The Learning and Assessment Framework for Multiplicative Thinking (LAF)

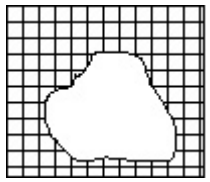
The information below describes the eight zones within the Learning and Assessment Framework for Multiplicative Thinking (LAF).

Zone	Key Characteristics	Teaching Implications	Learning Plans
Zone 1 - Primitive Modelling	Can solve simple multiplication and division problems involving relatively small whole numbers. For example, *Butterfly House* parts a and b. Tends to rely on drawing, models and count-all strategies. For example, draws and counts all pots for part a of *Packing Pots*. May use skip counting (repeated addition) for groups less than 5. For example, to find number of tables needed to seat up to 20 people in *Tables and Chairs*. Can make simple observations from data given in a task. For example, *Adventure Camp a.* Can reproduce a simple pattern. For example, *Tables*	**Teaching implications – consolidation and establishment** **Trusting the count for numbers to 10** For 6, as an example, this involves working with mental objects of 6 without having to model and/or count-all. Use flash cards to develop subitising (the ability to say how many without counting) for numbers to 5 initially and then to 10 and beyond using part-part-whole knowledge. For example: 8 is 4 and 4, or 5 and 3 more, or 2 less than 10 Practice regularly. **Simple skip counting** Use simple skip counting to determine how many are in a collection and to establish numbers up to 5 as countable objects. For example: count by twos, fives and tens, using concrete materials and a 0-99 Number Chart **Mental strategies for addition and subtraction facts to 20** Count on from larger. For example:	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 1 – Learning Plans Consolidating (PDF - 602Kb) Zone 1 – Learning Plans Introducing (PDF - 41Kb)

	and Chairs, parts a to e. Multiplicative Thinking (MT) not really apparent as there is no indication that groups are perceived as composite units, dealt with systematically, or that the number of groups can be manipulated to support a more efficient calculation.	for 2 and 7, think: 7, 8, 9. Double and near doubles. Use ten-frames and a 2-row bead-frame to show that 7 and 7 is 10 and 4 more, 14. Make-to-ten. For example: for 6 and 8, think: 8, 10, 14, scaffold using open number lines Explore and name mental strategies to solve subtraction problems such as 7 take 2, 12 take 5, and 16 take 9. **2 digit place-value** Work flexibly with ones and tens by making, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming (see Booker et al, 2004). Play the Place-Value Game (PDF - 26Kb). **Teaching implications – introduction and development** **Doubling (and halving) strategies** Use doubling and halving strategies for 2-digit numbers that do not require renaming. For example: 34 and 34, half of 46 Build to numbers that require some additional thinking. For example: to double 36, think: double 3 tens, double 6 ones, 60 and 12 ones, 72 **Extended mental strategies for addition and subtraction** Use efficient, place-value based strategies. For example: 37 and 24, think: 37, 47, 57, 60, 61	

		Scaffold thinking with open number lines. **Efficient and reliable strategies for counting large collections** Use strategies for large collections, for example: counting a collection of 50 or more by 2s, 5s or 10s Focus on how to organise the number of groups to facilitate the count. For example, by arranging the groups systematically in lines or arrays and then skip counting. **Make, name and use arrays or regions** Explain how to solve simple multiplication or sharing problems using concrete materials and skip counting. For example: 1 four, 2 fours, 3 fours … Lead up to more efficient counting strategies based on reading arrays in terms of a consistent number of rows. For example: 4 rows of anything, that is, 4 ones, 4 twos, 4 threes, 4 fours … **3 digit place-value** Work flexibly with tens and hundreds by making with MAB, naming, recording, comparing, ordering, counting forwards and backwards in place-value parts, and renaming (see Booker et al, 2004). **Strategies for unpacking and comprehending problem situations** Read, re-tell and ask questions such as ‘What is the question asking?’ or ‘What do we need to do?’. Use realistic word problems to explore different ideas for multiplication and division. For example:	

