Department of Education and Early Childhood Development

Key Characteristics of Effective Numeracy Teaching 7-10

Differentiating support for all students

Published for Student Learning Division
Department of Education and
Early Childhood Development
Melbourne
December 2009

Authorised by the Department of Education
and Early Childhood Development,
2 Treasury Place, East Melbourne, Victoria, 3002.

ISBN 978-0-7594-0595-0

This document is also available on the internet at
www.education.vic.gov.au/studentlearning/litnum

Contents

Introduction

The international evidence-base has reached consensus about the significant influence teachers have in the classroom and the knowledge and the skill base required to teach all students well. In order to improve teacher practice, teachers must not only understand what it looks like to improve in different domains of learning but they must also believe they are capable of improving their practice.
(*e*^{5} *Instructional Model*, DEECD, 2009)

The Key Characteristics of Effective Numeracy Teaching 7–10 provides a framework to support improvement in teaching and learning. The purpose of this document is to:

- Articulate effective practice in numeracy teaching that supports differentiation within the classroom
- Build knowledge and capacity in numeracy teaching and learning with a focus on student improvement
- Establish a common, shared language to describe effective practice in numeracy teaching

The document is applicable to a range of audiences:

- Classroom teachers: to enable them to guide their planning and instruction, based on student assessment information; to build discipline and pedagogical content knowledge; to focus discussion in professional learning teams
- Principals with the leadership team: to enable substantive conversations to take place with teachers on effective practice in numeracy teaching, to inform Strategic and Annual Implementation Plans including professional learning opportunities for the whole staff, teams of teachers and individual teachers
- Regional Network Leaders and other regional staff: to support articulation of clear and consistent messages on numeracy teaching and learning

Students within each class have a wide range of numeracy experiences, abilities and backgrounds. This provides a significant challenge as teachers need to ensure that all students develop the knowledge and skills to continually improve their numeracy.

International research indicates that teachers need to consider systematically 'two phases of teaching' to respond effectively to the diversity of student learning needs and aspirations at all stages of learning:

- The **first** phase - quality differentiated classroom teaching for all students, and
- The **second** phase - additional short-term intervention for students not achieving the expected level in numeracy in order to accelerate their learning.

The Key Characteristics of Effective Numeracy Teaching 7-10 specifies first phase, quality differentiated numeracy teaching for all students. This requires that ***the teacher proactively plans and carries out varied approaches to content, process, and product in anticipation of and response to student differences in readiness, interest, and learning needs*** (*e*^{5} *Instructional Model*, DEECD, 2009).

To plan effectively for differentiated teaching, teachers draw on their knowledge and understanding of key theories and knowledge of the Victorian Essential Learning Standards (VELS) Mathematics standards. This is important for responsive and focused teaching for students at all levels. Teachers also require knowledge of their students, including their interests and prior knowledge, English language proficiency, and identified learning strengths and areas for improvement.

Teachers identify a specific focus for teaching using assessment data. This includes using assessment information to determine purposeful teaching strategies that meet the identified needs of individual students.

They use of the *e*^{5} *Instructional Model* informs the delivery of the planned curriculum. Teacher's assess and monitor students' progress, adjusting their instruction to meet the needs of students when appropriate.

The Key Characteristics of Effective Numeracy Teaching 7-10 is a companion document to the *Victorian Literacy and Numeracy Statement* and the other three *Key Characteristics of Effective Literacy Teaching P-6 and 7-10 and Key Characteristics of Effective Numeracy Teaching P-6*. These documents were informed by

research and developed through consultation with classroom practitioners, literacy and numeracy experts, coaches and regional personnel.

In development is the Numeracy Teaching within Domains document. The purpose of the document is to raise awareness of the numeracy demands within identified VELS domains. This is new work and as such any feedback or advice is welcome.

For the Mathematics domain it is broken into Years 7 and 8, and Years 9 and 10. Each containing:

- **Teacher knowledge** - disciplinary and pedagogical content knowledge that teachers require including VELS for each domain, Mathematics Developmental Continuum P-10, and information on particular cohorts of students
- **Numeracy focus** - advice on essential knowledge and skills on which teachers need to focus
- **Assessment** - advice on key assessments and their timing to inform teaching including Victorian Curriculum and Assessment Authority (VCAA) On Demand Tests, Mathematics Fractions and Decimals Interview, Scaffolding Numeracy and National Assessment Program Literacy and Numeracy (NAPLAN) tests
- **Planning and instruction** - advice on organisational structures, recommended teaching strategies. This section refers to the e^5 Instructional Model which provides a framework to support differentiated, purposeful teaching for students at all levels.

