many indicators of progress include 'more research' links to deepen teachers understanding of the research supporting the ideas in the Continuum. • To find out more about proportional reasoning, read: • Number - About proportional reasoning: 5.5 (http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/number/N55004G.htm) • Which famous researcher is referenced here?	
Review the 'related progression points' for 'Making better estimates: 1.0'. Where does estimating relate to mathematics development in the VELS?	
List at least 1 indicator of progress from each dimension that has a DigiLearn link. (**NB** only indicators up to Level 4 at this stage)	