		3 rows, 7 chairs in each row, how many chairs (array)? Mandy has three times as many…as Tom…, how many … does she have (scalar idea)? 24 cards shared among 6 students, how many each (partition)? Lollipops cost 5c each, how much for 4 ('for each' idea)? **How to explain and justify** Present strategies for developing a solution orally and in writing through words and pictures. This is important for mathematical literacy.	
Zone 2 - Intuitive Modelling	Trusts the count for groups of 2 and 5, that is, can use these numbers as units for counting. For example, *Tables & Chairs* part j, *Butterfly House* part d. Counts large collections efficiently and systematically keeps track of count (for example, may order groups in arrays or as a list) but needs to 'see' all groups. For example, *Tiles, Tiles, Tiles* part a, *Butterfly House* part e. May use list and/or doubling as follows: 2 butterflies 5 drops 4 butterflies 10 drops	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Teaching implications – introduction and development** **More efficient strategies for counting groups** Introduce strategies based on a change in focus from a count of equal groups (for example, 1 three, 2 threes, 3 threes, 4 threes …) to a consistent number of groups (for example, 3 ones, 3 twos, 3 threes, 3 fours, …) which underpin the more efficient mental strategies listed below and ultimately lead to the factor-factor-product idea. **Array/region-based mental strategies for multiplication facts to 100** For example: • doubling (for 2s facts) • doubling and 1 more group (for 3s facts) • double doubles (for 4s facts)	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF: Zone 2 – Learning Plans Introducing (PDF - 574Kb)

	6 butterflies 15 drops … 12 butterflies 30 drops Can share collections into equal groups/parts. For example, *Pizza Party* parts a and b. Recognises small numbers as composite units. For example, can count equal group and skip count by twos, threes and fives Recognises multiplication is relevant, but tends not to be able to follow this through to solution. For example, *Packing Pots* part c, *Speedy Snail* part a. Can list some of the options in simple Cartesian product situations. For example, *Canteen Capers* part a. Orders 2 digit numbers. For example, partially correct ordering of times in *Swimming Sports* part a. Some evidence of multiplicative thinking as equal groups or shares are seen as entities that can be	• relate to tens (for 5s and 9s facts). For further examples, see There's More to Counting Than Meets the Eye (PDF - 72Kb). **Efficient strategies for solving problems where arrays and regions only partially observed** For example: If paint spill on a tiled floor, • How many tiles to replace? • How many altogether? • How do you know?  **Commutativity** Encourage commutativity by exploring the relationship between arrays and regions such as 3 fours and 4 threes. Play Multiplication Toss (PDF - 54Kb). **Informal division strategies** Tasks could include 'Think of Multiplication and Halving'. For example: 16 divided by 4, think: 4 'whats' are 16? 4; or half of 16 is 8, half of 8 is 4 **Extended mental strategies for multiplication**	

	counted systematically.	For example: for 3 twenty fives, think: double 25, 50, and twenty five more, 75 Use place-value based strategies such as 10 groups and 4 more groups for 14 groups. **Simple proportion problems** Involve non-numerical comparisons. For example, If Nick mixed less cordial with more water than he did yesterday, his drink would taste (a) stronger, (b) weaker (c) exactly the same, or (d) not enough information to tell. **How to recognise and describe simple relationships** Encourage recognition of patterns. For example: • 'double and add 2' from models, diagrams and tables • notice that a diagonal pattern on a 0-99 chart is a count of 11, 1 ten and 1 ones. **Language of fractions** Use practical experience with both continuous and discrete, 'real-world' fraction models. For example: • 3 quarters of the pizza • half the class. Distinguish between how many and how much. For example: in 2 thirds the numeral indicates how many, the name indicates how much **Halving partitioning strategy** Apply thinking involved to help children create their own fraction diagrams through activities such as:	