Suggested Readings

- Mathematics domain page

Access information on the Victorian Essential Learning Standards and Mathematics Development Continuum P–10 plus support for numeracy teachers including research, digital learning resources, assessment maps and sample tasks, web links, professional learning tools and other numeracy resources.

- Research eLert – Numeracy in practice: teaching, learning and using mathematics (2009)

Report focusing on research and links related to the characteristics of effective numeracy teaching and addressing key classroom issues including: what to teach, how to teach numeracy, how to cater for diversity and how to make best use of technology.

- Researching Numeracy Teaching Approaches in Primary Schools (2002-2003)

Research identifying twelve scaffolding practices that contribute to improved student learning outcomes and describing a range of communicative practices that teachers use to support students' mathematics learning.

- Middle Years Numeracy Research Project (MYNRP)

Commissioned to inform the development of a strategic and coordinated approach to the teaching and learning of numeracy for students in Years 5–9, the final report for this research was published in 2001.

Years 7 and 8

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain learning focus statements and standards and progression points – levels 3, 4, 5 and 6 to support planning for differentiated teaching
- knowledge of the Mathematics Developmental Continuum P–10, the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 3–6 of the Developmental Overviews
- knowledge of the Big Ideas Linked to the Fractions and Decimals Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from Koorie, ESL and or low SES backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- develop students' understanding of the meaning and use of digits, natural numbers, integers and rational numbers and the importance of the placement of zero (especially in decimals) and teach students to identify complete factor sets for natural numbers and to express natural numbers as products of powers of primes
- develop students' knowledge of and skill in using fractions and their reciprocals given in the simplest form (e.g. $\frac{1}{2} = \frac{5}{10} = \frac{15}{30}$) and knowledge of decimal equivalents for the unit fractions (e.g. $\frac{1}{4}$ = 0.25, $\frac{1}{5}$ = 0.2) and engage students in evaluating natural numbers and simple fractions given in base exponent form, in calculating the equivalent decimals, ratios and percentages and in using symbols to represent rational numbers
- use students' knowledge of perfect squares when calculating and estimating squares and square roots of numbers and cube and cube roots to a specified degree of accuracy, increase student knowledge of the simple powers of 2, 3 and 5 and teach students to generalise from perfect square and difference of two square number patterns
- engage students in identifying collections of numbers as subsets of integers, natural, rational and real numbers, teach students number sets (empty, power and finite), list the elements of the power set (set of all subsets) of a given finite set and to comprehend the partial-order relationship between these subsets with respect to inclusion, develop students' understanding of ratios as set:set and subset:set comparisons and the use of diagrams and graphs to illustrate the relationships between sets (intersection, union, inclusion and complement)
- teach students to use variables in general mathematical statements and substitute numbers for variables, to identify the correspondence of a function between two sets (one-to-one or many-to-one) and to represent it by a table of values, a graph and by a rule, to describe and specify the independent variable (and its domain) and the dependent variable (and its range) of a function and to construct tables of values and graphs for linear functions and to model various situations
- teach students to apply number properties (commutative, associative and distributive) in mental and written computations, to use exponent laws for multiplication and division of power terms and to become fluent at recognising and manipulating symbols, formulae and algebraic expressions and solving simple equations using tables, graphs and inverse operations
- engage students in building efficient mental, written and technology-based strategies for arithmetic computation and in using technology for a range of purposes
- introduce students to binary notation and the addition and subtraction of natural numbers in binary form