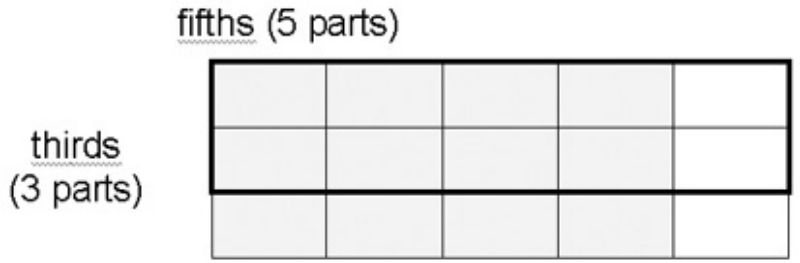
		• paper folding (kinder squares and streamers) • cutting plasticine 'cakes' and 'pizzas' • sharing collections equally (counters, cards etc). Focus on making and naming parts in the halving family (for example, 8 parts, eighths) including: • mixed fractions (for example, "2 and 3 quarters") • informal recording (for example, 3 eighths) • no symbols. **Key fraction generalisations** Use strategies that support the key fraction generalisation that equal parts are necessary and that the number of parts names the part.	
Zone 3 - Sensing	Demonstrates intuitive sense of proportion and partitioning. For example, *Butterfly House* part f (partial solution), *Missing Numbers* part b. Works with 'useful' numbers such as 2 and 5, and strategies such as doubling and halving. For example, *Packing Pots* part b, *Pizza Party* part c. May list all options in a simple Cartesian product situation but cannot explain or justify solutions. For example, *Canteen Capers* part b.	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Teaching implications – introduction and development** Introduce and develop the following strategies. **Place-value based strategies** Problems involving single-digit by two-digit multiplication are informally solved, either mentally or in writing. For example: 3 twenty-eights, think: 3 by 2 tens, 60 and 24 more, 84 **Initial recording to support place-value for multiplication facts** For more information, see:	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 3 – Learning Plans Introducing (PDF - 2.5Mb)

	Uses abbreviated methods for counting groups. Uses doubling and doubling again to find 4 groups of, or repeated halving to compare simple fractions. For example, *Pizza Party* part c. Beginning to work with larger whole numbers and patterns but tends to rely on count all methods or additive thinking to solve problems. For example, *Stained Glass Windows* parts a and b, *Tiles, Tiles, Tiles* part c.	• Booker et al, 2004 • There's More to Counting Than Meets the Eye (PDF - 72Kb). **More efficient strategies for solving number problems involving simple proportion** For example, recognising that problems involve several steps: • What do I do first? Find value for common amount • What do I do next? Determine multiplier/factor and apply • Why? **Rename number of groups** Demonstrate how to rename number of groups. For example: 6 fours, think: 5 fours and 1 more four Practice by using tasks such as Multiplication Toss (PDF - 54Kb). Re-name composite numbers in terms of equal groups. For example: 18 is 2 nines, 9 twos, 3 sixes, 6 threes **Cartesian product** For each idea use concrete materials and relatively simple problems such as: • 3 tops and 2 bottoms: how many outfits? • how many different types of pizzas, given choice of small, large, medium and 4 varieties. Discuss how to: • recognise problems of this type • keep track of the count such as draw all options, make a list or a table.	

		Tree diagrams appear to be too difficult at this zone and these are included in Zone 5. **How to interpret problem situations and solutions relevant to context** Students should ask, for example: • What operation is needed? • Why? • What does it mean in terms of original question? **Interpretation of remainders** Introduce simple, practical division problems that require the interpretation of remainders relevant to context. **Practical sharing situations** Names for simple fractional parts beyond the halving family (for example, thirds for 3 equal parts/shares, sixths for 6 equal parts etc) are introduced through practical sharing situations. These situations will also help build a sense of fractional parts, for example: • 3 sixths is the same as a half and 50% • 7 eighths is nearly 1 • “2 and 1 tenth” is close to 2. Use a range of continuous and discrete fraction models including mixed fraction models. **Thirding and fifthing partitioning strategies** Apply thinking involved to help children create their own fraction diagrams (regions) and number line representations through:	

		• paper folding (kinder squares and streamers) • cutting plasticine 'cakes' and 'pizzas' • sharing collections equally (counters, cards etc). For more information, please see Partitioning – The Missing Link in Building Fraction Knowledge and Confidence (PDF - 103Kb) Focus on making and naming parts in the thirding and fifthing families (for example, 5 parts, fifths). This includes: • mixed fractions (for example, "2 and 5 ninths") • informal recording (for example, 4 fifths), no symbols. Revisit key fraction generalisations from Zone 2. Include: • whole to part models (for example, partition to show 3 quarters) • part to whole (for example, if this is 1 third, show me the whole). Use diagrams and representations to rename related fractions. **Extend partitioning strategies** Construct number line representations by extending partitioning strategies. Use multiple fraction representations. **Key fraction generalisations** Use strategies that support the key fraction generalisation that greater the number of parts, the smaller they are, and conversely, the fewer the parts the larger they are.	
Zone 4 - Strategy Exploring	Solves more familiar multiplication and division problems involving two-digit numbers. For example:	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone.	The following learning plans have been developed for this zone, to support targeted teaching after students

	• *Butterfly House* parts c and d • *Packing Pots* part c • *Speedy Snail* part a. Tends to rely on additive thinking, drawings and/or informal strategies to tackle problems involving larger numbers and/or decimals and less familiar situations. For example: • *Packing Pots* part d • *Filling the Buses* parts a and b • *Tables & Chairs* parts g and h • *Butterfly House* parts h and g • *Speedy Snail* part c • *Computer Game* part a • *Stained Glass Windows* parts a and b. Tends not to explain their thinking or indicate working. Able to partition given number or quantity into equal parts and describe part formally (for example *Pizza*	**Teaching implications – introduction and development** **Multiplying and dividing larger whole numbers** Introduce more efficient strategies for multiplying and dividing larger whole numbers. These strategies work independently of models and are based on: • doubling • renaming the number of groups • factors • place-value • known addition facts. For example: for dividing 564 by 8, think: 8 what's are 560? 8 by 7 tens or 70, so 70 and 4 remainder For example: for 3908 divided by 10, rename as: 390 tens and 8 ones, so 390.8 **Tenths as a new place-value part** Introduce by making or representing, naming and recording ones and tenths (see Booker et al, 2004) Consolidate by: • comparing • ordering • sequencing counting forwards and backwards in ones and/or tenths • renaming. **Halving, thirding, fifthing strategies**	have been assessed and located on the LAF. Zone 4 – Learning Plans Introducing (PDF - 57Kb)

	Party parts a and b). Can locate familiar fractions (for example, *Missing Numbers* part a). Beginning to work with simple proportion. For example, can make a start, represent problem, but unable to complete successfully or justify their thinking (for example, *How Far* part a, *School Fair* parts a and b).	Show students how to partition continuous quantities more generally using the halving, thirding, fifthing strategies. For more information, see Siemon (2004) and Partitioning – The Missing Link in building Fraction Knowledge and Confidence (PDF - 103Kb) For example, recognise that: • sixths can be made by halving and thirding (or vice versa) • tenths can be made by fifthing and halving. Use this knowledge to construct fraction diagrams (for example, region models) and representations (for example, number line) for common fractions and decimals including mixed numbers. **Renaming simple unlike fractions** Introduce informal, partition-based strategies for renaming simple unlike fractions. For example, recognise that thirds and fifths can be renamed by thirding and then fifthing (or vice versa) on a common diagram. An example of this type of diagram is shown below.  Link to region model of multiplication (in this case 3 fives, or 3 parts by 5 parts) to recognise that thirds by fifths are fifteenths, so 2 thirds can be renamed as 10 fifths and 4 fifths can be renamed as 12 fifteenths. Use strategies to informally add and subtract like and related fractions.	