- teach students to identify parallel lines and use the transversals of these lines to calculate alternate, supplementary, corresponding and allied angles
- teach students about the properties of quadrilaterals and congruent and similar triangles and engage students in solving geometrical problems, applying these properties and justifying their results and in explaining geometric propositions
- engage students in visualising and constructing simple 3-D objects from 2-D nets and in using single-point perspective to make a 2-D representation of a simple 3-D object
- teach students to recognise and apply simple geometric transformations of the number plane (translation, reflection, rotation and dilation) as well as combinations of these (including their inverses) and to use precise map references, contour lines, bearings and Cartesian coordinates and more complex map scales
- teach students to measure using suitable units and estimate the accuracy of measurements, to give suitable lower and upper bounds for measurement values in context and to calculate absolute percentage error of estimated values
- engage students in using measurement formulas to calculate area and perimeter of circles, triangles and parallelograms and simple composite shapes and to calculate surface area and volume of prisms and cylinders
- teach students to identify empirical probability as long-run relative frequency, to calculate theoretical probabilities by dividing the number of possible successful outcomes by the total number of possible outcomes and to use tree diagrams to investigate the probability of outcomes in simple multiple event trials
- teach students to use appropriate technology to generate random numbers in the conduct of simple simulations and engage students in analysing the reasonableness of points of view, procedures and results according to given criteria and in identifying limitations and/or constraints in context
- teach students to tabulate, display and organise discrete and continuous data using technology for larger data sets, to represent uni-variate data in appropriate graphical forms, to calculate summary statistics for measures of centre and to make simple inferences based on this data
- teach students to test the validity of statements formed by the use of the connectives (and, or, not) and quantifiers (none, some, all) and to apply these to sets (with one and two attributes) and database searches
- teach students to formulate conjectures, to follow simple mathematical deductions, to develop simple models for real situations, to develop generalisations by abstracting the features from situations and expressing these in words and symbols and to predict using interpolation and extrapolation.

For students achieving above the expected level, teachers should refer to the appropriate VELS standards and progression points.

For students needing additional assistance, teachers should refer to earlier VELS standards and progression points.

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- use information from transition statements and data including VELS teacher judgements, VCAA On Demand testing and Year 7 NAPLAN results to understand the starting point for each student
- administer and analyse VCAA On Demand Adaptive Testing – a range of computer-based assessments which identify a student's achievement level (use of this tool is to identify the spread of achievement within the class and then to use the progress test to gain more detailed information about individual students)
- administer and analyse Fractions and Decimals Online Interview.

For students achieving below the expected level:

- administer the assessment materials to assess student's multiplicative thinking (Scaffolding Numeracy in the Middle Years) and analyse the assessment outcomes using the Learning and Assessment Framework

for Multiplicative Thinking (LAF) and plan for future learning by using learning plans.

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- administer and analyse VCAA On Demand Progress tests which are linear tests designed to measure outcomes against the VELS
- administer and analyse the Assessment for Common Misunderstandings for identified students. This assessment task addresses the key areas of number:
    - LEVEL 5 – Proportional reasoning, extending what is known about multiplication and division beyond rule-based procedures to solve problems involving fractions, decimals, percent, ratio, rate and proportion
    - LEVEL 4 – Partitioning, the missing link in building common fraction and decimal knowledge and confidence
    - LEVEL 3 – Multiplicative thinking, the key to understanding rational number and developing efficient mental and written computation strategies in later years
- regularly observe students working mathematically across all dimensions within the class context, particularly during independent numeracy, and document progress against the VELS standards and progression points
- provide students with regular opportunities for self assessment and self reflection
- use the VCAA mathematics assessment maps to help moderate student work
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the e^5 Instructional Model
- dedicate four hours weekly (as a minimum) to explicit numeracy teaching
- use a range of flexible student groupings ensuring appropriate level of differentiated teacher support including whole class focus, small groups, independent activities and whole class reflection and analysis
- organise learning spaces to support differentiated teaching and collaborative learning
- provide independent time so students can practise what they know and act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic numeracy tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning and feedback by the teacher and explanation of reasoning and methods by the student.

Recommended teaching strategies

Effective numeracy teachers:

- explicitly teach students strategies to approach mathematical problems
- select appropriate teaching strategies including classroom activities after analysis of the Fractions and Decimal Online Interview

- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the twelve scaffolding practices that meet the learning needs of all students in the most appropriate way possible. These are used throughout the five phases of instruction as described in the e^5 Instructional Model.

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

Years 9 and 10

Teacher knowledge

Effective teachers require:

- thorough knowledge of VELS mathematics domain learning focus statements and standards and progression points – levels 4, 5 and 6 to support planning for differentiated teaching
- knowledge of the Mathematics Developmental Continuum P–10, the indicators of progress, teaching strategies and activities within the Continuum
- knowledge of key mathematical concepts in levels 4–6 of the Developmental Overviews
- knowledge of the Big Ideas Linked to the Fractions and Decimals Interview
- knowledge of the particular needs that students may have in relation to English language and numeracy, including students from Koorie, ESL and or low SES backgrounds.