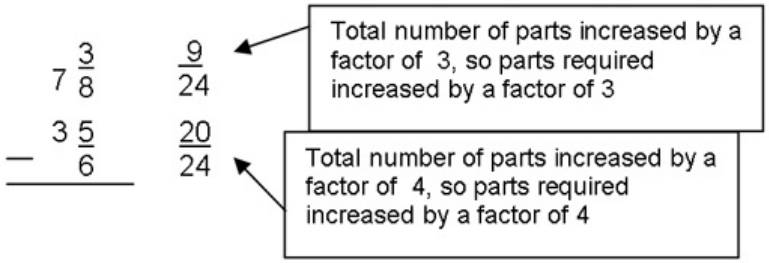
		Key fraction generalisations Use strategies that support the key fraction generalisations that recognise that: • equal parts are necessary • the total number of parts names the part • as the total number of parts increases they get smaller. This idea is crucial for the later development of more formal strategies for renaming fractions (see Zone 5) which relate the initial number of parts (3, thirds) to the final number of parts (15, fifteenths). In terms of factors, the number of parts has been increased by a factor of 5. **Metacognitive strategies** Introduce strategies to support: • problem comprehension • problem representation • strategy monitoring and checking • interpretation of outcomes relevant to context. For more information, see the Siemon and Booker (1990) paper on Teaching and Learning For, About and Through Problem Solving (PDF - 462Kb). **Simple proportion problems** Introduce techniques for dealing with these situations with simple proportion problems. For example: • find for 1 then multiply or divide as appropriate • using scale diagrams • interpreting distances from maps.	

Zone 5 - Strategy Refining	Systematically solves simple proportion and array problems, suggesting Multiplicative Thinking. For example: • Butterfly House part e • Packing Pots part a • *How Far* part a. May use additive thinking to solve simple proportion problems involving fractions. For example: • *School Fair* part a • *Speedy Snail* part b. Able to solve simple, two-step problems using a recognised rule or relationship (for example, *Fencing the Freeway* Part A). However, finds this difficult for larger numbers. For example: • *Tables & Chairs* parts k and l • *Tiles, Tiles, Tiles* part c • *Stained Glass Windows* part c. Able to order numbers	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Place-value** Introduce ideas and strategies for 5 digits and beyond if not already developed and decimal fractions to hundredths including renaming. **Multiplying and dividing by multiples of ten** Provide flexible, meaningful and efficient strategies for multiplying and dividing by multiples of ten. For example: 2.13 by 10, think: 21 ones and 3 tenths, 21.3. **The area idea and an expanded range of Cartesian product problems** Support multi-digit multiplication and formal recording (see Booker et al, 2004) through the area idea. Introduce more efficient strategies for representing and solving an expanded range of Cartesian product problems involving three or more variables and tree diagram representations. **Formal terminology** Introduce terminology associated with multiplication and division such as: • factor • product • divisor • multiplier • raised to the power of ….	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 5 – Learning Plans Introducing (PDF - 70Kb)

	involving tens, ones, tenths and hundredths in supportive context. For example, *Swimming Sports* part a. Able to determine all options in Cartesian product situations involving relatively small numbers, but tends to do this additively. For example: • *Canteen Capers* part a • *Butterfly House* parts l and i. Beginning to work with decimal numbers and percent but unable to apply efficiently to solve problems. For example, *Swimming Sports* parts a and b, *Computer Game* part b. Some evidence that multiplicative thinking being used to support partitioning. For example, *Missing Numbers* part b. Beginning to approach a broader range of multiplicative situations more systematically.	Play the Factor Cross (PDF - 57Kb) game. Use calculators to explore what happens with repeated factors. For example: • 4 × 4 × 4 × 4 … • factors less than 1 • negative factors. **Renaming an expanded range of unrelated fractions** Introduce informal, partition-based strategies for renaming an expanded range of unrelated fractions as a precursor to developing an efficient, more formal strategy for generating equivalent fractions (see below). For example: explore using paper folding, diagrams and line models how sixths and eighths could be renamed as forty-eighths but they can also be renamed as twenty-fourths because both are factors of 24. **Generalisation for renaming fractions** If the number of equal parts (represented by the denominator) increases or decreases by a certain factor then the number of parts required (indicated by the numerator) increases or decreases by the same factor. For example: the total number of parts increased by a factor of 3, so parts required increased by a factor of 3. **Addition and subtraction of unlike fractions** Introduce written solution strategies for the addition and subtraction of unlike fractions.	

For example:

think of a diagram showing sixths by eighths … forty-eighths… Is this the simplest? No, twenty-fourths will do, rename fractions by inspection



9 twenty-fourths can't take 20 twenty-fourths, trade 1 one for 24 twenty-fourths to get 6 and 33 twenty-fourths. Subtraction is then relatively straightforward

Link between multiplication and division and fractions

Explore the link between multiplication and division and fractions including decimals. For example:

3 pizzas shared among 4, 3 divided by 4 is 0.75 etc.

These links will help with understanding the fraction as operator idea. For example:

- ¾ of 120
- 75% of $48
- 250% of 458,239.

Use the Multiple Patterns (PDF - 30Kb) task. Establish benchmark equivalences. For example:

1 third =33 1/3 %.

Metacognitive strategies

		Introduce strategies to support: • problem comprehension • strategy monitoring and checking • interpretation of outcomes relevant to context. For more information, see Siemon and Booker (1990) paper on Teaching and Learning For, About and Through Problem Solving (PDF - 462Kb).	
Zone 6 - Strategy Extending	Can work with Cartesian Product idea to systematically list or determine the number of options. For example: • *Canteen Capers* part b • *Butterfly House* parts i and h. Can solve a broader range of multiplication and division problems involving two digit numbers, patterns and/or proportion. For example: • *Tables & Chairs* part h • *Butterfly House* part f • *Stained Glass Windows* parts b and c	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Hundredths as a new place-value part** Introduce strategies for hundredths as a new place-value part by making/representing, naming and recording ones, tenths and hundredths (see Booker et al, 2004). Consolidate by comparing, ordering, sequencing counting forwards and backwards in place-value parts and renaming. **Explain and justify solution strategies** For problems involving multiplication and division, introduce strategies that show students how to explain and justify solutions. This is particularly important in relation to interpreting decimal remainders appropriate to context. For example: How many buses will be needed to take 594 students and teachers to the school Speech night, assuming each bus hold 45 passengers and everyone must wear a seatbelt?"	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 6 – Learning Plans Introducing (PDF - 81Kb)

	• *Computer Game* parts a and b. However, may not be able to explain or justify solution strategy. For example: • *Fencing the Freeway* parts b and d • *Swimming Sports* part b • *How Far* part b • *Speedy Snail* part b. Able to rename and compare fractions in the halving family (for example, *Pizza Party* part c) and use partitioning strategies to locate simple fractions (for example, *Missing Numbers* part a). Developing sense of proportion (for example, sees relevance of proportion in *Adventure Camp* part a *and Tiles, Tiles, Tiles* part b), but unable to explain or justify thinking. Developing a degree of comfort with working mentally with multiplication and division facts.	For more information, see the multiplication workshop From Additive to Multiplicative Thinking – The Big Challenge of the Middle Years. **Proportion problems** Introduce more efficient and systematic processes that can be generalised for dealing with proportion problems. For example: • use of the 'for each' idea • formal recording • the use of fractions • percent to justify claims. For example: Jane scored 14 goals from 20 attempts. Emma scored 18 goals from 25 attempts. Which girl should be selected for the school basketball team and why? 6 girls share 4 pizzas equally. 8 boys share 6 pizzas equally. Who had more pizza, the girls or the boys? 35 feral cats were found in a 146 hectare nature reserve. 27 feral cats were found in a 103 hectare reserve. Which reserve had the biggest feral cat problem? Orange juice is sold in different sized containers: 5L for $14, 2 L for $5, and 500mL for $1.35. Which represents the best value for money? **Multiplication and division involving larger numbers based on sound place-value ideas** Introduce more efficient strategies and formal processes for working with multiplication and division involving larger numbers based on sound place-	