Numeracy focus

Effective teachers determine the numeracy focus by referencing the VELS mathematics standards and progression points.

For students achieving at the expected level, effective teachers:

- develop students' understanding of the set of real numbers (natural, integer, rational and irrational) and teach them to classify and describe the properties of the real number system and the subsets of rational and irrational numbers, identifying subsets (as discrete or continuous, finite or infinite), providing examples of their elements and applying these to functions, relations and the solution of related equations
- build students' skills at performing computations (using both mental and written algorithms) involving natural numbers, integers and finite decimals, fractions and irrational numbers (such as square roots) and build skills at solving percentage problems (such as adding and taking off a percentage) and solving ratio and proportion problems in a variety of ways
- engage students in using appropriate estimates to evaluate the reasonableness of the results of calculations involving rational and irrational numbers and the decimal approximations for them and extend students' skills at forming and testing conjectures
- teach students to apply the algebraic properties (closure, associative, commutative, identity, inverse and distributive) to computations with number, to rearrange formulas and simplify and verify the equivalence or otherwise of algebraic expressions involving real variables (linear, square, cube, exponent, and reciprocal)
- engage students in representing rational numbers as fractions and decimals and specifying decimal approximations for the square root of primes, rational numbers that are not perfect squares, the golden ratio (θ) and simple fractions of π to a particular decimal place accuracy
- teach students to simplify surds (irrational numbers) and rationalise expressions with surds in the denominator and to recognise and calculate with the exact value of surds as a mathematical object rather than only considering its decimal approximation and engage students in using irrational numbers and common surds in calculations in both exact and approximate form
- teach students to use the Euclidean division algorithm to find the greatest common divisor (also called the highest common factor) of two natural numbers and to express relations between sets (membership, complement, intersection, union, subset) for up to three sets, illustrating this in a variety of ways
- teach students to identify and represent linear, quadratic and exponential functions by table, rule and graph using independent and dependent variables, domain and range and teach students to distinguish between, use and interpret these types of functions by testing for constant first or second difference or constant ratio between consecutive terms and modelling a range of contexts
- teach students to solve equations of the form f(x) = k, where k is a real and simultaneous linear equation in two variables using algebraic, numerical (systematic guess, check and refine or bisection) and graphical methods and teach students to recognise and explain the roles of the relevant constants in the relationships f(x) = a x + c, with reference to gradient and y axis intercept f(x) = $a(x + b)^2$ + c and f(x) = ca^x

- teach students to use perspective, isometric drawings, nets and computer-generated images to make representations, to recognise and describe boundaries, surfaces and interiors of common plane and 3-D shapes (including cylinders, spheres, cones, prisms and polyhedra) and to explore the effect of changing the scale of one characteristic of 2-D and 3-D shapes on related characteristics
- teach students to recognise features of circles (centre, radius, diameter, chord, arc, semi-circle, circumference, segment, sector and tangent) and to use associated angle properties and engage students in exploring properties of spheres
- teach students to calculate constant rates and interpret and use mensuration formulas for calculating the perimeter, surface area and volume of familiar 2-D and 3-D shapes and simple composites and engage students in estimating and measuring length, area, surface area, mass, volume, capacity, rates and angle (degrees and radians), selecting and using appropriate units, converting between units as required and deciding on acceptable levels of error in a given situation
- teach students to apply isometric and similarity transformations of geometric shapes in the plane, to identify points that are invariant under a given transformation and to use latitude and longitude to locate places on the Earth's surface and measure distances between places using great circles
- teach students to use Pythagoras' theorem and trigonometric ratios (sine, cosine, tangent) to obtain lengths of sides, angles and the area of right-angle triangles
- teach students to estimate probabilities based on data, to assign and justify subjective probabilities in familiar situations and to calculate probabilities for complementary, mutually exclusive and compound events (defined using and, or and not) and engage students in listing event spaces (for up to three events) by lists, grids, tree diagrams, Venn diagrams and Karnaugh maps (two-way tables) and classifying events as dependent or independent
- teach students to understand the difference between a population and a sample, to generate data using surveys, experiments and sampling procedures, to calculate summary statistics for centrality (mode, median, mean), spread (box plot, inter-quartile range, outliers) and association (by eye estimation of the line of best fit from a scatter plot), to distinguish informally between association and causal relationship in bi-variate data and to make predictions based on an estimated line of best fit for scatter-plot data with strong association between two variables
- engage students in choosing, using and developing models and procedures to investigate and solve problems set in a wide range of practical, theoretical and historical contexts and engage students in formulating and testing conjectures, generalisations and mathematical arguments in natural language and symbolic form (generalising from one situation to another) then in investigating further by changing the initial constraints or other boundary conditions and judging the reasonableness of their results
- engage students in selecting and using technology in various combinations to assist in inquiry, to manipulate and represent data, to analyse functions and to carry out symbolic manipulation and engage students in using geometry software or graphics calculators to create geometric objects and transform them, taking into account invariance under transformation.