		value ideas. For example: 3486 × 21 can be estimated by thinking about 35 hundreds by 2 tens, 70 thousands, and 1 more group of 35 hundred, ie, 73,500 or 3486 × 21 can be calculated by using factors of 21, ie, 3486 × 3 × 7 Two digit multiplication can be used to support the multiplication of ones and tenths by ones and tenths. For example: for 2.3 by 5.7, rename as tenths and compute as 23 tenths by 57 tenths, which gives 1311 hundredths hence 13.11 Consider a broader range of problems and applications. For example, Average gate takings per day over the World Cricket cup Series. Matt rode around the park 8 times. The odometer on his bike indicated that he had ridden a total of 15 km. How far was it around the park? After 11 training sessions, Kate's average time for 100 metres butterfly was 61.3 seconds. In her next 2 trials, Kate clocked 61.21 and 60.87 seconds. What was her new average time? **Integers** Use real-world examples to explain integers, such as: • heights above and below sea-zone • temperatures above and below zero • simple addition and difference calculations. **Notion of variable and pattern recognition and description**	

		Introduce strategies for understanding the notion of variable and how to recognise and formally describe patterns involving all four operations. Use the Max's Matchsticks (PDF - 28Kb) task to explore how patterns may be viewed differently leading to different ways of counting and forms of representation.	
Zone 7 - Connecting	Able to solve and explain one-step problems involving multiplication and division with whole numbers using informal strategies and/or formal recording. For example: • *Filling the Buses* part a • *Fencing the Freeway* part d • *Packing Pots* part d. Can solve and explain solutions to problems involving simple patterns, percent and proportion. For example: • *Fencing the Freeway* part c • *Swimming Sports* part b • *Butterfly House* part g • *Tables & Chairs*	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Comparing, ordering, sequencing, counting forwards and backwards in place-value parts** Introduce strategies for: • comparing, ordering, sequencing, counting forwards and backwards in place-value parts • renaming large whole numbers, common fractions, decimals and integers. For example: a 3 to 4 metre length of rope, appropriately labelled number cards and pegs could be used to sequence numbers from 100 to 1,000,000, from -3 to +3, from 2 to 5 and so on The metaphor of a magnifying glass can be used to locate numbers involving hundredths or thousandths on a number line as a result of successive *tenthing*. For more information, see Siemon (2004) Partitioning – The Missing Link in Building Fraction Knowledge and Confidence (PDF - 103Kb).	The following learning plans have been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 7 – Learning Plans Introducing (PDF - 59Kb)

	parts g and l • *Speedy Snai*l part c • *Tiles, Tiles, Tiles* parts b and c • *School Fair* part a • *Stained Glass Windows* part a • *Computer Game* part b • *How Far* part b. May not be able to show working and/or explain strategies for situations involving larger numbers. For example: • *Tables & Chairs* parts m and k • *Tiles, Tiles, Tiles* part c. May not be able to show working and/or explain strategies for less familiar problems. For example: • *Adventure Camp* part b • *School Fair* part b • *How Far* part c. Locates fractions using efficient partitioning strategies. For example,	**Inverse and identity relations** Build an appreciation of inverse and identity relations through strategies such as recognising which number when added leaves the original number unchanged (zero) and how inverses are determined in relation to this. For example: the inverse of 8 is -8 as -8 + 8 = 0 and 8 + -8 = 0 In a similar fashion, recognise that 1 is the corresponding number for multiplication, where the inverse of a number is defined as its reciprocal. For example: the inverse of 8 is 1/8 **Index notation** Introduce strategies for representing multiplication of repeated factors. For example: $5 \times 5 \times 5 \times 5 \times 5 \times 5 = 5^6$ **Place-value** A more generalised understanding of place-value and the structure of the number system in terms of exponentiation should be developed. For example: 10^{-3}, 10^{-2}, 10^{-1}, 10^0, 10^1, 10^2 10^3 … **Multiplication and division in a broader range of situations** Strategies to recognise and apply multiplication and division in a broader range of situations can include: • ratio • proportion • unfamiliar, multiple-step problems.	

	Missing Numbers part a. Beginning to make connections between problems and solution strategies and understand how to communicate this mathematically.	See the Orange Juice (PDF - 25Kb) task for an example. **Formal recognition and description of number patterns** Develop an awareness of how to recognise and describe number patterns more formally. For example: • triangular numbers • square numbers • growth patterns. See the 'Garden Beds' task from Maths 300 and Super Market Packer (PDF - 30Kb) task for examples. **Notation to support general arithmetic** Notation for simple algebra or general arithmetic can be used, for example, to recognise and understand the meaning of expressions such as: $x+4$, $3x$, $5x^2$, or $^{x-1}/_{3}$ **Ratio as the comparison of any two quantities** For example: the comparison of the number of feral cats to the size of the national park Recognise that ratios can be used to compare measures of the same type. For example: the number of feral cats compared to the number of feral dogs Recognise that within this, two types of comparison are possible. For instance, one can compare: • the parts to the parts (for example, cats to dogs), or • the parts to the whole (for example, cats to the total number of cats	

		and dogs). Ratios can be also used to compare measures of different types. In other words, they are generally described as a rate. For example: the number of feral cats per square kilometre Ratios are not always rational numbers. For example: the ratio of the circumference of a circle to its diameter **Proportion problems involving larger numbers and/or fractions** Introduce strategies for recognising and representing proportion problems involving larger numbers and/or fractions. For example: • problems involving scale such as map calculations, increasing/reducing ingredients in a recipe • simple problems involving derived measures such as volume, density, speed and chance.	
Zone 8 - Reflective Knowing	Can use appropriate representations, language and symbols to solve and justify a wide range of problems involving unfamiliar multiplicative situations including fractions and decimals. For example: • *Adventure Camp* part b • *Speedy Snail* part b. Can justify partitioning. For example, *Missing Numbers* part b.	**Teaching implications – consolidation and establishment** Consolidate or establish the ideas and strategies introduced or developed in the previous zone. **Teaching implications – introduction and development** Expand the range of multiplicative situations Introduce a broader range of multiplicative situations. For example, problems involving: • the calculation of area or volume • derived measures and rates • variation • complex proportion.	The following learning plan has been developed for this zone, to support targeted teaching after students have been assessed and located on the LAF. Zone 8 – Learning Plans Introducing (PDF - 178Kb)

	Can use and formally describe patterns in terms of general rules. For example, *Tables and Chairs*, parts m and k. Beginning to work more systematically with complex, open-ended problems. For example: • *School Fair* part b • *Computer Game* part c.	Multiple step problems involving large whole numbers, decimals and fractions should also be introduced. For example: Find the volume of a cylinder 4 cm in diameter and 9 cm long. Find the surface area of a compound shape. Foreign currency calculations. Determine the amount of water lost to evaporation from the Hume Weir during the summer. **Simplifying expressions** Strategies for simplifying expressions include: • adding and subtracting like terms • justifying and explaining the use of cancellation techniques for division through the use of common factors. For example: $^{42a}/_{7} = 6a$ because $^{42a}/_{7} = {}^{7 \times 6a}/_{7}$ and $^{7}/_{7} = 1$ **Algebraic reasoning and representation strategies** To solve problems involving multiplicative relationships, students will require algebraic reasoning and representation strategies. For example: If 2 T-shirts and 2 drinks cost $44 and 1 T-shirt and 3 drinks cost $30, what is the price of each? 5 locker keys are returned at random to the students who own them. What is the probability that each student will receive the key that opens their locker? A scientist has a collection of beetles and spiders. The sensor in the floor of the enclosure indicated that there were 174 legs and the infra-	

		red image indicated that there were 26 bodies altogether. How many were beetles and how many were spiders? 365 is an extraordinary number. It is the sum of 3 consecutive square numbers and also the sum of the next 2 consecutive square numbers. Find the numbers referred to. **Numbers and operations expressed in exponent form** Introduce strategies for working with numbers and operations expressed in exponent form. For example: Why $2^3 \times 2^6 = 2^9$. **More abstract problem solving situations** These situations require an appreciation of problem solving as a process and an awareness of the value of recognising problem type. Student will be required to develop a greater range of strategies and representations, including the manipulation of symbols. For example: • tables • symbolic expressions • rule generation • testing.	