For students achieving above the expected level, teachers should refer to the appropriate VELS standards and progression points.

For students needing additional assistance, teachers should refer to earlier VELS standards and progression points.

Assessment

Effective teachers continuously monitor and track the progress of individual students.

At the beginning of the year, effective teachers:

- use data including: VELS teacher judgements, VCAA on demand testing and Year 9 NAPLAN results to understand the starting point for each student
- administer and analyse VCAA On Demand Adaptive Testing – a range of computer-based assessments which identify a student's achievement level (use of this tool is to identify the spread of achievement within the class and then to use the progress test to gain more detailed information about individual students).

For students achieving below the expected level:

- administer and analyse Fractions and Decimals Online Interview
- administer the assessment materials to assess students' multiplicative thinking. Scaffolding Numeracy in the Middle Years, analyse the assessment outcomes using Learning and Assessment Framework for Multiplicative Thinking and plan for future learning by using learning plans.

Throughout the year, effective teachers:

- schedule and document ongoing assessment to track individual student's progress
- administer and analyse VCAA On Demand Progress tests which are linear tests designed to measure outcomes against the VELS
- administer and analyse the Assessment for Common Misunderstandings for identified students. This assessment task addresses key areas of number:
    - LEVEL 6 – Generalising, skills and strategies to support equivalence, recognition of number properties and patterns, and the use of algebraic text without which it is impossible to engage with broader curricula expectations at this level
    - LEVEL 5 – Proportional reasoning, extending what is known about multiplication and division beyond rule-based procedures to solve problems involving fractions, decimals, percent, ratio, rate and proportion
    - LEVEL 4 – Partitioning, the missing link in building common fraction and decimal knowledge and confidence
- regularly observe students working mathematically across all dimensions within the class context, particularly during independent numeracy, and document progress against the VELS standards and progression points
- provide students with regular opportunities for self assessment and self reflection
- use the VCAA mathematics assessment maps to help moderate student work
- provide timely and frequent feedback on assessment tasks.

At the end of each semester, effective teachers:

- integrate evidence collected throughout the semester to make on-balance judgements against VELS mathematics standards and progression points.

Planning and instruction

Assessment data is the starting point for curriculum planning and differentiated instruction.

Effective teachers:

- demonstrate capabilities as described in the e^5 Instructional Model
- dedicate four hours weekly (as a minimum) to explicit numeracy teaching
- use a range of flexible student groupings ensuring appropriate level of differentiated teacher support, including whole class focus, small groups, independent activities and whole class reflection and analysis
- organise learning spaces to support differentiated teaching and collaborative learning
- provide independent time so student can practise what they know and act on reflection and feedback, giving opportunities to make knowledge and skills automatic
- develop mathematical language by explicitly introducing new terms and symbols and expecting and encouraging correct use, making connections between language, symbols and materials
- provide opportunities and resources for students to manipulate concrete materials
- structure purposeful, authentic tasks that allow different possibilities, strategies and products to emerge and encourage higher order thinking skills
- develop numeracy understanding through strategic questioning and feedback by the teacher and explanation of reasoning and methods by the student

Recommended teaching strategies

- explicitly teach students strategies to approach mathematical problems
- select appropriate teaching strategies including classroom activities after analysis of the Fractions and Decimal Online Interview
- engage students in discussion, reflection and active construction throughout sessions to extend their thinking by building on their contributions and questions and to resolve misconceptions
- use a range of practices selected from the twelve scaffolding practices that meet the learning needs of all students in the most appropriate way possible. These are used throughout the five phases of instruction as described in the e^5 Instructional Model.

Selecting materials

Effective teachers select a range of materials that are:

- based on the students' developmental stages and the mathematics being explored
- appropriate to the learning context
- socially and culturally inclusive.

Bibliography

Australian Association for Mathematics Teaching 2002, *Standards for excellence in teaching mathematics in Australian schools,* AAMT Inc, viewed 25 September, 2009 <http://www.aamt.edu.au/Standards>.

Charles, R I 2005, 'Big ideas and understandings as the foundation for elementary and middle school mathematics'*, NCSM journal*, vol. 8, no.1, viewed 25 September, 2009 <http://www.authenticeducation.org/bigideas/sample_units/math_samples/BigIdeas_NCSM_Spr05v7.pdf>.

Commonwealth of Australia 2008, *National numeracy review report*, Human Capital Working Group, Council of Australian Governments, Canberra.

Department of Education and Early Childhood Development 2006, *Developmental overviews,* Office for Government School Education, Department of Education and Early Childhood Development, viewed 11 September, 2009 <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/overview.htm>.

Department of Education and Early Childhood Development 2009, *Education for global and multicultural citizenship – A strategy for Victorian government schools 2009 – 2013*. Office for Government School Education, Department of Education and Early Childhood Development, viewed on 4 November, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/commrel/policy/multicultural-ed-strategy.pdf >

Department of Education and Early Childhood Development 2009, *e*^{5} *Instructional model*, Office for Government School Education, Department of Education and Early Childhood Development, viewed 8 August, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/innovation/e5/E5_A1PosterTable4.pdf>

Department of Education and Early Childhood Development 2009, *Energising science and mathematics in Victoria*, Education Policy and Research Division, Office for Policy, Research and Innovation, Department of Education and Early Childhood Development, viewed 25 August, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/govrel/Policy/energising-scimaths-ed.pdf>.

Department of Education and Early Childhood Development 2007, *Fractions and decimals online interview,* Office Government School Education, Department of Education and Early Childhood Development, viewed 25 August, 2009 <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/interview/fractions.htm>.

Department of Education and Early Childhood Development 2009, Language Support program professional learning guide, Office for Government School Education, Department of Education and Early Childhood Development, Melbourne.

Department of Education and Early Childhood Development 2006, *Mathematics developmental continuum P–10*, Office for Government School Education, Department of Education and Early Childhood Development, viewed 10 August 2009 <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/mathscontinuum/>.

Department of Education and Early Childhood Development 2009, *Numeracy in practice: teaching, learning and using mathematics,* Education Policy and Research Division, Office for Policy, Research and Innovation, Department of Education and Early Childhood Development, viewed 14 August, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/publ/research/nws/Numeracy_in_practice_Paper_No_18.pdf>.

Department of Education and Early Childhood Development 2006, *Scaffolding numeracy in the middle years,* Office for Government School Education, Department of Education and Early Childhood Development, viewed 11 September, 2009 <http://www.education.vic.gov.au/studentlearning/teachingresources/maths/snmy/default.htm>.

Department of Education and Early Childhood Development 2009, *Victorian literacy and numeracy statement*, Education Policy and Research Division, Office for Policy, Research and Innovation,

Department of Education and Early Childhood Development, viewed 11 December, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/litnumstmt.pdf >

Department of Education and Early Childhood Development 2008, *Wannik: Learning together – Journey to our future*, Office for Policy, Research and Innovation, Department of Education and Early Childhood Development, viewed 10 September, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/govrel/Policy/wannik.pdf> .

National Centre for Excellence in the Teaching of Mathematics (NCETM) 2008, *Mathematics matters: Final report,* National Centre for Excellence in the Teaching of Mathematics, viewed 25 September, 2009 <https://www.ncetm.org.uk/files/309231/Mathematics+Matters+Final+Report.pdf>

Siemon, D 2007, *Developing the 'big' ideas in number*, viewed 20 October, 2009 <http://www.eduweb.vic.gov.au/edulibrary/public/teachlearn/student/devbigideas.pdf>.

Victorian Curriculum Assessment Authority 2005, *Victorian Essential Learning Standards*, viewed 21 July, 2009 <http://www.vcaa.vic.edu.au/>.

Vygotsky, L. 1978, *Mind in society: The development of higher psychological processes* (M. Cole, V. John-Steiner, S. Scribner, & E. Souberman, Eds. & Trans). Harvard University Press, Cambridge, MA:.